4.5 RÉCAPITULATION

Par (1.18), (4.2), (4.16) et (4.42), il vient

$$Q(N,\lambda) = \alpha(\lambda)N + \widetilde{R}(N,\lambda)$$

avec

$$\widetilde{R}(N,\lambda) = -Q_1(N,\lambda) + Q_3(N,\lambda) + Q_4(N,\lambda) + \varepsilon(N,\lambda) \tag{4.49}$$

et (1.8) résulte de (4.48), (4.35), (4.41) et (1.32).

La méthode utilisée pour obtenir (4.48), (4.35) et (4.41) montre que, pour tout λ fixé, $\lambda > \sqrt{2}$, on a $Q_1(N,\lambda) = O_\lambda(N^{7/8}(\log N))$, $Q_3(N,\lambda) = O_\lambda(N^{1/2}(\log N))$, $Q_4(N,\lambda) = O_\lambda(1)$, et, par (1.31), on a $\varepsilon(N,\lambda) = O_\lambda(1)$; ainsi, par (4.49), (1.6) est démontré, ce qui achève la preuve du théorème 1.

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