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41

COHOMOLOGY OF PURE AUTOMORPHISM GROUPS OF FREE PRODUCTS OF FINITE GROUPS

by Craig JENSEN

Recently, John McCammond, John Meier and I verified [2] the Brownstein–Lee conjecture [1]. It concerns the cohomology of the *pure symmetric automorphism group of a free group*, $P\Sigma_n$, namely the group which has to take each of the preferred generators of free group F_n to a conjugate of itself.

THEOREM 41.1 (The Brownstein–Lee Conjecture). *The cohomology ring $H^*(P\Sigma_n, \mathbf{Z})$ is generated by one-dimensional classes α_{ij}^*, where $i \neq j$, subject to the relations*

(1) $\alpha_{ij}^* \wedge \alpha_{ij}^* = 0$

(2) $\alpha_{ij}^* \wedge \alpha_{ji}^* = 0$

(3) $\alpha_{kj}^* \wedge \alpha_{ji}^* = (\alpha_{kj}^* - \alpha_{ij}^*) \wedge \alpha_{ki}^*$

and the Poincaré series is $\mathfrak{p}(z) = (1 + nz)^{n-1}$.

What can we say about the cohomology of similar groups ? A first question might be:

QUESTION 41.2. *What is the cohomology of $PAut(\mathbf{Z}/p * \cdots * \mathbf{Z}/p)$? That is, what is the cohomology of the pure (meaning each $\mathbf{Z}/p$ in the free product has to be taken to a conjugate of itself) automorphism group of a free product of n copies of $\mathbf{Z}/p$?*

After we get this, a more generalized question would be:

QUESTION 41.3. *Let $G_1, \ldots, G_n$ be finite abelian groups. What is the cohomology of $PAut(G_1 * \cdots * G_n)$?*

or perhaps

QUESTION 41.4. *Let $G_1, \dots, G_n$ be finite abelian groups or* $\mathbf{Z}$. *What is the cohomology of* $PAut(G_1 * \cdots * G_n)$ *?*

or perhaps even

QUESTION 41.5. *Let $G_1, \dots, G_n$ be finite groups or* $\mathbf{Z}$. *What is the cohomology of* $PAut(G_1 * \cdots * G_n)$ *?*

REFERENCES

[1] BROWNSTEIN, A. and R. LEE. Cohomology of the group of motions of n strings in 3-space. In: *Mapping Class Groups and Moduli Spaces of Riemann Surfaces (Göttingen, 1991 and Seattle, WA, 1991)*, 51–61. Contemp. Math. *150*. Amer. Math. Soc., 1993.

[2] JENSEN, C. A., J. MCCAMMOND and J. MEIER. The integral cohomology of the group of loops. *Geom. Topol. 10* (2006), 759–784.

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