

## §2. Type 1 case

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## § 2. TYPE 1 CASE

In this section we consider the case where a meridian of  $L^n$  in  $M^{n+2}$  has infinite order in  $H_1(M-L; \mathbf{Z})$ . We shall denote by  $[m]$  the homology class in  $H_1(M-L; \mathbf{Z})$  represented by a meridian  $m$  of  $L$  in  $M$ . For a manifold pair  $(X, Y)$  of codimension 2 and an epimorphism  $\gamma$  from  $\pi_1(X-Y)$  to a finite group, let  $(X, Y)_\gamma$  be the branched covering of  $(X, Y)$  corresponding to  $\gamma$ . Each knot group  $\pi_1(S^{n+2}-K)$  has a natural epimorphism to  $\mathbf{Z}_p$  for any positive integer  $p$ , and the corresponding  $p$ -fold branched cyclic covering of  $(S^{n+2}, K)$  is denoted by  $(S^{n+2}, K)_p$ .

LEMMA 2.1. *Suppose  $[m]$  is of infinite order. Then if  $(S^{n+2}, K) \in I(M, L)$  then  $(S^{n+2}, K)_p$  is a homotopy  $(n+2)$ -sphere for any positive integer  $p$ .*

*Proof.* Since  $[m]$  represents a nontrivial element in the finitely generated free abelian group  $B_1(M-L) \equiv H_1(M-L; \mathbf{Z})/\text{Tor } H_1(M-L; \mathbf{Z})$ , there is a positive integer  $r$  and a primitive element  $x$  in  $B_1(M-L)$  such that  $[m] = rx$  in  $B_1(M-L)$ . For each positive integer  $p$ , let  $\gamma_p$  be the canonical epimorphism  $\pi_1(M-L) \rightarrow B_1(M-L) \otimes \mathbf{Z}_{pr}$ . Noting the naturality of the homomorphism  $\gamma_p$ , we can see the following:

$$\begin{aligned} (M, L)_{\gamma_p} &= ((M, L) \# (S^{n+2}, K))_{\gamma_p \circ f_*} \\ &= (M, L)_{\gamma_p} \# d_p(S^{n+2}, K)_p \end{aligned}$$

Here  $f$  is a diffeomorphism  $(M, L) \# (S^{n+2}, K) \rightarrow (M, L)$  and  $d_p$  is the order of  $B_1(M-L) \otimes \mathbf{Z}_{pr}$  divided by  $p$ . Hence  $H_*((S^{n+2}, K)_p; \mathbf{Z}) \simeq H_*(S^{n+2}; \mathbf{Z})$  and  $\pi_1((S^{n+2}, K)_p) \simeq 1$  by the existence of prime decompositions of finitely generated groups into free products [Wg]. Q.E.D.

It is conjectured that those knots which satisfy the conclusion of the above lemma are trivial. In fact, for  $n = 1$ , it follows from the Smith conjecture [MB]. As a supporting evidence for higher dimensional cases, we have

LEMMA. *Suppose that  $(S^{n+2}, K)_p$  is a homotopy  $(n+2)$ -sphere for every positive integer  $p$ . Then the Alexander modules of  $K$  are trivial.*

*Proof.* Let  $\tilde{E}(K)$  be the infinite cyclic cover of the exterior  $E(K)$  of  $K$  in  $S^{n+2}$ , and let  $t$  denote the automorphism of the homology group of  $\tilde{E}(K)$  induced by the action of a meridian. Then, by the arguments of [Sm1],

we can see that  $t^p - 1: H_q(\tilde{E}(K); \mathbf{Z}_r) \rightarrow H_q(\tilde{E}(K); \mathbf{Z}_r)$  is an isomorphism for any positive integers  $p, q$ , and  $r$ . Assume  $r$  is prime. Then  $H_q(\tilde{E}(K); \mathbf{Z}_r)$  is a finite abelian group, since it is a finitely generated torsion module over the principal ideal domain  $\mathbf{Z}_r\langle t \rangle$  (see [Le3, p. 8]). So the automorphism  $t$  on  $H_q(\tilde{E}(K); \mathbf{Z}_r)$  has a finite order, say  $d$ , and we have  $t^d - 1 = 0$ . Hence  $H_q(\tilde{E}(K); \mathbf{Z}_r) = 0$ , and by the universal coefficient theorem, the following holds for any prime  $r$  and any positive integer  $q$ :

$$(2.3) \quad H_q(\tilde{E}(K); \mathbf{Z}) \otimes \mathbf{Z}_r = 0$$

$$(2.4) \quad \text{Tor}(H_q(\tilde{E}(K); \mathbf{Z}), \mathbf{Z}_r) = 0$$

By (2.4),  $H_q(\tilde{E}(K); \mathbf{Z})$  has no nontrivial elements of finite order; so it has a square presentation matrix  $M(t)$  as a  $\mathbf{Z}\langle t \rangle$ -module by [Le3, Proposition 3.5]. By (2.3) the  $q$ -th Alexander polynomial  $\det M_q(t) (\in \mathbf{Z}\langle t \rangle)$  is a unit mod.  $r$  for any prime  $r$ . Hence it is a unit in  $\mathbf{Z}\langle t \rangle$ , and we have  $H_q(\tilde{E}(K); \mathbf{Z}) = 0$  for any positive integer  $q$ . Q.E.D.

Thus, as a consequence of Lemmas 2.1 and 2.2 and the results of [Le2] and [T], we have the following:

**PROPOSITION 2.5.** *Suppose  $[m]$  is of infinite order. Then any knot in  $I(M, L)$  has trivial Alexander modules and is null cobordant.*

Hence the only obstruction for a knot  $(S^{n+2}, K)$  in  $I(M, L)$  to be trivial lies in the knot group  $\pi_1(S^{n+2} - K)$ . For the special case where  $[m]$  generates  $H_1(M - L)$ , we can apply the result of Maeda [Ma] (cf. [DF]), and obtain the following:

**THEOREM 2.6.** *Suppose  $n \geq 3$  and  $H_1(M - L)$  is the infinite cyclic group generated by  $[m]$ . Then  $I(M, L)$  is trivial.*

*Proof.* Let  $(S^{n+2}, K)$  be a knot in  $I(M, L)$ . Note that  $\pi_1(M - L)$  is isomorphic to the amalgamated free product  $\pi_1(M - L) \underset{\langle m \rangle}{*} \pi_1(S^{n+2} - K)$ .

Then we can conclude  $\pi_1(S^{n+2} - K) \simeq \mathbf{Z}$  by the result of [Ma] (cf. [DF]) which asserts the existence of a prime decomposition of a finitely presented group  $G$  with  $G/[G, G] \simeq \mathbf{Z}$  with respect to such amalgamated free products. Combined with Proposition 2.5, we see  $S^{n+2} - K$  is homotopy equivalent to a circle. Hence  $(S^{n+2}, K)$  is trivial by [Le1].