cannot have more than $c' \sigma \log \sigma$ zeros in a disc $|z| \leqslant c''$, where c', c'' depend only on p. We are indebted for the above details to D. W. Masser (for a problem involving zeros of polynomials in several variables see his [13]).

To extend our method to a class of functions wider than that given by (44) is practical provided only that one can usefully estimate the determinants arising in lemma 2. This can certainly be done in the case

$$F(z) = \sum_{h=1}^{m} \sum_{t=1}^{\rho(h)} a_{ht} (\log z)^{t-1} z^{\alpha_h},$$

for details see van der Poorten [22]. A similar argument should allow one to deal with functions

$$\sum_{h=1}^{\sigma} b_h f_{\mu_h}(z),$$

where f_μ is given by (45); now lemma 5 allows one to consider rather surprising functions. There are further, rather isolated cases where one can deal with the determinants; for some examples, and further references see [21].

REFERENCES

[1] Balkema A. A. and R. Tijdeman. Some estimates in the theory of exponential sums. *Acta Math. Acad. Sci. Hungar. 24* (1973), pp. 115-133.

[2] Brownawell, D. Gel'fond's method for algebraic independence. *Trans. Amer. Math. Soc. 205* (1975), pp. 1-26.

[3] Cudnovskii, G. V. Algebraic independence of some values of the exponential function. *Mat. Zametki 15* (1974), pp. 661-672 = *Math. Notes 15* (1974), pp. 391-398.

[4] Cijsouw, P. L. On the simultaneous approximation of certain numbers. *Duke Math. J. 42* (1975), pp. 249-257.

[5] Cijsouw P. and R. Tijdeman. An auxiliary result in the theory of transcendental numbers II. *Duke Math. J. 42* (1975), pp. 239-247.

[6] Dickson, D. G. Asymptotic distribution of zeros of exponential sums. *Publ. Math. Debrecen 11* (1964), pp. 295-300.

[7] Gel'fond, A. O. *Transcendental and algebraic numbers.* Dover New York, 1960.

[8] Hayman, W. K. Differential inequalities and local valency. *Pacific J. of Maths. 44* (1973), pp. 117-137.

[9] Knuth, D. E. *Surreal Numbers* (how two ex-students turned onto pure mathematics and found total happiness) Ad-Wess., 1975.

[10] Mahler, K. On a class of entire functions. *Acta Math. Acad. Sci. Hungar 18* (1967) pp. 83-86.

[11] Makai, E. The first main theorem of P. Turán. *Acta Math. Acad. Sci. Hungar 10* (1959), pp. 405-411.

[12] —— On a minimum problem. *Ann. Univ. Sci. Budapes . Eötvös Sect. Math. 3-4* (1960-61), pp. 177-182.

[13] Masser, D. W. *Elliptic functions and transcendence.* Lecture Notes in Math. 437 Springer Verlag.

[14] Muir, T. *A treatise on the theory of determinants.* Dover, New York, 1960.

[15] PÓLYA, G. Geometrisches über the Verteilung der Nullstellen gewisser ganzer transzendenter Funktionen. *Münchener Sitzungsberichte 50* (1920), pp. 285-290.
[16] VAN DER POORTEN, A. J. *Simultaneous algebraic approximation of functions.* Ph. D. thesis, University of N.S.W., Sydney, Australia, 1968.
[17] —— Generalisations of Turan's main theorems on lower bounds for sums of powers. *Bull. Austral. Math. Soc. 2* (1970), pp. 15-38.
[18] —— A generalization of Turan's main theorems to binomials and logarithms. *Bull. Austral. Math. Soc. 2* (1970), pp. 183-196.
[19] —— Perfect approximation of functions. *Bull. Austral. Math. Soc. 5* (1971), pp. 117-126.
[20] —— Generalizing Turan's main theorems on lower bounds for sums of powers. *Acta Math. Acad. Sci. Hungar 24* (1973), pp. 93-96.
[21] —— Some determinants that should be better known. *J. Austral. Math. Soc. 21* (1976), pp. 278-288.
[22] —— On the distribution of zeros of exponential polynomials. *Math. Slovaca. (to appear).*
[23] —— Zeros of exponential polynomials. *Indag. Math. 38* (1976) pp. 46-49.
[24] —— Hermite interpolation and p-adic exponential polynomials. *J. Austral. Math. Soc. 22* (1977).
[25] SHOREY ,T. N. P-adic analogue of a theorem of Tijdeman and its application. *Indag. Math 34* (1972), pp. 436-442.
[26] TIJDEMAN, R. *On the distribution of the values of certain functions.* Ph.D Thesis, Universiteit van Amsterdam, 1969.
[27] —— On the number of zeros of general exponential polynomials. *Indag. Math 33* (1971), pp. 1-7.
[28] —— On the algebraic independence of certain numbers. *Indag. Math 33* (1971), pp. 146-162.
[29] —— An auxiliary result in the theory of transcendental numbers, *J. Numb. Theory 5* (1973), pp. 80-94.
[30] TURÁN, P. *Eine neue Methode in der Analysis und deren Anwendungen.* Adadémiai Kiadó Budapest, 1953.
[31] VOORHOEVE, M. On the oscillation of exponential polynomials. *(to appear).*
[32] —— and A. J. VAN DER POORTEN. Wronskian determinants and the zeros of certain functions. *Indag. Math. 37* (1975), pp. 417-424.
[33] —— A. J. VAN DER POORTEN and R. TIJDEMAN. On the number of zeros of certain functions. *Indag. Math 37* (1975), pp. 407-416.
[34] WALDSCHMIDT, M. Propriétés arithmétiques des valeurs de fonctions méromorphes algébriquement indépendantes. *Acta Arith 23* (1973), pp. 19-88.
[35] —— Indépendance algébrique par la méthode de G. V. Cudnovskij. *Sém. Delange-Pisot-Poitou* (Groupe d'étude de théorie des nombres) 16e année 1974/75 no G8, 18 pp.
[36] —— *Nombres Transcendants.* Lecture Notes in Math., 402 Springer Verlag.

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