$S \in H^0(\bar{M}, K^{-1})$. This would imply that either K is trivial on M or $H^0(\bar{M}, K^n) = 0$ for every $m > 0$ and hence the Kodaira dimension of $\bar{M}^2$ would either be $-\infty$ or 0. This is because if $t \in H^0(\bar{M}, K^n)$, then $t \cdot S^n$ is a holomorphic function on M and hence constant; since S is zero somewhere unless K is trivial, we have $t \cdot S^n = 0$, so that $t = 0$ unless K is trivial on M.

Since M is Kähler and simply connected, the minimal model of $\bar{M}$ is a Kähler surface with $K = 0$ or $-\infty$ and $b_1 = 0$. When $K = 0$, it is either a $K-3$ surface or Enriques' surface. When $K = -\infty$ it is either a rational surface or a ruled surface of genus zero, $\bar{M}^2$ is equal the minimal model blown up successively at a finite number of points, and $M = \bar{M} \backslash \{s=0\}$ for some $0 \neq s \in H^0(\bar{M}, K^{-1})$. Conversely, if $M = \bar{M} \backslash \{s=0\}$ with $s \in H^0(\bar{M}, K^{-1})$ and $\bar{M}$ is as above, then M should admit a Ricci flat, complete, Kähler metric. In higher dimensions, the situation is much more complicated.

In physics, the following question has been studied. Is a Ricci flat metric with a suitable locally asymptotic property actually unique? This is the case when the metric is asymptotically flat. One would also like to know what happens when the metric is locally asymptotic to a cone. Perhaps assuming that the metric is Kähler may make this problem easier.

The existence of Ricci flat metrics has many applications. For example, using Ricci flat metrics, Siu [S1] proved that any surface M^2 with $c_1(M) = 0$ and $H^1(M, \mathbf{R}) = 0$ must be Kähler. See also Todorov [To] for higher dimensions. One can also ask the following question: Let M^{2n} be a simply-connected, compact, complex manifold where $n \geqslant 2$. If there exists a non-degenerate 2-form $\omega \in H^{2,0}(M)$, is M then Kähler? Todorov claimed that M is Kähler under an additional assumption: $\dim H^{2,0}(M) = 1$.

REFERENCES

[Al] ALMGREN, F. J. Jr. The homotopy groups of the integral cycle groups. *Topology 1* (1962), 257-299.

[Au1] AUBIN, T. Equations différentielles non linéaires et Problème de Yamabe concernant la courbure scalaire. *J. Math. Pures et appl. 55* (1976), 269-296.

[Au2] —— *Nonlinear Analysis on Manifolds, Monge-Ampère Equations.* Springer-Verlag, New York, 1982.

[Au3] —— Equations du type Monge-Ampère sur les variétés kählériennes compactes. *C.R.A.S 283A* (1976), 119-121.

[A] ANDERSON, M. T. The Dirichlet problem at infinity for manifolds with negative curvature. *J. Diff. Geom. 18* (1983), 701-722.

[A-S] ANDERSON, M. and R. SCHOEN. Positive harmonic functions on complete manifolds of negative curvature. To appear in *Ann. of Math.*

[B1] BANDO, S. On the classification of the three-dimensional compact Kähler manifolds of nonnegative bisectional curvature. *J. Diff. Geom. 19* (1984), 283-297.

[B2] —— An obstruction for Chern class forms to be harmonic. Preprint.

[Be] BEAUVILLE, A. Variétés rationnelles et unirationnelles. *Lecture Notes in Math. V. 997,* 16-33.

[Bo] BOGOMOLOV.

[Br1] BRYANT, R. L. Conformal and minimal immersions of compact surfaces into 4-sphere. *J. Diff. Geom. 17* (1982), 455-473.

[Br2] —— Submanifolds and special structures on the octonians. *J. Diff. Geom. 17* (1982), 185-232.

[B-B] BAILY, W. L., Jr. and A. BOREL. Compactification of arithmetic quotients of bounded symetric domains. *Ann. Math. 84* (1966), 442-528.

[BGM] BERGER, M., P. GAUDUCHON et E. MAZET. *Le Spectre d'une variété Riemannienne.* Lecture Notes in Math. No. 194, Springer-Verlag, 1971.

[B-M] BOCHNER, S. and W. T. MARTIN. *Several complex variables.* Princeton University Press, 1948.

[Ca1] CALABI, E. Minimal immersions of surfaces in Euclidean spheres. *J. Diff. Geom. I* (1967), 111-125.

[Ca2] —— Extremal Kähler metric. I. *Seminar in Diff. Geom.,* 1982 (edited by Yau), pp. 259-290; II. *Diff. Geo. and Complex Analysis,* 1985. (edited by I. Charel and H. M. Farkas; dedicated to H. E. Rauch).

[Ca3] —— Construction and properties of some 6-dimensional almost complex manifolds. *Trans. AMS 87* (1958), 407-438.

[Ch1] CHENG, S. Y. Eigenvalues comparison theorems and its applications. *Math. Zeit. 143* (1975), 289-293.

[Ch2] —— Eigenfunctions and nodal sets. *Comm. Math. Helv. 51* (1976), 43-55.

[Co] CHO, K. Positivity of the curvature of Weil-Peterson metric on the moduli space of stable vector bundles. Preprint.

[Cr] CHRISTODOULOU, D. A mathematical theory of gravitation collapse.

[CS] CHOI, H. I. and R. SCHOEN. The space of minimal embeddings of a surface into a three-dimensional manifold of positive Ricci curvature. *Invent. Math. 81* (1985), 387-394.

[CW] CHOI, H. I. and A. N. WANG. A first eigenvalue estimate for minimal hypersurfaces. *J. Diff. Geom. 18* (1983), 559-562.

[CC] CAO, H.-D. and B. CHOW. Compact Kähler manifolds with nonnegative curvature operator. *Inv. Math. 83* (1986), 553-556.

[C-V] CALABI, E. and E. VESENTINI. On compact local symmetric Kähler manifolds. *Ann. Math. 71* (1960), 472-507.

[C-Y1] CHENG, S.-Y. and S.-T. YAU. On the existence of complete Kähler-Einstein metric on noncompact complex manifolds and regularity of Fefferman's equation. *Comm. Pure. Appl. Math. 32* (1980), 507-544.

[C-Y2] —— Inequality between Chern numbers of Singular Kähler surfaces and characterization of orbit space of Discrete group of $SU(2, 1)$. Preprint.

[D1] DONALDSON, S. K. A new proof of a theorem of Narasimhan and Seshadri. *J. Diff. Geom. 18* (1983), 269-277.

[D2] —— Inequality between Chern numbers of singular Kähler anti-self dual Yang-Mills connections over complex algebraic surfaces and stable vector bundles. *Proc. London. Math. Soc. 1985.*

[D3] —— An application of gauge theory to the topology of 4-manifolds. *J. Diff. Geom. 18* (1983), 269-316.

[D4] —— Connections, cohomology and the insertion forms of 4-manifolds. Preprint.

[DGMS] DELIGNE, P., P. A. GRIFFITHS, MORGAN and D. SULLIVAN.

[Ei] EISEMAN

[ESa] EELLS, J. and S. SALAMAN. Twistor construction of harmonic maps of surfaces into four manifolds. Preprint.

[E-S] EELLS, J. and J. H. SAMPSON. Harmonic mappings of Riemannian manifolds. *Amer. J. Math. 86* (1964), 1009-1060.

[E-W1] EELLS, J. and J. C. WOOD. Restrictions on harmonic maps of surfaces. *Topology 15* (1976), 263-266.

[E-W2] —— Harmonic maps from surfaces to complex projective spaces. *Adv. Math. 49* (1983), 217-263.

[Es] ESCOBAR, H. Spectrum of the Laplacian on complete Riemannian manifolds. *Comm. in Part. Diff. Equat. 11 (1)* (1986), 63-85.

[Fe] FEFFERMAN, C. The Bergman kernel and biholomorphic mapping of pseudoconvex domain. *Invent. Math. 26* (1974), 1-65.

[Fr] FRANKEL, T. T. Manifolds with positive curvature. *Pacific J. Math. 11* (1961), 165-174.

[Fu1] FUTAKI, A. An obstruction to the existence of Einstein-Kähler metrics. *Invent. Math. 73* (1983), 437-443.

[Fu2] —— On a character of the automorphism group of a compact complex manifold. Preprint.

[F-M] FUTAKI, A. and S. MORITA. Invariant polynomial characterization to compact complex manifold and compact group actions. Preprint.

[G] GIESEKER, D. Global moduli for surfaces of general type. *Invent. Math. 43* (1977), 233-282.

[G-H] GRIFFITHS, P. A. and J. HARRIS. *Principles of Algebraic Geometry.* New York, 1978.

[Gr] GROMOV, M. Curvature, diameter and Betti numbers. *Comm. Math. Helv. 56* (1981), 179-195.

[GW] GORDON, C. S and E. N. WILSON. Isospectral deformations of compact solvmanifolds. *J. Diff. Geom. 19* (1984), 241-256.

[Ha] HAMILTON, S. *Harmonic Maps of Manifolds with Boundary.* Lecture Notes in Math. No. 471 (1975), Springer-Verlag.

[Hi] HITCHEN.

[Ho] HORIKAWA, H. Algebraic surfaces of general type with small C_1^2, I. *Ann. Math. 104* (1976), 357-387.

[Hr] HARTMAN, P. On homotopic harmonic maps. *Canad. J. Math. 19* (1967), 693-687.

[H-K] HIRZEBRUCH, F. and I. KODAIRA. On the complex projective spaces. *J. Math. Pures & Appl. 36* (1957), 201-266.

[H-L] HARVEY, R. and H. B. LAWSON. An intrinsic characterization of Kähler manifolds. *Invent. Math. 74* (1983), 169-198.

[H-M] HARRIS, J. and D. MUMFORD. On the Kodaira dimension of the moduli space of curves. *Invent. Math. 67* (1982), 23-86.

[HW] HULL and WITTEN.

[Ii] IITAKA, S. J. *Math. Soc. Japan. 24* (1972), 384-396.

[Is] ISKOVSKIH, V. A. Fano 3-folds, I, II. *Math USSR Izv. 11* (1977) No. 3, 485-527; *12* (1978) No. 3, 469-506.

[Iv] IVRII, V. Ya. Second term of the spectral asymptotic expansion of the Laplace-Beltrami operator on manifolds with boundary. *Funct. Analy. Appl. 14 (2)* (1980), 98-105.

[I-S] ISHIKAWA, K. and Y. SAKANE. On complex projective bundles over a Kähler C-space. *Osaka J. Math. 16* (1979), 121-132.

[Ja] JACOBOWITZ, H. Local isometric embeddings of surfaces into Euclidean four space. *Indiana Univ. Math. J. 21* (1971), 249-254.

[Jo] JOST, J. Univalency of harmonic maps between surfaces. *J. Reine Angew. Math. 324* (1981), 141-153.

[J-S] JOST, J. and R. SCHOEN. On the existence of harmonic diffeomorphisms between surfaces. Preprint.

[J-Y1] JOST, J. and S.-T. YAU. Harmonic mappings and Kähler manifolds. *Math. Ann. 262* (1983), 145-166.

[J-Y2] —— A strong rigidity theorem for a certain class of compact complex surfaces. *Math. Ann. 271* (1985), 143-152.

[Ka] KAWATAMA.

[Ko] KODAIRA, K. On the structure of compact analytic surface. I. *Amer. J. Math. 86* (1969), 751-798.

[Ku] KUIPER, N. H. On conformally flat spaces in the large. *Ann. Math. 50* (1949), 916-924.

[Ky] KOBAYASHI, S. Hyperbolic Manifold and Holomorphic Mappings.

[Kz] KAZHDAN, D.

[Lm] LEMAIRE, L. Applications harmoniques de surfaces riemanniennes. *J. Diff. Geom. 13* (1978), 51-78.

[Le1] LEMPERT, L. La métrique de Kobayashi et la représentation des domaines sur la boule. *Bulletin Soc. Math. de France 109* (1981), 427-474.

[Le2] —— Holomorphic retracts and intrinsic metrics in convex domains. *Analysis Mathematica 8* (1982), 257-261.

[Lü] LÜBKE, M. Stability of Einstein-Hermitian vector bundles. *Man. Math. 42* (1983), 245-257.

[Lv] LEVINE.

[L-M] LAWSON, H. B. and M. L. MICHELSOHN. Clifford bundles, immersions of manifolds and the vector field problem. *J. Diff. Geom. 15* (1980), 237-267.

[L-S] LAWSON, H. B. and J. SIMONS. On stable currents and their applications to global problems in real and complex geometry. *Ann. Math. 98* (1973), 427-450.

[LuS] LUSTERNIK, L. and L. SCHNIRELMANN. Sur le problème de trois géodésiques fermées sur les surfaces de genre 0. *C. R. Acad. Sci. Paris 189* (1929), 269-271.

[Lv] L'VOVSKIĬ, S.M. Boundedness of the degree of Fano threefolds. *Math. USSR Izv. 19* (1982), No. 3, 521-558.

[LT] LI and TAM.

[L-Y1] LI, P. and S.-T. YAU. Eigenvalues of a compact Riemannian manifold. *AMS Symposium on Geometry of the Laplace Operator,* XXXVI. Hawaii (1979), 205-240.

[L-Y2] —— A new conformal invariant and its applications to the Willmore conjecture and the first eigenvalue of compact surfaces. *Invent. Math. 69* (1982), 269-291.

[Ln1] LIN, C. S. The local isometric embedding in R^3 of 2-dimensional Riemannian manifolds with nonnegative curvature. *J. Diff. Geom. 21* (1985), 213-230.

[Ln2] —— The local isometric embedding in R^3 of two-dimensional Riemannian manifolds with Gaussian curvature changing sign cleanly. Preprint.

[Ln3] —— Preprint.

[M1] MIYAOKA, Y. Kähler metrics on elliptic surfaces. *Proc. Japan Acad. 50* (1974), 533-536.

[M2] —— On the Chern numbers of surfaces of general type. *Invent. Math. 42* (1977), 225-237.

[M3] —— The maximal number of quotients singularities on surfaces with given numerical invariants. *Math. Ann. 268* (1984), 159-171.

[Mc] MICALIF, M. Preprint.

[MD] MOORE, D. Minimal two-spheres and the topology of manifolds with positive curvature on totally isotropic two-planes. Preprint.

[M-K] MORROW, J. and K. KODAIRA. *Complex manifolds.* Holt, Rinehart, Winston, Inc., 1971.

[Mi] MILNOR, J. Eigenvalues of the Laplace operator of certain manifolds, *Proc. Nat. Sci. USA 51* (1964), 542.

[Mk1] MOK, N.-M. An embedding theorem of complete Kähler manifolds of positive bisectional curvature onto affine algebraic varieties. *Bull. Soc. Math. de France 112* (1984), 197-258.

[Mk2] —— To appear.

[Mc] MICHELSOHN, M. L.

[Mo1] MORI, S. Projective manifolds with ample tangent bundles. *Ann. Math. 110* (1979), 593-606.

[Mo2] —— Threefolds whose canonical bundles are numerically effective. *Proc. Nat. Acad. Sci. USA 77* (1980), 3125-3126.

[Mr] MORROW, J. A. A survey of some results on complex Kähler manifolds. *Global Analysis,* Papers in Honor of K. Kodaira, Univ. of Tokyo Press and Princeton Univ. Press, 1969, 315-324.

[Mu] MUMFORD, D.

[M-M] MORI, S-MUKAI. Classification of Fano 3-folds with $B_2 \geq 2$. *Man. Math. 36* (1981), 147-162.

[M-S] MOSTOW, G. D. and Y.-T. SIU. A compact Kähler surface of negative curvature not covered by the ball. *Ann. Math. 112* (1980), 321-360.

[M-S-Y] MOK, N.-M., Y.-T. SIU and S.-T. YAU. The Poincaré-Lelong equation on complete Kähler manifolds. *Compositio Math. 44* (1981) (1-3), 183-218.

[M-Y] MEEKS, W. III and S.-T. YAU. Topology of three-dimensional manifolds and the embedding problem in minimal surface theory. *Ann. Math. 112* (1980), 441-484.

[Mk-Y] MOK, N.-M. and S.-T. YAU. Completeness of the Kähler-Einstein metric on bounded Riemann domains and the characterization of domains of holomorphy by curvature conditions. Sympos. on Math. Heritage of Henri Poincaré. (Indiana Univ. 1980), *Proc. Sympos. Pure Math. 39,* Part 1. AMS, (1983), 41-59.

[M-Z] MOK, N.-M. and J.-Q. ZHONG. Curvature characteristics of compact hermitian symmetric spaces. To appear in *J. Diff. Geom.*

[N-S] NARASIMHAN, M. S. and C. S. SESHADRI. Stable and unitary vector bundles on compact Riemann surfaces. *Ann. Math. 82* (1965), 540-567.

[Pe] PETERNELL, T. Algebraic structures on certain 3-folds. *Math. Ann. 274* (1986), 133-156.

[Pg] POGORELOV, A. V. An example of a two-dimensional Riemannian metric admitting no local realization in E_3. *Dokl. Akad. Nauk SSSR 198* (1971), 42-43.

[Pi] PITTS, J. T. Existence and regularity of minimal surfaces in Riemannian manifolds. Mimeograph (1979).

[Po] PÓLYA. On the eigenvalues of a vibrating membrane. *London Math. Soc. Ser. 3., 11* (1961), 414-433.

[P-P] PINKHAM, H. and U. PERSSON. Degeneration of surfaces with trivial canonical bundle. *Ann. Math. 113* (1981), 45-66.

[S1] SIU, Y.-T. Every K-3 surface is Kähler. *Invent. Math. 73* (1983), 139-150.

[S2] —— The complex-analyticity of harmonic maps and the strong rigidity of compact Kähler manifolds. *Ann. Math. 112* (1980), 73-111.

[S3] —— Curvature characteristics of hyperquadrics. *Duke Math. J. 47* (1980), 641-654.

[S4] —— A simple proof of the surjectivity of the period map of $K3$ surfaces. *Man. Math. 35* (1981), 311-321.

[S5] —— A vanishing theorem for semipositive line bundles over non-Kähler manifolds. *J. Diff. Geom. 19* (1984), 431-452.

[Sc] SCHOEN, R. Conformation deformation of a Riemannian metric to constant scalar curvature. *J. Diff. Geom. 20* (1984), 479-496.

[Sa] SAMPSON, J. H. Applications of harmonic maps to Kähler geometry. Preprint.

[Sg] SIEGEL.

[Si] SIMON, L.

[Sk1] SAKANE. On compact Einstein-Kähler manifolds with aboundant holomorphic transformations. *Manifolds and Lie groups,* Papers in Honor of Matsushima, progress in Math. Boston (1981), pp. 337-358.

[Sk2] —— On nonsingular hyperplane sections of a hermitian symmetric space. Preprint.

[St] SATAKE, J. On compactifications of the quotient spaces of arithmetically defined discontinuous groups. *Ann. Math. 72* (1980), 555-580.

[Sr] STROMINGER, A.

[Sa-U] SACKS, J. and K. UHLENBECK. The existence of minimal immersions of 2-spheres. *Ann. Math. 113* (1981), 1-24.

[Sc-U1] SCHOEN, R. and K. UHLENBECK. A regularity for harmonic maps. *J. Diff. Geom. 17* (1982), 307-335.

[Sc-U2] —— Boundary regularity and the Dirichlet problem for harmonic maps. *J. Diff. Geom. 18* (1983), 253-268.

[Sc-U3] —— Approximation theorem for Sobolev mappings. Preprint.

[Sc-Y1] SCHOEN, R. and S.-T. YAU. Existence of incompressible minimal surfaces and the topology of three dimensional manifolds with non-negative scalar curvature. *Ann. Math. 110* (1979), 127-142.

[Sc-Y2] —— Compact group actions and the topology of manifolds with non-positive curvature. *Topology 18* (1979), 361-380.

[Sc-Y3] —— Harmonic maps and the topology of stable hypersurfaces and manifolds with non-negative Ricci curvature. *Comm. Math. Helv. 39* (1981), 333-341.

[Sc-Y4] —— On the univalent harmonic maps between surfaces. *Invent. Math. 44* (1978), 265-278.

[ScY5] —— To appear.

[ScY6] —— To appear.

[S-Y1] SIU, Y.-T. and S.-T. YAU. Compact Kähler manifold of positive bisectional curvature. *Invent. Math. 59* (1980), 189-204.

[S-Y2] —— Complete Kähler manifolds with nonpositive curvature of faster than quadratic decay. *Ann. of Math. 105* (1977), 225-264.

[S-Y3] —— Compactification of negatively curved Kähler manifolds of finite volume. *Seminar on Diff. Geom.* (edited by S.-T. Yau), Princeton University Press, 1982, 363-380.

[SWYY] SINGER, I. M., B. WONG, S.-T. YAU and Stephen S.-T. YAU. An estimate of the gap of the first two eigen values in Schrödinger operator. To appear.

[Sm-S] SMITH, and L. SIMON. Unpublished.

[SS] SCHOEN, R. and L. SIMON. Regularity of stable minimal hypersurfaces.

[Su] SULLIVAN, D. The Dirichlet problem at infinity for a negatively curved manifold. *J. Diff. Geom. 18* (1983), 723-732.

[T1] TAUBES, C.

[T2] ——

[To] TODOROV, A. N. Applications of the Kähler-Einstein-Calabi-Yau metric to moduli of *K*3 surfaces. *Invent. Math. 61* (1980), 251-265.

[Tr] TRUDINGER, N. Remarks concerning the conformal deformation of Riemann structures on compact manifolds. *Ann. Scuola. Norm. Sup. Pisa. 3* (1968), 265-274.

[U1] UHLENBECK, K. Connections with L^p bounds on curvature. *Comm. Math. Phys. 83* (1982), 31-42.

[U2] —— Removable singularities in Yang-Mills Fields. *Comm. Math. Phys. 83* (1982), 11-29.

[U3] UHLENBECK, K. Harmonic Maps into Lie groups. Preprint.

[U-Y] UHLENBECK, K. and S.-T. YAU. On the existence of Hermitian-Yang-Mills connections on stable vector bundles. Preprint.

[UCSD] *Proceedings of the UCSD conference at La Jolla,* 1985. To appear.

[V1] VAN DE VEN, A. On the Chern numbers of certain complex and almost complex manifolds. *Nat. Acad. Proc. 55* (1966), 1624-1627.

[V2] —— On the Chern numbers of surface of general type. *Invent. Math. 36* (1976), 285-293.

[Wh] WHITE, B. Homotopy classes in Sobolev spaces of mappings. Preprint.

[We] WEYL, H. Das asymptotische Verteilungsgesetz der Eigenwerte linearer partieller Differentialgleichungen. *Math. Ann. 71* (1911), 441-479.

[Wi] WITTEN, E.

[Wo1] WONG, B. On the holomorphic sectional curvature of differentiable Kobayashi metric and Caratheodory metric. *Trans. AMS.*

[Wo2] —— Characterization of the unit ball by its automorphism group. *Invent. Math. 41* (1977), 253-257.

[Yg] YANG, P. On Kähler manifolds with negative holomorphic bisectional curvature. *Duke Math. J. 43* (1976), 871-874.

[Ya] YAMABE, H. On the deformation of Riemannian Structures on compact manifolds. *Osaka Math. J. 12* (1960), 21-37.

[Y1] YAU, S.-T. Calabi's conjecture and some new results in algebraic geometry. *Proc. Natl. Acad. Sci. USA 74* (1977), No. 5. 1798-1799.

[Y2] —— On the Ricci curvature of a compact Kähler manifold and the complex Monge-Ampère equation, I. *Comm. Pure. Appl. Math. 31* (1978), 339-411.

[Y3] —— On the curvature of compact Hermitian manifolds. *Invent. Math. 25* (1974), 213-239.

[Y4] —— A general Schwarz lemma for Kähler manifolds. *Amer. J. Math. 100* (1978), 197-203.

[Y5] —— Some function-theoretic properties of complete Riemannian manifolds and their applications to geometry. *Indiana Math. J. 25* (1976), 659-670.

[Y6] —— Problem Section. *Seminar on Diff. Geom.,* edited by S. T. Yau, Princeton Univ. Press, 1982.

[Y7] —— Harmonic functions on complete Riemannian manifolds. *Comm. Pure Appl. Math. 28* (1975), 201-228.

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(Reçu le 6 mai 1986)

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