4. Main theorem

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THEOREM 3. (Guillemin [10], Losik [17]). *$C^*_\triangle(L_M, \Omega_M)$ is a model for a bundle E with fiber F_n, base space M, associated to the tangent bundle of M.*

More precisely, a model for $C^*_\triangle(L_M, \Omega_M)$ is the DG-algebra $\Omega_M \otimes WU_n$ over Ω_M, where

$$d(1 \otimes c_i) = 0 \quad d(1 \otimes h_i) = 1 \otimes c_i - p_{i/2} \otimes 1$$

where $p_{i/2}$ is zero if i is odd and is a form representing the Pontrjagin class of M of degree $2i$ if i is even.

Note that if a foliation F on $X \times M$ transverse to the fibers $\{x\} \times M$ is given, one has a characteristic homomorphism

$$C^*(L_M, \Omega_M) \to \Omega_{X \times M}$$

One has also a morphism

$$WO_n \to C^*_\triangle(L_M, \Omega_M)$$

(or $WU_n \to C^*(L_M, \Omega_M)$ in case M has trivial Pontrjagin classes) whose composition with the previous one is the usual characteristic homomorphism for the foliation F (cf. [3], [12]).

4. MAIN THEOREM

THEOREM 1. *$C^*(L_M)$ is a model for the space Γ of continuous sections of the bundle E described in the theorem above.*

This result, first conjectured by Bott (and also Fuks), has been proved by several people (Bott-Segal [1]), Fuks-Segal, Haefliger [13], Ph. Trauber, and others).

Suppose that G is a compact connected Lie group acting on M. Then it also acts on the bundle E and on its space of sections. Let us denote by Γ_G the total space of the bundle with fiber Γ associated to the universal G-bundle with base space BG.

THEOREM 1'. *$C^*(L_M; G)$ is a model for the space Γ_G.*

The way I proved theorem 1 was to construct first a tentative algebraic model A for Γ following ideas of R. Thom [20] and D. Sullivan [18], and

[1]) Added on proof: *Topology 16* (1977), pp. 285-298.

a morphism of A in $C^*(L_M)$. Then one proves directly that it induces an isomorphism in cohomology. The fact that A is also a model for Γ was proved in a similar way (cf. [14]).

When M has a finite dimensional model, one can construct a model for Γ which is finite dimensional in each degree, and with it one can make explicit calculations.

Note that the inclusion $C^*_{\triangle}(L_M, \Omega_M) \to C^*(L_M, \Omega_M)$ is a model for the evaluation map $\Gamma \times M \to E$ associating to a section s and a point x of M the element $s(x)$ of E.

For computations along the lines of the spectral sequence of Gelfand-Fuks, see Cohen and Taylor [22].

The proof of theorem 1′ is very similar to the proof of theorem 1. In the next paragraph, we explain the construction of an algebraic model for Γ_G suitable for computations. In § 6, we indicate briefly why this is a model for Γ_G.

5. Construction of an algebraic model for the space of sections of a fiber bundle ([20], [18], [13]).

As a guide, consider first the geometric situation. Let $p: E \to M$ be a fiber bundle with base space M, fiber F and let Γ be the space of continuous sections of E.

We have the commutative diagramm

1)
$$\begin{tikzcd}
M \times \Gamma \arrow[rr, "e"] \arrow[d] \arrow[dr] & & E \arrow[dl, "p"] \\
\Gamma \arrow[dr] & M \arrow[d] & \\
& * &
\end{tikzcd}$$

where e is the evaluation map associating to the point x of M and the section s the point $s(x)$ of E. The other maps are natural projections ($*$ is a point).