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Autor(en): **Martínez-Pérez, Conchita / Nucinkis, Brita E. A.**

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SOLUBLE GROUPS OF TYPE VF

by Conchita MARTÍNEZ-PÉREZ and Brita E. A. NUCINKIS

Cohomological finiteness conditions for soluble groups are very well understood for torsion-free soluble groups, but there remains a gap for soluble groups with torsion. This can be summarised by the following conjecture:

CONJECTURE 51.1. *Every soluble group G of type VF admits a cocompact model for \underline{EG} .*

A group is said to be *of type VF* if it has a finite index subgroup admitting a finite $K(G, 1)$.

Finiteness conditions for soluble groups have been attracting wide attention ever since the celebrated result by Stammbach [9], that for a torsion-free soluble group the homological dimension $\text{hd } G$ is equal to the Hirsch length $\text{h } G$ of the group. For polycyclic groups the Hirsch length is equal to the cohomological dimension, $\text{cd } G$. (For countable groups, the homological dimension and the cohomological dimension differ by at most one.)

This result was extended by Gruenberg, see [1], who showed that a torsion-free nilpotent group is finitely generated if and only if $\text{cd } G = \text{h } G$. This led to the question, which cohomological finiteness condition describes torsion-free soluble groups with $\text{cd } G = \text{h } G$.

There were partial answers to this question by numerous authors including Bieri, Gildenhuys and Strebel and it was finally answered by Kropholler [3]. Kropholler's later result [4] meant that this result could be phrased as follows:

THEOREM 51.2 ([3,4]). *Let G be a soluble group. Then the following are equivalent:*

- (1) G is of type FP_∞ ,
- (2) G is virtually of type FP ,
- (3) $\text{vcd } G = \text{h } G < \infty$,
- (4) G is virtually torsion-free and constructible.

The fact that G is constructible implies that G is finitely presented. Thus any torsion-free group satisfying the conditions of the Theorem is of type F , i.e. it has a finite $\text{K}(G, 1)$ or equivalently a cocompact model for EG .

In case G has torsion it cannot admit a finite dimensional model for EG , which is equivalent to saying that $\text{cd } G = \infty$. We say a G -CW-complex X is a model for $\underline{\text{EG}}$ if X^H is contractible for all finite $H < G$ and empty otherwise. The cohomological counterpart is Bredon (co)homology. It was shown [8] that for countable groups the Bredon cohomological dimension $\underline{\text{cd}} G$ and the Bredon homological dimension $\underline{\text{hd}} G$ differ by at most one. By using a spectral sequence of Martínez-Pérez [7], Flores and Nucinkis [2] proved the analogue to Stambach's result, namely that for soluble groups, $\underline{\text{hd}} G = \text{h } G$. This led us to pose the following conjecture:

CONJECTURE 51.3. *Let G be a soluble group. Then the following are equivalent:*

- (1) G is of type $\underline{\text{FP}}_\infty$,
- (2) $\underline{\text{cd}} G = \text{h } G < \infty$,
- (3) G is of type FP_∞ .

$\underline{\text{FP}}_\infty$ denotes the Bredon analogue to FP_∞ . It is not hard to see that (1) \Rightarrow (2) \Rightarrow (3), see [2]. There are, however, examples by Leary and Nucinkis [5] showing that generally groups of type VF do not necessarily admit a cocompact model for $\underline{\text{EG}}$, but all available evidence leads us to believe that Conjecture 51.1 still holds for soluble groups. Lück [6] showed that a group admits a model of finite type for $\underline{\text{EG}}$ if and only if it is finitely presented of type FP_∞ , has finitely many conjugacy classes of finite subgroups and all centralisers of finite subgroups are of type FP_∞ . But even with this reduction, an answer to both conjectures remains frustratingly elusive.

REFERENCES

- [1] BIERI, R. *Homological Dimension of Discrete Groups*. Queen Mary College Mathematics Notes, London, 1976.
- [2] FLORES R.J. and B.E.A. NUCINKIS. On Bredon homology of elementary amenable groups. *Proc. Amer. Math. Soc.* 135 (2007), 5–11.
- [3] KROPHOLLER, P.H. Cohomological dimensions of soluble groups. *J. Pure Appl. Algebra* 43 (1986), 281–287.
- [4] ——— On groups of type FP_∞ . *J. Pure Appl. Algebra* 90 (1993), 55–67.
- [5] LEARY, I.J. and B.E.A. NUCINKIS. Some groups of type VF. *Invent. Math.* 151 (2003), 135–165.
- [6] LÜCK, W. The type of the classifying space for a family of subgroups. *J. Pure Appl. Algebra* 149 (2000), 177–203.
- [7] MARTÍNEZ-PÉREZ, C. A spectral sequence in Bredon (co)homology. *J. Pure Appl. Algebra* 176 (2002), 161–173.
- [8] NUCINKIS, B.E.A. On dimensions in Bredon homology. *Homology Homotopy Appl.* 6 (2004), 33–47.
- [9] STAMMBACH, U. On the weak homological dimension of the group algebra of solvable groups. *J. London Math. Soc.* (2) 2 (1970), 567–570.

Conchita Martínez-Pérez

Universidad de Zaragoza
E-50009 Zaragoza
Spain
e-mail: conmar@unizar.es

Brita E.A. Nucinkis

University of Southampton
Southampton SO17 1BJ
England
e-mail: B.E.A.Nucinkis@soton.ac.uk