ALTERNATION AND THE ACKERMANN CASE OF THE DECISION PROBLEM [1]

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ABSTRACT. The Ackermann prefix class is the set of all formulas of predicate calculus (first order logic without function symbols) with quantifier prefix $\exists \ldots \exists\forall\exists \ldots \exists$. This is one of the few prefix classes for which satisfiability is decidable. Lower bounds for the computational complexity of this decision problem and the $\forall\exists$ sub-problem are presented. The tool to get the main result is the alternating Turing machine. An introduction to alternating Turing machines is given, because they are probably the most remarkable new subject of automata theory, and are well known only to computer scientists.

1. INTRODUCTION AND HISTORICAL BACKGROUND

From the beginning of this century to the thirties, the problem of deciding universal validity of first order formulas, moved slowly to the center of interest of mathematical logic. Especially Hilbert considered it to be a fundamental problem. As it seemed too hard to solve the decision problem (or Entscheidungsproblem) in general, the main approach was to restrict the class of formulas (for which a decision algorithm should work) by very simple syntactic criteria. An earlier example of this kind of restriction was the decidability result of Löwenheim [29] for the monadic (only unary predicate symbols) predicate calculus. Later the main such criterion was the form of the quantifier sequence for formulas in prenex form (see [14], [28], [43] for other syntactically defined classes). There is a duality between universal validity and satisfiability. A closed formula (i.e. a formula of predicate calculus without free variables) is universally valid, iff its negation is not satisfiable. Around 1930 the decidability of the satisfiability

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