

# The Kosterlitz-Thouless transition in Josephson junction arrays

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Objektyp: **Article**

Zeitschrift: **Helvetica Physica Acta**

Band (Jahr): **60 (1987)**

Heft 1

PDF erstellt am: **25.09.2024**

Persistenter Link: <https://doi.org/10.5169/seals-115848>

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THE KOSTERLITZ-THOULESS TRANSITION

IN JOSEPHSON JUNCTION ARRAYS

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Abstract : A study of the ac response of square two-dimensional arrays of proximity effect Josephson junctions as a function of temperature, frequency and applied transverse magnetic field is presented. As a function of magnetic field, both the real and the imaginary part of the array's conductance exhibit clear structures at fields corresponding to rational numbers of flux quanta per unit cell of the array. At an integer number of flux quanta per unit cell, the temperature and frequency dependence of the conductance show that the superconducting to normal transition of the array can be described by the Kosterlitz-Thouless theory and by its extension to finite frequencies.

I. Introduction

The Kosterlitz-Thouless (KT) theory of phase transitions [1,2], applied to a two-dimensional (2D) superconductor [3], is based on a description of the superconductor in terms of fluctuations in the phase of its order parameter. At low temperatures the relevant phase fluctuations are slowly varying functions of position. As the temperature increases, thermal fluctuations in the phase result in topological excitations in the form of bound pairs of vortices of opposite circulation (vortex-antivortex pairs). The transition to the normal state is triggered by the unbinding of these vortex pairs, i.e. the creation of free vortices at a critical temperature  $T_c$ . The most obvious consequence of such a transition is the appearance of a dc resistance with a characteristic temperature dependence [4-8] above  $T_c$ . Another mani-

festation of the KT transition is the crossover in behavior one observes in the sample's current-voltage characteristics in the vicinity of  $T_c$  [7,9]. In the transition region the dynamics of vortex excitations has unique features which can be seen in experiments probing the response of 2D systems exposed to ac driving fields [10,11]. In particular, measurements of the complex dynamic sheet conductance allow to verify two important theoretical predictions : the temperature dependence of the complex dielectric constant, and the anomalous temperature dependence of the free vortex correlation length above  $T_c$ . In this paper we report measurements of the complex conductance of proximity-coupled 2D arrays of Josephson junctions [12]. In section II we introduce the basic concepts of the KT transition. Section III is dedicated to array physics. We discuss the relationship between arrays and the XY model, some vortex dynamics and magnetic field effects. Our experimental procedures and results are presented in section IV. The results are analyzed within the theoretical framework of sections II and III. Section V contains our conclusions.

## II. Basic features of the KT transition

Consider the classical XY model : a regular lattice of spins on the XY-plane. The spins are subject to nearest neighbor interactions and are free to rotate about an axis perpendicular to the XY-plane. The  $i$ th spin is described by  $\phi_i$ , the angle it makes with a fixed direction in the XY-plane. The Hamiltonian of the system is given by

$$H = -J \sum_{\langle ij \rangle} \cos (\phi_j - \phi_i) \quad , \quad (1)$$

where the sum is over all pairs of spins and  $J$  (positive) is the coupling energy. Except at zero temperature, where all the spins are aligned, there is no conventional long-range order in the spin system and thus a vanishing spontaneous magnetization [13]. In fact, at any non-zero temperature spin waves (i.e. long wavelength excitations of the spins) lead, at large enough distances, to uncorrelated spins. On the other hand, at sufficiently low temperatures, we are not in the presence of liquid-like short range order. Wegner

[14] found that the spin-spin correlation function decays algebraically, with a temperature dependent exponent  $\eta(T) = k_B T / 2\pi J$  :

$$\langle \cos(\phi_j - \phi_i) \rangle \propto r_{ij}^{-\eta(T)}, \quad (2)$$

where  $r_{ij}$  is the distance between the lattice sites  $i$  and  $j$ . Intuitively, one expects some type of phase transition from a low temperature phase characterized by the quasi long-range order described by (2) to a high temperature liquid-like phase characterized by an exponentially decaying correlation function. This phase transition was investigated by Kosterlitz and Thouless in 1972 [1] by taking into account the effects of thermally excited vortices. A vortex (antivortex) is defined as a configuration of the  $\phi_i$  such that the sum of the phase changes along a closed path is  $2\pi$  ( $-2\pi$ ). The energy  $E_V$  of an isolated vortex can be computed from (1) and is given by

$$E_V = \pi J \log L/a, \quad (3)$$

where  $L$  is the system size and  $a$  the lattice parameter. For the interaction energy  $E_p$  of a vortex-antivortex pair with cores separated by a distance  $r$ , one finds

$$E_p = 2\pi J \log r/a. \quad (4)$$

Notice that in general  $E_p \ll E_V$ , the thermal excitation probability is therefore larger for bound pairs than for single vortices. A rough estimate [1] of the KT transition temperature  $T_C$  can be obtained by computing the free energy  $F = E_V - TS$  of a single vortex excitation and by requiring that at  $T = T_C$  there is a spontaneous nucleation of free vortices, i.e.  $F(T_C) = 0$ . This leads to  $k_B T_C \approx \pi J / 2$ . The correct value for  $T_C$  is obtained by taking into account the presence of bound pairs and their interaction. The interaction between the constituents of a pair of size  $r_0$  is reduced ("renormalized") by an amount  $\epsilon(r_0)$ , due to the presence of pairs of size  $r < r_0$ . The calculation of renormalized quantities, which is the main scope of the theory, is based on the KT scaling equations [1]. The physical interpretation of  $\epsilon(r)$  becomes obvious by making an analogy with the 2D Coulomb

gas, where electric charges also interact logarithmically [1,6,15]. In the 2D Coulomb gas analogue,  $\epsilon(r)$  describes the scale dependent screening properties of a dielectric medium consisting of electric dipoles (corresponding to the vortex-antivortex pairs in the XY model) of different size.

Thus we have the following picture for the KT-transition : at low temperatures there are thermal excitations in the form of spin waves and bound vortex-antivortex pairs. At a transition temperature  $T_c$  given by

$$2k_B T_c = \pi J_R, \quad (5)$$

where  $J_R = J/\epsilon_c$  is the renormalized coupling energy and  $\epsilon_c$  the dielectric constant at infinite scale  $\epsilon(\infty)$ , pairs of largest separation ( $r \rightarrow \infty$ ) unbind. The resulting free vortex excitations (corresponding to free electric charges in the 2D Coulomb gas analogue) destroy the quasi long-range order existing below  $T_c$ . Above  $T_c$  one is dealing with a liquid-like phase characterized by a correlation function of the form

$$\langle \cos(\phi_j - \phi_i) \rangle \propto e^{-r_{ij}/\xi_+(T)}. \quad (6)$$

The correlation length  $\xi_+(T)$  has an unusual temperature dependence reflecting the peculiar nature of the KT transition. It is given by [2]

$$\xi_+(T) \approx a e^{b[T - T_c]^{-1/2}}, \quad (7)$$

where  $b$  is a nonuniversal constant of order unity. Physically,  $\xi_+(T)$  is a measure of the average separation of free vortices. The free vortex areal density  $n_f$  is therefore approximately given by  $n_f \approx \xi_+^{-2}(T)$ .

### III. 2D Arrays

#### a) Connection with the XY model

Large two-dimensional arrays of superconducting weak links constitute a very appealing physical realization of the XY model. With modern photolitho-

graphic techniques it is possible to fabricate large regular lattices of superconducting (S) islands. The individual islands are Josephson coupled through an insulator (I) forming arrays of SIS junctions [16] or through a normal metal (N), forming proximity effect SNS arrays [17-19]. In arrays where the geometrical and physical properties of the individual junctions are sufficiently uniform, fluctuations in the magnitude of the superconducting order parameter are largely suppressed well below the BCS transition of the individual islands. On the other hand, 2D fluctuations in the phase of the order parameter are still important. The phase difference ( $\phi_j - \phi_i$ ) between two sites and its time evolution are governed by the Josephson equations. The supercurrent flowing between islands  $i$  and  $j$  is given by

$$i_s = i_c(T) \sin(\phi_j - \phi_i) \quad ,$$

where  $i_c(T)$  is the critical current of the junction. The voltage across the barrier is

$$V = \frac{\hbar}{2e} \frac{\partial}{\partial t} (\phi_j - \phi_i) \quad .$$

With these two expressions the interaction energy  $E_{ij} = \int i_s V dt$  of the islands  $i$  and  $j$  becomes :

$$E_{ij} = \frac{\hbar i_c(T)}{2e} [1 - \cos(\phi_j - \phi_i)] \quad . \quad (8)$$

Summing over all pairs  $\langle ij \rangle$  we obtain the same Hamiltonian as in (1), with a coupling energy

$$J = \frac{\hbar i_c(T)}{2e} \quad . \quad (9)$$

The phase of the superconducting order parameter corresponds to the spin-angle variable of the XY model. With (5), the universal KT prediction for the transition temperature becomes

$$\frac{i_c(T_c)}{\epsilon_c T_c} = \frac{8ek_B}{h} \approx 27 \text{ nA/K} \quad . \quad (10)$$

Since the coupling energy in (9) is temperature dependent, the statistical

mechanics of the system is conveniently described in terms of a dimensionless temperature parameter  $\tilde{T} = k_B T / J = 2ek_B T / (\hbar i_c(T))$  [20]. In particular it is  $\tilde{T}$  and not  $T$ , which enters expression (7) for  $\xi_+$ .

There is one important limitation to the isomorphism between the XY model and 2D arrays. In 2D arrays, as in all 2D superconductors, the vortex-antivortex interaction no longer depends logarithmically on the separation distance at distances larger than the effective penetration depth  $\Lambda$  [21]. It turns out, however, that at  $T_c$ ,  $\Delta T_c \approx 2\text{cmK}$  [20]; in the interesting temperature region  $\Lambda$  is therefore a macroscopic length scale.

#### b) Vortex dynamics in arrays

The dynamical properties of the KT transition were studied by Ambegaokar et al [22,23]. An important result of their model is that the vortex response to an applied field of angular frequency  $\omega$  is controlled by a frequency dependent length  $r_\omega = (14D/\omega)^{1/2}$ , where  $D$  is the vortex diffusion constant. Bound vortex pairs of size larger than  $r_\omega$  do not respond to the applied field, whereas the response of the smaller pairs ( $r < r_\omega$ ) is described by a complex dielectric constant which is derived from the static KT dielectric constant  $\epsilon(r)$  in the following way [22,23] :

$$\begin{aligned} \text{Re } \epsilon(\omega) &= \epsilon(r_\omega) \\ \text{Im } \epsilon(\omega) &= \frac{\pi}{4} \left( r \frac{d\epsilon}{dr} \right) \Big|_{r=r_\omega} \end{aligned} \quad (11)$$

According to the physical interpretation of  $\xi_+(\tilde{T})$ , at finite frequencies the vortex unbinding transition will be seen at a temperature  $T_\omega$  such that

$$r_\omega = \xi_+(\tilde{T}_\omega) \quad . \quad (12)$$

By making use of the 2D Coulomb gas analogue, the contribution,  $\epsilon_f$ , of the free vortex charges to the dielectric constant above  $T_c$  can be written in the form

$$\epsilon_f = i \frac{4\pi \sigma_V}{\omega} \quad , \quad (13)$$

where  $\sigma_v = (e/h)r_n i_c(T) [a^2/\xi_+^2(T)]$  is the free vortex conductivity [12], proportional to  $n_f \approx \xi_+^{-2}(T)$ , and  $r_n$  is the resistance of an individual junction.

In the experiments reported below (section IV), the physical quantity of interest is the complex sheet impedance,  $Z_{\square}(\omega, T)$  of the array. It is related to the complex vortex dielectric constant  $\epsilon(\omega, T)$  by

$$Z_{\square}(\omega, T) = i\omega L_{K_{\square}} \epsilon(\omega, T) \quad (14)$$

where  $L_{K_{\square}} = \hbar/(2ei_c(T))$  is the sheet kinetic inductance of the array and  $\epsilon(\omega, T) = \epsilon'(\omega, T) + i\epsilon''(\omega, T)$  with, according to Eqs. (11) and (12),  $\epsilon'(\omega, T) = \epsilon(r_{\omega})$  and  $\epsilon''(\omega, T) = \pi/4 [r_{\omega} (d\epsilon/dr)|_{r=r_{\omega}}] + 4\pi \sigma_v/\omega$ .

### c) Magnetic field effects

A magnetic field  $\vec{B}$ , perpendicular to the plane of the array, introduces vortices with a tendency to form a regular 2D lattice, the lattice parameter being controlled by the magnitude of  $\vec{B}$ . The interaction of the field-induced vortices with the periodic pinning potential provided by the array leads to commensurate (C) and incommensurate (I) vortex phases. The array can now be described by a uniformly frustrated lattice spin model [24-27], with a Hamiltonian

$$H = -J \sum_{\langle ij \rangle} \cos(\phi_j - \phi_i - A_{ij}) \quad (15)$$

where the argument of the cosine is the gauge-invariant phase difference between islands  $i$  and  $j$ ,  $\phi_0 A_{ij}/(2\pi)$  is the line integral of the vector potential  $\vec{A}$  from site  $i$  to site  $j$  and  $\phi_0$  is the superconducting flux quantum. The  $A_{ij}$ 's satisfy the condition  $\oint A_{ij} = 2\pi f$ , where the sum is over all the links in an elementary cell and  $f$ , the frustration parameter, represents the magnetic flux per elementary cell in units of  $\phi_0$ :  $f = Ba^2/\phi_0$ . If the magnetic field is uniform,  $f$  is a constant over the entire array. Quite clearly, the Hamiltonian (15) is periodic in  $f$  with period 1. Furthermore, as  $f$  changes, the energy of the system goes through a series of local minima, corresponding to rational values of  $f$  (C-vortex phases). As a consequence, the transition temperature and the critical currents show a complex periodic dependence on  $f$  [24, 27-29].



#### IV. Experimental results and discussion

Arrays consisting of  $N \times N$  Pb/Cu proximity-effect junctions, with  $N \approx 10^3$ , were fabricated on sapphire substrates using standard evaporation techniques, photolithography and sputter-etching. Fig. 1 shows a scanning electron micrograph of a typical array. The square lead islands, 200 nm thick, form a square lattice on a 200 nm thick copper film. The lattice parameter  $a$  is 8  $\mu\text{m}$ , the distance  $L$  between the squares is 1.7  $\mu\text{m}$ . Fig. 2 shows the resistance of an array as a function of temperature. There are two distinct transitions, as observed by other groups [17-19]: the proximity-effect reduced BCS transition of the lead islands at 6.8 K and a transition to zero resistance at  $T_c \approx 3.9$  K. In the temperature region  $3.9 \text{ K} < T < 6.8 \text{ K}$  the coherence length in the copper increases with decreasing temperature leading to an increase of the effective size of the superconducting islands and thereby a decrease in resistance [17].

The complex sheet conductance  $G_{\square} = Z_{\square}^{-1}$  of the arrays was measured using a variation of the mutual inductance technique devised by Fiory and Hebard [11,30]. Two coaxial cylindrical coils consisting of an external drive coil of diameter 4 mm and an internal astatic pair of receive coils, 2 mm in diameter, were immersed in stycast. After appropriate machining, the coil assembly was positioned directly on the sample, the distance between the sample and the first winding of the detection coil being of the order of 10  $\mu\text{m}$ . An

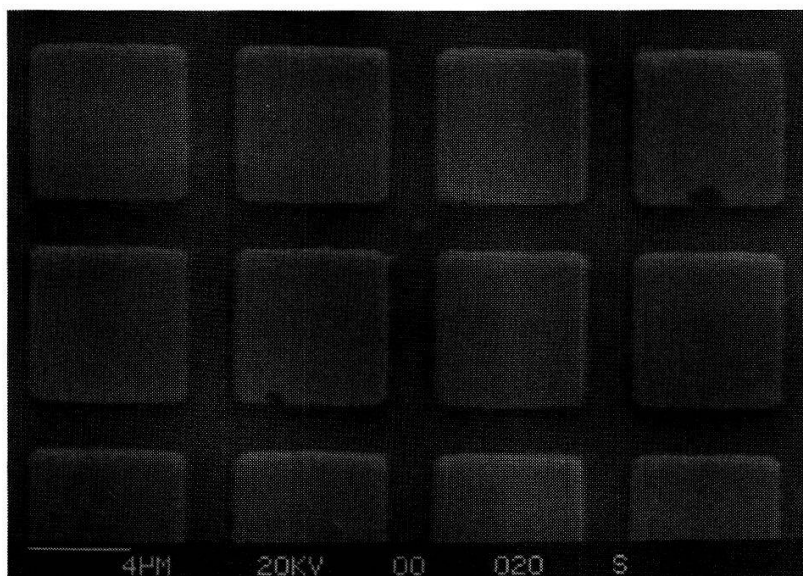


Figure 1. Scanning electron micrograph of an array. Lighter colored squares are the Pb islands on the top of the darker Cu film.

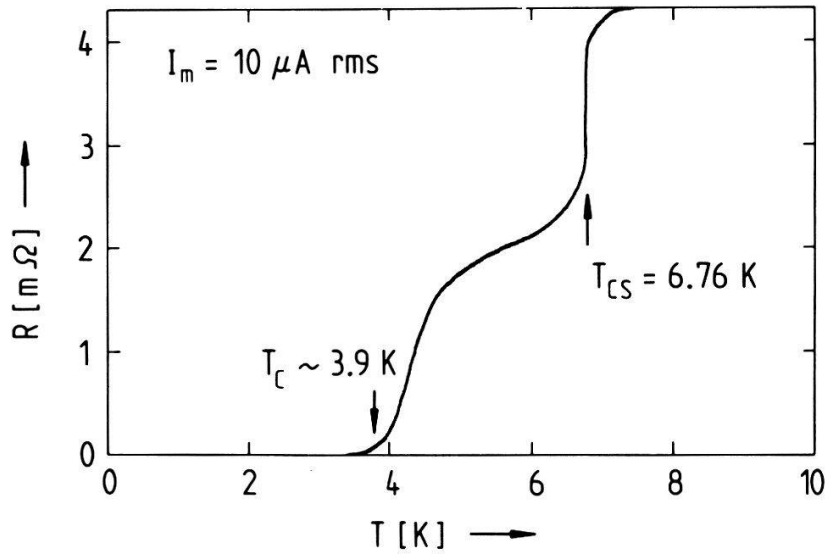


Figure 2. Dc resistance vs temperature of a 2D array of Josephson junctions.

$T_{CS}$  : BCS-transitions of the Pb islands.  $T_C$  : Vortex unbinding transition.

ac current of amplitude  $I_{D\omega}$  and angular frequency  $\omega$  flowing through the drive coil will induce screening currents in the array proportional to  $I_{D\omega}$  and  $\omega$  and, in the weak screening limit [30], also to  $G_{\square}$ , the complex sheet conductance of the array. These currents will, in turn, induce a voltage  $\delta V_{\omega} \propto \omega^2 I_{D\omega} G_{\square}$  at the receive coil which can be phase-sensitively detected. Using Eq. (14) for  $G_{\square} = Z_{\square}^{-1}$ , in the weak screening limit appropriate to discuss our experiments in the transition region, the signal voltage can be written as :

$$\delta V_{\omega}(T) = iC\omega I_{D\omega} \frac{i_c(T)}{\varepsilon(\omega, T)}, \quad (16)$$

where  $C$  is a constant depending on the sample-coil geometrical configuration, whose numerical value was estimated to be  $\sim 0.74 \text{ Vs/A}^2$ . At low temperatures ( $T \ll T_C$ ) the weak screening condition is no longer satisfied and Eq.(16) must be modified to include the geometrical inductance of the sample.

Since  $i_c(T)$  is a monotonically decreasing function of temperature, any unusual behaviour of the signal voltage  $\delta V_{\omega}$  in the transition region will be determined by  $\varepsilon^{-1}(\omega, T)$ . Calculations of  $\varepsilon^{-1}(\omega, T)$  based on Eqs. (11)

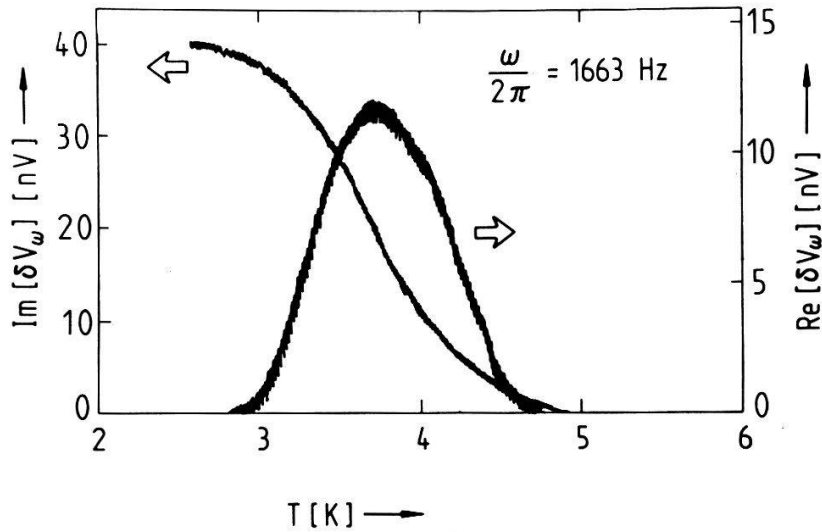


Figure 3. Temperature dependence of the receive coil signal  $\delta V_\omega$  proportional to the array's sheet conductance at a frequency of 1663 Hz.

and (13) show that a peak in  $\text{Im}(\epsilon^{-1})$  and a roll-off in  $\text{Re}(\epsilon^{-1})$  are expected in the neighborhood of  $T_c$ . This is quite clearly demonstrated by the measurements shown in Fig. 3 which exhibit a peak in dissipation [ $\text{Re}(\delta V_\omega)$ ] and a drop in superfluid density [ $\text{Im}(\delta V_\omega)$ ]. Structures similar to those shown in Fig. 3 were observed in uniform 2D superconductors [11] and in superfluid helium films [10].

The evolution of the signals with increasing frequency is shown in Fig. 4, where the signal voltage is normalized with respect to the angular frequency of the driving current. Notice that, with increasing frequency, the structures in both  $\text{Re}(\delta V_\omega)$  and  $\text{Im}(\delta V_\omega)$  shift to higher temperatures. This is consistent with the theoretical prediction implied by Eq. (12): as the frequency increases, the probing length  $r_\omega$  becomes smaller and the vortex unbinding transition is observed at a higher temperature  $T_\omega$ .

Also shown in Fig. 4 are the signals measured in a magnetic field corresponding to one flux quantum per unit cell of the array, the  $f = 1$  case. The response for  $f = 1$  is similar to that for  $f = 0$ , in qualitative agreement with the conclusion of section III, that the Hamiltonian of the system is periodic in  $f$  with period 1. We conjecture that in the  $f = 1$  case the thermal excitations are highly mobile positive and negative vacancies, which can be viewed as vortex-antivortex excitations superposed on a pinned commensurate background of one field-induced vortex per unit cell of the array. Notice however, that the transition for  $f = 1$  in Fig. 4 occurs at a slightly lower

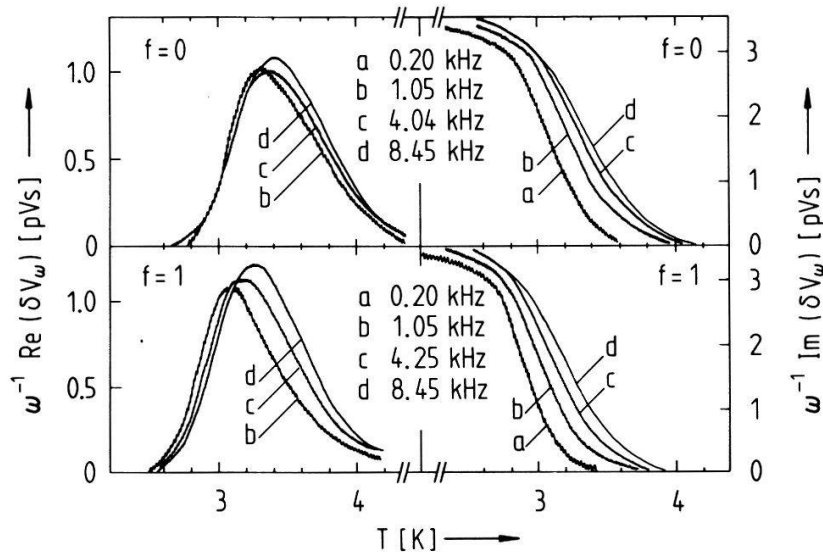


Figure 4. Temperature dependence of the complex ac response of a 2D array at different frequencies for  $f=0$  and  $f=1$ . Signal voltages are normalized with respect to angular frequency.

temperature than in the  $f = 0$  case. The finite size of the junctions causes the critical current of the individual junctions to be slightly reduced by the magnetic field. The  $KT$  transition is thus also expected to occur at a slightly reduced temperature in the case  $f = 1$ .

The frequency dependence of the signals, through Eq. (12), can be used to verify the unusual temperature dependence of the vortex correlation length  $\xi_+(T)$  given in Eq. (7). The  $r_\omega$ -values were calculated with  $D = (c/\phi_0)^2 r_n a^2 k_B T$  for the vortex diffusivity, as derived in Ref. 20, and  $r_n = 2.2 \text{ m}\Omega$ , inferred from the array sheet resistance at  $T_{CS}$ . The temperatures  $T_\omega$  were deduced from the  $\text{Im}(\Delta V_\omega)$  vs  $T$  curves by extrapolating the steep portions to zero. In order to determine  $\tilde{T} = k_B T/J = 2ek_B T/(\hbar i_c(T))$ , low temperature measurements of the array's critical current in zero field,  $i_c(T,0)$ , and in a  $f = 1$  field,  $i_c(T,1)$ , were fitted to the expression

$$i_c(T,f) = i_0(f) [1 - (T/T_{CS})]^2 \exp[-L/\xi_N(T)] , \quad (17)$$

yielding  $\xi_N(T_{CS}) = 85 \text{ nm}$  for the Cu coherence length,  $i_0(0) = 0.78 \text{ A}$  and  $i_0(1) = 0.26 \text{ A}$ . Finally, introducing the scale parameter  $\lambda_\omega = \ln(r_\omega/a)$ ,

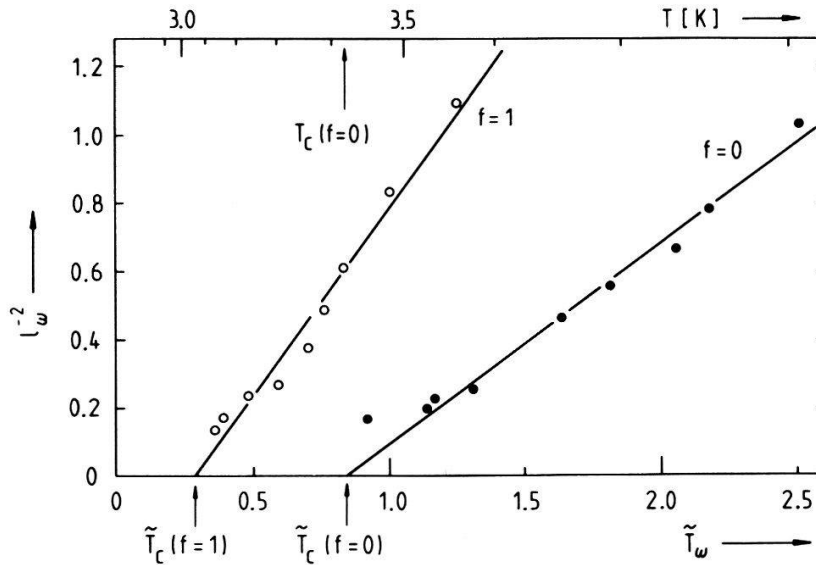


Figure 5. Dependence of the scale parameter  $\lambda_\omega = \lambda n(r_\omega/a)$  on the dimensionless temperature  $\tilde{T}_\omega$ . Solid lines are fits according to Eq. (18). On the upper axis the real temperature for  $f = 0$  is shown.

Eq. (12) can be cast into the form

$$\lambda_\omega^{-2} = b^{-2}[\tilde{T}_\omega - \tilde{T}_c(f)] \quad , \quad (18)$$

As can be seen in Fig. 5, our measurements confirm the linear relationship between  $\lambda_\omega^{-2}$  and  $\tilde{T}_\omega$ . By extrapolating the fitted straight lines to infinite scale, corresponding to the limit  $\omega \rightarrow 0$ , we find  $i_c(T_c, 0)/T_c(0) = 49$  nA/K and  $i_c(T_c, 1)/T_c(1) = 143$  nA/K, which leads, with Eq. (10), to  $\epsilon_c(0) = 1.81$  and  $\epsilon_c(1) = 5.3$ . Our  $\epsilon_c(0)$  value is in excellent agreement with a Monte Carlo calculation performed by Tobochnik and Chester [31], who found  $\epsilon_c = 1.75$ . The  $\epsilon_c(1)$  value, on the other hand, seems somewhat large, even though additional screening by the commensurate vortex background is expected to enhance  $\epsilon_c$ .

At this point, having studied the temperature dependence of the response at integer values of  $f$ , we would like to consider the dynamic response of the array also at rational and irrational values of  $f$ . Real and imaginary parts of the signal as a function of the frustration parameter  $f$  are shown in Fig. 6 at four different temperatures. The structures occurring at integer  $f$

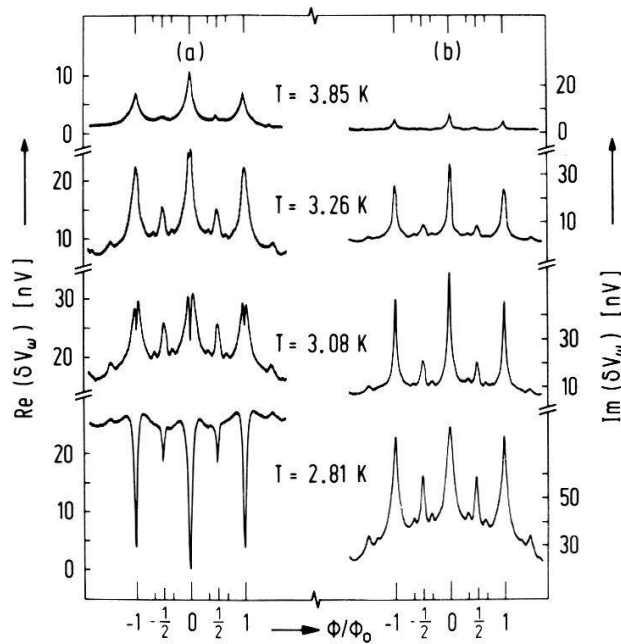


Figure 6. (a) Real and (b) imaginary parts of the ac response at 4033 Hz as a function of the frustration parameter  $f = \phi/\phi_0$  at four temperatures

values, as well as half-integer  $f$  values ( $f=p/2$ ) and at  $f=p/3$ , reveal the presence of low energy commensurate vortex phases. At integer  $f$  values the evolution of the structures with temperature is the signature of the vortex-unbinding transition discussed above. The temperature evolution of the signals at low-order rational  $f$  values is similar to that observed at integer  $f$  values, leading to speculations about the possibility of a KT-like phase transition at rational values of  $f$ . However, a theory describing the dynamics of field-induced vortices and their interaction with topological excitations such as thermal vortices and domain walls at non-zero temperatures is not available. A detailed analysis of the data shown in Fig. 6 is therefore not yet possible.

## V. Conclusions

Our measurements of the dynamic response of 2D arrays, in magnetic fields corresponding to integer values of the frustration parameter  $f$ , as a function of temperature verify the qualitative behavior of the dielectric constant  $\epsilon(\omega, T)$  predicted by the KT theory for phase transitions in two

dimensions. The analysis of the frequency dependence of the measurements provides a quantitative check of the exponential-inverse-square-root temperature dependence of the free vortex correlation length above  $T_c$ .

The nature of the phase transition in magnetic fields corresponding to non-integer  $f$  values is an unsolved and challenging problem. In particular, the case  $f = 1/2$ , corresponding to the fully frustrated XY model, has recently received a considerable amount of attention and some progress is being made, both theoretically and with numerical computer simulations [27, 32, 33]. Our experimental work to-date on 2D arrays cannot unambiguously settle the question of the nature of the phase transition, KT-like or Ising-like at  $f = 1/2$ .

This work was supported by the Swiss National Science Foundation.

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