$$(16)\quad a_{n+1} = |S_{n+1}|g^2(n+1) \leq (k-1)|S_n|g^2(n+1) = \left(1 + \frac{k-2}{(k-2)n+k}\right)^2 a_n \,.$$

We have to show that

$$(17)\qquad \frac{\sum_{v \in S_{n+1}} f^2(v)}{\sum_{v \in B_n} f^2(v)} = \frac{a_{n+1}}{a_1 + \cdots + a_n} \to_{n \to \infty} 0 \,.$$

It is a standard exercise to show that (16) implies (17). □

Let f_n be the sequence of functions which are restrictions of f to the vertices that are at a distance not greater than n:

$$f_n = f|_{B_n} \,.$$

By Lemma 6 and Lemma 7 it follows that

$$\limsup_{n \to +\infty} \frac{\|Pf_n\|_{l^2(X,N)}}{\|f_n\|_{l^2(X,N)}} \geq \frac{2\sqrt{k-1}}{k} \,,$$

which proves Theorem 8. □

Some examples of upper bounds on the norm of the simple random walk operator on graphs and their comparison with the lower bound from Theorem 8 can be found in [22].

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