$$(25.13) \qquad 5i \sum_{k=0}^{\infty} \frac{1}{(2k+1)^2 + 10(2k+1)i - 50} = \frac{4689}{11{,}890},$$

or equivalently that

$$(25.14) \qquad 50 \sum_{k=0}^{\infty} \frac{2k+1}{(2k+1)^4 + 2500} = \frac{4689}{11{,}890},$$

since (25.12) obviously implies that the imaginary part of the left side of (25.13) is zero. To show (25.14), write

$$50 \sum_{k=0}^{\infty} \frac{2k+1}{(2k+1)^4 + 2500}$$

$$= \frac{5}{2} \sum_{k=0}^{\infty} \left\{ \frac{1}{(2k+1)^2 - 10(2k+1) + 50} - \frac{1}{(2k+1)^2 + 10(2k+1) + 50} \right\}$$

$$= \frac{5}{2} \left\{ \sum_{k=0}^{\infty} \frac{1}{(2k+1)^2 - 10(2k+1) + 50} - \sum_{k=5}^{\infty} \frac{1}{(2k+1-10)^2 + 10(2k+1-10) + 50)} \right\}$$

$$= \frac{5}{2} \sum_{k=0}^{4} \frac{1}{(2k+1)^2 - 10(2k+1) + 50} = \frac{4689}{11{,}890},$$

and the proof of (25.14), and hence (25.11), is complete.

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(Reçu le 5 janvier 1979)

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