2-dimensional S-stable subspace W_1 of V, and by the preceding argument $W_1{}^1$ is also S-stable. Thus V is the orthogonal sum of 2-dimensional S-stable subspaces $W_1, ..., W_r$. Let y_i, z_i be an orthonormal basis for W_i, then relative to the basis $y_1, z_1, ..., y_r, z_r$ the matrix of each A in S is block diagonal and the blocks are positive multiples of 2×2 rotation matrices. Thus we have determined the structure of real normal operators.

Finally, return to Proposition 2 and suppose that $y_1, ..., y_n$ is a fan (flag) basis for an S-stable fan (flag) and that E has an inner product. The Gram-Schmidt process applied to this basis yields an orthonormal fan (flag) basis $z_1, ..., z_n$ for the same S-stable fan (flag). Thus the matrix P in the Corollary can be taken to be unitary or real orthogonal. Moreover, if S contains A and A^* for some A (hence A is normal) we get directly that the matrices of A and A^* relative to the basis $z_1, ..., z_n$ are both (nearly) super-diagonal, and since one is the adjoint of the other, they are both diagonal (block diagonal with blocks at most 2×2 in size). This observation could be used to give another proof for Proposition 4 and the structure of real normal operators. In either case, the argument can be simplified a bit if S is assumed to be *-closed.

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