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COMMISSION INTERNATIONALE
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A COMPARATIVE STUDY OF METHODS OF INITIATION INTO GEOMETRY

Report submitted on behalf of the International Commission on Mathematical Instruction (I.C.M.I.) at the International Congress of Mathematicians held at Edinburgh, 1958

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(Reçu le 28 janvier 1959.)

1. At its Geneva meeting on July 2, 1955, the Executive Committee of the I.C.M.I. adopted a working plan consisting of three subjects to be studied by the national subcommittees and to be discussed at the Edinburgh Congress. This working plan was communicated to the national committees by circular letters in August 1955, again in January 1956, and in August 1957. On the strength of our experience in the Netherlands I venture to claim that a working period of three years is imperative for projects such as the ones we adopted in Geneva three years ago. I have the impression that in some countries the delegates or committees that had to report on the third subject, had not been designated before the end of 1957. In one case they were informed as late as March 1958 that they were expected to report on that subject. It is to be regretted that apparently the national committees did not sufficiently appreciate the difficulty of the task to be fulfilled. Most of the national reporters have suffered from a serious lack of time which could easily have been avoided. This explains and excuses the national reporters who finally gave excellent reports though the time available was too short for more detailed expositions.

National reports on the third theme of the I.C.M.I. have been submitted to the general reporter by ten countries: Belgium, Canada, Finland, Germany (Federal Republic), Italy, Japan, the Netherlands, Poland, U.S.A., and Yugoslavia [1]). Most of these reports cover only a few pages. The German, the Polish and the Yugoslavian reports are one printed sheet long. The report of the Netherlands Subcommittee has been printed; it contains 120 pages. Of course it should not be concluded from this data that Dutch educators can tell more about teaching geometry than those of other countries. Our whole secret is that we started our work as early as 1955 and we were therefore able to take advantage of the whole three year period.

2. As a project of international educational collaboration the third subject of the I.C.M.I. is rather unusual. From the beginning of this century programmes of mathematical instruction have often been discussed on the international level. In 1955 at the Geneva meeting a few of us strongly advocated the need of an international exchange of experiences in the field of teaching methods. On the strength of our arguments the third subject has been adopted. Nevertheless the title of the subject seems to have been open to misinterpretations. Some reporters gave an account of *programmes* of geometrical instruction. If possible in the time still available these reports have been replaced by accounts on teaching methods as desired by the I.C.M.I. I hope that in the new four year period the cooperation will be better so that reporters will know what they are expected to do. It would be wise to add detailed instructions to the new themes.

You may have the impression that I am somewhat disappointed. In a certain sense this is true. All national reports, even the shortest of them, contain so many extremely valuable details that I greatly regret that they could not have been longer. I am convinced that educators of the world can learn a good many things from the experiences of their colleagues. This conviction

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1) It is a pity that owing to personnel circumstances no report on the present subject has been delivered by the Committee of the United Kingdom. I could, however, draw some information from two reports of the Mathematical Association of 1922 (ed. of 1956) and of 1937 (ed. of 1957) on " The Teaching of Geometry in Schools ".

has been confirmed by the reports I have studied when preparing this general report. I hope all national reports will be printed and made accessible to a broader public. It will be a good thing to proceed in this manner. In all sciences we take advantage of worldwide experiences. The exchange of teaching experiences should not be hampered by political or linguistic frontiers.

3. Comparative studies in education have to account for a large diversity of educational systems which is caused and maintained by different opinions, in the past and nowadays, about the social task of the school. This might be illustrated by one, particularly striking example. Nearly the same kind of geometry (mainly mensuration) is taught in Canada to 15 or 16 year olds [2]) as in Germany and Poland to 10 or 11 year olds. This is not a reason for one country to boast and for the other to feel inferior. In European countries children are given a widely diverging amount of intellectual education according to the school they attend. In Western Europe the percentage of youths who are educated on the highest intellectual level, though ever increasing, is still very small; in the countries of Eastern Europe it is much higher. In U.S.A., Canada and Japan, and in some other extra-European countries education is more uniform. Intellectual people in Western Europe are usually proud of the level of their education which is the same as that of their children. They are exposed to the danger of disregarding all attempts of giving a broader part of the youth an education that is not equivalent to the highest level of European education. When European people speak of mathematical education, they will often be inclined to attach the highest importance to that kind of school which prepares students for scientific training. The majority of youths will not receive the educational attention they deserve. So the large part played by prescientific education in the European reports is not surprising. Nevertheless I am convinced that the aims of teaching geometry, as seen by educators all over the world, are essentially the same. All educators

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[2]) This statement was based on a geometry course for 15-16 olds that was added to the Canadian report. The Canadian reporters inform me that this course does not reflect geometrical instruction in Canada. Deductive geometry starts with the 14-15 agr group.

will admit that in our world operational and creative skill is more valuable than a stock of permanent knowledge. They will prefer teaching children the principles of comparing areas to telling them Pythagoras' theorem. They will judge it more important, if children can find the area of a circle by rational means than that they know by heart that π equals $3\frac{1}{7}$. From Greek times onward geometry has proven its value not as a sum of disconnected experiences, but as a deductive system of knowledge. Teachers will try to develop the sense of scientific geometry as much as possible in the minds of their pupils. Teachers will not give the same answer to the question why children should be taught deductive geometry, but all will be convinced of its educative value. Probably they will differ as to the age t_0 at which deductive geometry can and should be taught, and on the initial conditions of training which must be fulfilled, so as to make the teaching of deductive geometry fruitful.

4. I have the impression that in the U.S.A. teaching practice the point t_0 is fixed at the age of fifteen (10th grade). Japanese education seems to share this opinion, though our Japanese reporter advocates an earlier introduction of deductive methods. In Canada the point t_0 seems to lie still higher. The Canadian report contains a mensuration course for 15 or 16 year olds, as an introduction to deductive geometry. Among the European countries Italy fixes t_0 as late as the age of fourteen, but the reporter remarks that during the preparatory period, which precedes this point, the pupil is gradually led or should be led to adopting the deductive method.

In most European countries the traditional t_0 is the age of twelve (7th grade), but I doubt whether this tradition is as old as people usually think, for not until the end of the last century did mathematics become an important teaching subject. European educators will be inclined to divide youth in two groups, the one for which $t_0 = 15$ is much too high, and the other for which no t_0 exists, i.e. which never reaches the maturity for deductive geometry. They will refuse to delay deductive geometry on behalf of the second group. On the other hand educators who are inclined to question traditional opinions, will often decide that twelve years is too early to teach deductive geometry.

5. It is much more important to know which initial conditions of training must be fulfilled at the time point t_0.

For many years the answer to this question has been very simple in European educational practice. Geometry is a rational science, so it can be taught to children as soon as they have matured into rational beings. Definitions, axioms, theorems, and proofs were engraved into the mental tabula rasa of children who did not grasp the meaning and the aim of the deductive method. Euclidean rigour has been the principle of teaching geometry right from the beginning, but neither the authors of textbooks nor the teachers realized that the hotchpotch of definitions, axioms, theorems and proofs they dealt with in the first chapters of geometry did not at all correspond to the exalted ideal of mathematical rigour. The results have been disappointing, but there are still many teachers who oppose new methods. In Belgium the traditional method of beginning geometrical studies has been officially abjured. The greater part of the teachers have adopted a new point of view.

In the Netherlands the most progressive teachers have succeeded in convincing the government of the need of new teaching methods. The reader of the Netherlands report will notice that the struggle against the traditional system is still far from finished in our country. The adoption of new methods will be a rather slow process, in which new textbooks will play a decisive role. In our country the government has not the right to prescribe or to forbid text-books.

In Belgium and in the Netherlands the system of confronting the pupil in the first grade of secondary schools with Euclid has been particularly bad, as in these countries no geometry whatsoever is taught in the primary schools (apart from computations of areas of rectangles and of volumes of rectangular parallelepipeds). In Germany there is a long tradition of " Raumlehre ", preparatory geometry in the 5th and 6th grade, but it seems that this subject seriously suffers from the indifference of teachers who maintain that there is no time available for

" Raumlehre " and from the opposition of those who object to any kind of mathematical preparations. England has a very short period of preparatory geometry. In Italy three years of preparatory geometry precede the deductive geometry course, but nevertheless the reporter complains that the 14 year olds are struggling to understand the notions of the deductive system with the same difficulties as the 12 year olds in Belgium and Holland. There is a preparatory course, but the teacher of the deductive course refuses to derive advantage from its results. (From this experience we can learn that the value of a preparatory course does not consist in its existence, but in the fact that its results will be used and can be used as initial conditions for a higher course. We shall come back to this important point.) The same complaint about discontinuities between the different levels of geometrical instruction is also heard in the Yugoslavian report, though it does not specify at what age the pupil will pass from the preparatory to the deductive course. A high degree of continuity is met in the Polish system; in the 6th and 7th grade geometrical instruction develops gradually from a preparatory into a deductive course. U.S.A. and Japan have a long lasting preparatory course covering the 7th, 8th and 9th years. I am not sure, however, whether this course is really preparatory. A small minority of youth continue with deductive geometry in the 10th grade. It is therefore probable that the geometrical instruction in the preceding grades is not directed to the goal of creating favourable initial conditions for teaching deductive geometry. Canada has a mensuration course in the 10th or 11th grade in which a few preparatory elements can be found [3]).

6. All modern educators agree that geometrical instruction cannot start with an exposition of the deductive system. Teaching deductive grometry may be the first intermediate aim of geometrical instruction, but the foundations must be laid in a preparatory course. As to the length, the depth and the material contents of this course there is a large diversity of opinions. In the course sketched by some contributors to the

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3) See the note to p. 3.

Netherlands report the deductive level is reached in a few months, in the more individual system of other Dutch contributors the duration of the introductory period would depend on the ability of the pupil. I am sure they would set two years as a limit — after two years it would be evident whether a child could switch over to the deductive course. The longest preparatory course, that of three years, is used in the American and in the Japanese system.

All educators agree that one of the initial conditions to be fulfilled at the time t_0 is a conscious acquaintance with the intuitive properties of concrete space and of the figures in it. Yet the actual interpretations of this demand cover a wide range. Some contributors to the Netherlands report deny the need of special measures. They hold that the stock of incidental spatial experiences of a twelve year old is broad enough as a basis for geometry. Obviously they identify geometry with plane geometry, but even under this restriction other teachers will deny the sufficiency of the incidental experiences of the twelve year olds. They will point out that often children and even grown-up people who have learnt mathematics do not know what are congruent and similar figures other than triangles, and that they cannot discern central and axial symmetries in plane figures. But the first group of teachers will not yield to this argument because in a more or less classical course of geometry symmetries or other geometrical transformations and general congruency and similarity do not play any essential part. So the desirable extent of intuitive properties of concrete space known to the pupil at t_0 will depend on the pattern of deductive geometry the teacher has in mind.

The question whether the pupil should be acquainted with three-dimensional space as a substrate for solid geometry at a very early stage is crucial. It will not arise if the teacher is inclined to switch over from preparatory geometry to a classical pattern of deductive geometry as soon as possible. Therefore some of the Dutch contributors do not mention solid geometry at all. On the other hand in three contributions to the Dutch report solid geometry plays an even more important part than plane geometry. Acquaintance with space is the leading idea of the preparatory courses of Mrs. Ehrenfest and of Van Albada. The latter goes as far as to deal with perspective, shadow constructions, and descriptive geometry besides solids and making models. Mrs. and Mr. van Hiele pay more attention to preparing the deductive structure, but nevertheless two thirds of the time available in the first year is devoted to solid geometry. Modern Belgian methods lay strong stress on solid geometry, particularly on making models. Sections of solid bodies are used as the most natural means of introducing plane figures and their relations (congruency and similarity). German educators often plead for the need of a " fusion " of plane and solid geometry, but it is doubtful to what degree this fusion has been realized. The Italian reporter does not mention solid geometry. Also in the consulted English report little attention is paid to it. The Polish reporter says to hesitate with regard to solid geometry, he raises objections, but nevertheless solid geometry occupies a rather important place in his preparatory geometry. The importance of solid geometry is also stressed in the Yugoslavian report. Solid geometry comes rather late in the American preparatory programmes; it seems to occupy a place of minor importance. The same seems to be true in the Japanese systems. No solid geometry is mentioned in the Canadian report.

The problem of solid geometry should be seriously reconsidered by all those who are interested in teaching geometry. Some teachers hold that early acquaintance with solid geometry is the best preventive against the usual difficulties experienced by many children when deductive solid geometry starts. They are afraid of exclusive plane geometry killing spatial imagination.

7. Acquaintance with manual techniques will improve acquaintance with space, but it has also merits of its own, and it may also be considered as one of the initial conditions to be fulfilled at t_0. Some Dutch contributors pay little attention to these techniques. Others stress their importance, but they do not go further than teaching the technique of using the usual instruments (ruler, compasses, and so on) and of drawing and constructing. Three contributors teach a variety of techniques, such as cutting, matching, making models, paperfolding, measuring, and so on. The same or even more stress is laid on manual

techniques, particularly on making geometrical and mechanical models in the aforesaid Belgian school. A strong collaboration between the teachers of mathematics and of handicraft is characteristic of Belgian instruction. It is not surprising that in Germany, which is the cradle of the " Arbeitsunterricht ", much attention is paid to teaching a variety of geometrical techniques. In Poland and Yugoslavia they occupy an important place in the programme, though in Poland drawing techniques prevail. In Italy, England and Canada the techniques to be taught are mainly restricted to drawing and constructing. In U.S.A. and Japan we meet again with a rich variety of techniques to be taught. In U.S.A. handicraft and mathematics are strongly related.

8. The question may be raised to what degree applied geometry should be incorporated into a preparatory course. When I say " applied geometry ", I do not mean a series of computations of volumes, in which the words rectangle and parallelepiped are replaced by " garden " and " swimming pool ", or a series of exercises on triangles which are supposed to contain towers or to cross rivers. I shall speak of applied geometry, if real problems are to be solved by geometrical means. Real problems can considerably improve the conditions of transfer of training. They can also be a powerful means of motivation. It is, however, not easy to find real problems, that is to say problems which, in a given classroom situation, ask for a solution. The project method is an excellent idea, but I never found reports on projects in which geometry was integrated strongly enough and at the same time on a sufficiently high level. In one Dutch system of lower technical education (cp. a contribution by Krooshof in our report), in each phase of the instruction all subjects are centred around one piece of handicraft made by the pupil. But the relation between the piece of work and the mathematical subject is often too loose or too artificial.

The need of stronger relations between mathematics and the other teaching subjects is vividly felt by some reporters, especially by the Yugoslavians. Valuable proposals are found in the famous " Übungenbuch " by Mrs. Ehrenfest-Afanassjewa and in several reports, e.g. in van Albada's contribution to the Dutch report, and in the American and Japanese report. Outdoor activities can be stimulating. Handicraft can be a rich source of real geometrical problems. This is perhaps less true in the Belgian system, where handicraft is closely knitted with mathematics and making models prevails, than in U.S.A., where handicraft is selfconsistent. In the American report an instruction paper for making a school transit was included. Such a piece of work, if directed by a good teacher, can be a rich source of applied mathematics. (Note that this might be less true in the European system, where children in the preparatory phase are too young to make pieces of handicraft like those mentioned above.)

9. We have already pointed out the influence which a prospective deductive course can have on the preceding preparatory course. The teacher will be reluctant to adopt a preparatory theory of parallels based on the existence of rectangles, or of similarity, as long as he believes that the Euclid-Hilbert definition of parallels as non-intersecting lines and the Euclid-Hilbert form of the axiom of parallel lines is the only possible one in a deductive course at school. It has been one of the merits of K. Fladt to point out that the Euclid-Hilbert approach is less suitable for initiation into geometry. Nevertheless I have the impression that this approach still prevails in mathematical instruction.

Another example is the use of geometrical transformations, first advocated by F. Klein as a consequence of his so-called " Erlanger Program " [4]), but still far from being generally adopted, even in Germany, so it seems. In the higher forms of some Belgian schools geometrical transformations, illustrated by a rich variety of mobile models, are a substantial part of the subject matter, more substantial than in any other country, as far as I know. But it is quite another thing to teach transformations or to make them the fundamentals of geometry. In the initiating phase the Polish program shows the strongest influence of the " Erlanger Program ". The Polish experiences are

4) The teaching problem of " Erlanger Program " is quite differently viewed by Professor Servais. As it is not possible to incorporate his profound remarks, they will be annexed to this report.

diametrically opposed to those expounded by the German reporter. I shall come back to this point.

How to explain this failure of " Erlanger Program " in school geometry ? One of the causes is the badly-understood authority of Hilbert's " Grundlagen der Geometrie ", which can be said to have lengthened the life of Euclid's methods for half a century. There is still another point. " Erlanger Program " has been used as a slogan, but the problem of transformation geometry in mathematical instruction has not been seriously enough faced. There are text-books in which transformations are explained, but the basis is always classical. There does not exist any systematic course of school geometry based on the transformation idea. Even the problem of how to introduce and how to teach transformations, has not received due attention. It is perhaps worthwhile to expound the cardinal problem of teaching geometrical transformations. There is a danger that a child understands that transformation is just picking up a figure and laying it down elsewhere, without changing its shape or applying some similarity or affinity. Of course this is a serious misapprehension. Free mobility of figures is much less than geometrical transformation. Geometrical transformation means that the whole plane (or the whole space) is picked up and put down elsewhere. Free mobility of figures is a much more intuitive notion than geometrical transformation. The less intuitive notion is very likely blocked by the more intuitive one, especially if moving models are used, as it is done in many Belgian schools. This is not a mere hypothesis of mine, but a real danger. I have seen textbooks, in which that fundamental mistake has been made by the author himself and it is continuously being made in Piaget's work. Of course one cannot arrive at a consistent notion of transformation groups, if one starts from " mobility of figures " instead of " geometrical transformation ".

This is the fundamental problem of teaching geometrical transformation: how to fight against " free mobility " [5]). I have

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[5]) This does not mean that " free mobility of figures " is bad as a teaching subject. I only assert that in an instruction system based on " Erlanger Program " we shall have to fight against it. But I do not claim that geometrical instruction must be based on " Erlanger Program ".

found too few indications in the literature that this difficulty has clearly been realized. So I can very well understand the sceptical attitude towards transformations and the " Erlanger Program " as a subject matter and as a basis of geometrical instruction.

Nevertheless I think there is some hope left. There is one non-trivial geometrical transformation that is immediately seen as a transformation of the whole plane, not as the movement of a figure in the plane. That transformation is axial symmetry. Central symmetry and rotation are much more difficult; translations are the most difficult cases. If the teacher starts with symmetry and if rotations and translations are introduced as products of symmetries, there is a real chance, I believe, that the child will grasp the notion of transformation, even in the initiating phase.

This is exactly what happens in preparatory geometry according to the Polish report starting with symmetries and dealing with rotations and translations as generated by symmetries. (6th grade.) The lengthy digression on mobility, transformations, and " Erlanger Program " you have listened to, was meant as a plea for the Polish view. The Polish argument is different. Too much stress is laid on the idea of mobility of figures, and symmetry is preferred to other transformations because it does not depend on the notion of parallel lines and because symmetries produce the whole group of plane movements. In my opinion this argument is less decisive. I believe that symmetry is the didactic key of transformation geometry, because it exhibits a transformation of the whole plane.

I have mentioned that the point of view of the German reporter is diametrically opposed to that of the Polish reporter. The German reporter has carefully analyzed the situation of teaching symmetry. He has indicated some difficulty and a way out of it, a way which I cannot explain in a small compass. In any case he prefers starting with translations. In view of our former exposition it is extremely instructive to see how he proposes to introduce translations (5th grade): by means of the square lattice (squared paper seems to be a rather popular device in German geometrical instruction). I share this view. I believe that the only way to introduce translations early is to use the square lattice. Not however because the pupil is familiar with it, but because it is the most (perhaps the only) natural *infinite* figure. Free mobility of the square lattice can suggest translation or rotation over the whole plane. So it might be a device that prevents the wrong reduction of " geometrical transformation " to " free mobility " of a single figure. Nevertheless I think that there are some other reasons for preferring symmetry to translation as a starting point.

Symmetries are more interesting than translations and rotations. To a young child congruent figures are the same. The child will not hit upon the idea that something has happened if a figure is carried to another place. To an unsophisticated mind movement is not a transformation. In this regard rotation is somewhat better than translation. If a cube is translated, nothing has happened; if it is turned and put upon an edge or a corner, something has been changed. But mirror reflection gives the strongest feeling of an important event. Symmetry as a transformation is more attractive, more abundant, and more problematic than translation and rotation. So one can understand why it appears in nearly all reports, and why its usefulness as a teaching subject is often stressed. Symmetry has even been integrated in some rather classical systems as exposed in contributions to the Dutch report. There is already a rich abundancy of examples how to teach symmetry and how to use it. Nevertheless this theme is far from exhausted. In more recent literature one is often struck by the many new versions. In the Dutch report symmetry plays an important part in the contributions of van Albada and of the van Hieles. In the first case stress is laid on acquaintance with space, in the second case it is subordinated to the general aim of preparing deductive geometry. As mentioned above in the Polish report symmetry is even the base of congruence. It should be added that in the Polish system homothety is the other pillar on which transformation geometry rests.

10. I shall deal with two major subjects proposed for initiating into geometry: " square lattices " and " paving a floor with congruent tiles ". Square lattices have drawn the special attention of German teachers. (Cp. a paper of M. Enders in

"Der Mathematikunterricht 1955, 29-76".) Areas, parallelograms, proportions, similarity and other transformations, and coordinates are taught while using systematically squared paper. No doubt the square lattice, if used with not too many pretentions, will be a valuable device. Compared to symmetry it is too poor to derive a substantial part of preparatory geometry from it. In any case it is too rigid. The plane is analyzed here, and structurated by means of a fixed system of horizontal and vertical lines. Such a procedure is artificial. It does not match the attitude of synthetic geometry. It can lead the pupils the wrong way. Getting rid of this rigid substrate of the plane can become a difficult task.

Gattegno's geoplan is much better. This is a square lattice of nails on a wooden plate without joining lines drawn. On this plate figures are constructed with elastic strings. This is a more flexible and quicker method.

Paving is mentioned in the Dutch report only, though as a topic of instruction it has a rather long history. It was used by E. Borel in his booklets, and a few Belgian teachers have studied it. In the Dutch report it is one of the chapters of Van Albada's method. It has thoroughly been explored by Mrs. van Hiele. A detailed report on an experiment of teaching this subject during one trimester is to be found in her thesis. The children are given bags containing different kind of congruent cardboard polygones. They have to cover a portion of the plane with them and to copy these patterns by drawing. Parallelism, sum of the angles of a polygon, similarities, congruencies and some transformations arise in a quite natural way from working in this field. The paving patterns show a rich variety of relations. The children themselves ask why some relations hold and other relations do not hold, and they discover the logical linking of geometrical relations.

11. From teaching subjects we shall now pass to teaching methods. There is a general agreement that the first phase of teaching geometry must be concrete and intuitive. The abstract approach has unanimously been condemned. Yet the interpretations of what is concrete and intuitive differ widely. The demand for concreteness combined with modern psychological

ideas would suggest starting with global structures which should be gradually differentiated and refined, and not the reverse way, which is classical, of starting with the elements in order to build up gradually the global structures. Nevertheless the classical start " point, line, surface, body " is not yet out of use. An old-fashionned subject such as the generation of a line by a moving point, of a surface by a moving line, and so on, is even recommended in the otherwise rather progressive official Belgian programme [6]). Often concreteness and intuitiveness are treated as synonymous with the handling of rulers and compasses, e.g. as in the official Italian programme. Others are of the opinion that drawing is too narrow a base for intuitive geometry, and that it is too closely related to a rather abstract and sophisticated image of perception space. They use solid and mechanical models, global patterns like pavements, outdoor observations and so on. Still others argue that showing models is not enough. Not only the used material, but also the relation of the pupil to the material must be concrete in the sense that as long as concrete material is used it should be handled by the pupils themselves. This is said in various ways in the Belgian, German, Polish and Yugoslavian reports. From the Netherlands report van Hiele's intention " to give the pupil concrete material that he can handle " may be cited. In the opinions of the American and Japanese reporters this is a more or less self-evident attitude. Card-board, scissors, glue, adhesive tape, plexiglass, Meccano pieces, knitting needles have become or will become as powerful means of concrete geometrical expression as rulers and compasses have been in the past. Active learning is met with criticism by the Italian reporter, but it is evident that the caricature of active learning he paints does not aim at more serious procedures.

I would like to add a few words about the so-called experimental method in preparatory geometry. As long as experimenting simply means trying, there is no need for further observations. Sometimes, however, experimentation in geometry is

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[6]) Professor Servais informs me that the programme does not recommend verbalizing the dynamical generation of geometrical entities, but rather an active dynamical approach that matches the psychological dynamism of the child.

understood in the same sense as it is taken in the properly experimental sciences. In the Dutch report, Mrs. Ehrenfest and the van Hiele's, though advocating a concrete approach, turn against this interpretation. Measuring the perimeter of a circle or the volume of a pyramid or the sum of the angles of triangles might serve to illustrate the notions of the perimeter of a figure, volume of a body, sum of angles of a polygon. But if the problem is faced, how to approximate π, how to find the formula for the volume of a pyramid, how to make sure that the angles of a triangle are 180° together, the teaching value of this method is small or even negative. It is just the aim of the preparatory course to block this kind of approach.

From Mrs. van Hiele's experiment it appears that the child himself tends rather early not to rely upon the method of experimental science. When paving a " floor " with one kind of figure, he will finish the manual activity of fitting the pieces as soon as he has grasped the general pattern of the floor. It is natural for him to disregard bad fittings caused by incongruencies of the used material. If this stage is reached, it would be unwise to have the child fall back on more primitive methods.

12. There is a general agreement that children can learn the names of geometrical objects and of relations in the preparatory phase. Teaching names can be purely ostensive or more or less explanatory. (This is a rectangle — or — a rectangle is . . .) If the approach is concrete enough, the child will learn the names of even complicated geometrical objects and relations (regular polygon, similarity) in the same way as he formerly learnt the names of persons, animals, things, activities, and so on. In a less concrete approach the teacher will explain the meaning of the names. The first method has the advantage that after the children have grasped the meaning of some word, they can try to find verbal explanations on their own. Examples of this procedure are to be found in the mentioned thesis of Mrs. van Hiele. There is a rather general agreement that such explanations should not be learnt by heart. The teacher can check in a concrete situation whether a child knows the meaning of a name. Yet without doubt it is one of the

goals of preparatory geometry, that children can explain the words they are using, not by showing the related objects, but by verbalizing their properties. Some teachers go even further in the initiating phase: they are teaching and asking formal definitions, which is much more than informal explanations. This difference is stressed in several reports, especially in the Polish report and in van Hiele's contribution to the Dutch report. A child can explain a rhomb as a figure having four equal and parallel sides, orthogonal diagonals, bisecting diagonals, and halving diagonals, thus summing up all the properties of the rhomb he hits upon. The feature that a rhomb can be defined by a part of these properties, is a feature of deductive, not of preparatory geometry. A child cannot grasp the sense of definitions if it has not grasped the interrelationship of properties and the possibility to derive one property from another. Thus formulating definitions testifies to a rather high level of learning geometry. According to the van Hieles the level of being able to define what " definition " means, is still higher.

It is, however, possible, even in the initiating phase, to arrive at economical definitions. Often the so-called genetic definitions, advocated by the Polish reporters will be economical. A thing will be defined by telling how it can be made. I think that this procedure is rather dangerous. A thing can be actualized in many ways, whereas in a deductive system a thing is defined in a unique way. Furthermore if children are working with ruler and compasses congruent triangles are genetically defined by the equality of three elements, not of six. This misunderstanding can be avoided only by laying strong stress on congruency of figures other than triangles.

It goes without saying that already in the initiating phase theorems will be formulated. It does not matter whether the word " theorem " is used or not. It is quite another thing with the word axiom. Many teachers and reporters refuse to use it, and most of them do not even discuss the possibility of using it. A few wish to use axiom in the sense of " selfevident truth ". There is little to object as regards this usage, though it is not the modern meaning of the word, and though the child will meet with numerous self-evident propositions that are called theorems and not axioms. The use of the word " axiom " will be a rather harmless verbalism.

Various reporters insist on formulating theorems not in the hypothetical, but in the assertive mode. In some Dutch methods the implication arrow is systematically used. I think this must be preferred to artificial reductions of the linguistic level.

It is generally admitted that in the preparatory course selfevident truth will be adopted without any argumentation. The equality of opposite angles, proved in many textbooks, is such a selfevident truth. One of the contributors to the Dutch report, Vredenduin, has carefully accounted for the theorems which are adopted at sight.

It is also generally agreed that no theorems should be proved until the pupils feel the need of proving theorems. The habit of proving theorems must be gradually developed, while the concrete basis may not be left. This point has not been treated in detail in the different reports, except in the Dutch report. I cannot recapitulate the developments towards the deductive system, as sketched by the individual contributors. The most detailed exposition is to be found in Mrs. van Hiele's thesis. From her experiment and the theoretical work of both van Hieles it becomes clear that the acquisition of the logical faculties in learning geometry is a more complicated and less continuous process than it was usually thought to be. We shall come back to this point.

13. In the first paragraphs of this report I spoke of a moment t_0 at which the deductive phase of geometrical instruction starts. So I may expect the objection that the deductive attitude of the pupil develops so gradually that no sharp t_0 can be indicated. It is, however, a fact that such a t_0 exists in most teaching programmes. It is an urgent question whether such a discontinuity in teaching can be justified by discontinuities in the learning process. This question has been answered by several contributors to the Dutch report, especially by Vredenduin. At a certain moment we shall tell the children: "Up to now we have adopted some theorems at sight, and we have proved some other theorems. From now onwards we will define everything, and we will prove every theorem we shall pronounce."

If we transfer this statement from the teaching sphere into the learning sphere, we can say that at a certain moment the pupil should have grasped the sense, the possibility and the necessity of proving theorems. The pupil then enters individually the phase of deductive geometry. Of course it is desirable that the t_0 of the teaching process and the t_0 of every individual learning process coincide. In progressive schools in Holland (perhaps also in other countries) systems of individual or group education supplant more and more the rigid class system. This is particularly true in mathematical instruction. In this way pupils working in the same class-room will be allowed to pass the different discontinuities of their learning process at different times.

The discontinuity t_0 is particularly important. Teachers complain that when the deductive course starts, children shy off. Then, they conclude, children are not mature enough. But the same phenomenon is observed when deductive geometry starts at the age of 15 instead of 12 (as testified by the Italian reporter). So it cannot be a matter of maturity, but rather a discrepancy between the teaching and the learning process. In the same sense the van Hieles conclude that the discontinuities in the learning process are not to be interpreted as symptoms of maturing.

In order to overcome those difficulties, it is to be insisted upon that a preparatory course be really preparatory, i.e. that it prepares consciously for deductive geometry, and that the preparatory results of such a preparatory course are fully utilized in the deductive phase. To this point the Italian and the Yugoslavian reporters have paid special attention. Actual school systems, as outlined in the various reports, tend to have the transition from preparatory to deductive geometry coinciding with the changing from one kind of school to another or at least from one class to the next. Perhaps more continuity in the teaching situation would be an advantage just where discontinuities in the learning process are to be surmounted.

Of course, not all initiation into geometry must be preparatory. The greater part of youth never reaches the deductive level. For them an intuitive course may be the initial and final phase of geometrical instruction. In my opinion, which is perhaps not that of the Polish and the Yugoslavian reporters, there is no need to have these students attend a preparatory course that prepares for deductive geometry. Acquaintance with space and practical geometry will suffice. Among the reports of the Western-European countries, the Netherlands report is the only one which pays regard to this type of instruction.

On the other hand, if the destination is deductive geometry, the teacher of the initiating course may never lose sight of this goal. Particularly, if during two or three years the level of instruction is too low, the transition to a higher level can involve extraordinary difficulties.

Most of the present reports are not detailed enough to judge whether or not, and how the goal of deductive geometry is approached during the preparatory phase. I doubt whether a mensuration course like the Canadian one can be called preparatory. It would be particularly interesting to know how it is done in the school systems where children who have to prepare for a higher level of geometrical instruction are not separated from those who actually work on their final level.

In all contributions on secondary education in the Dutch report the efforts are steadily focussed on the goal of deductive geometry. The shortest way is the most popular. Obviously our teachers are anxious not to waste time with subjects that might be dispensed with. (Note that our children learn three or even five foreign languages, and that the whole school time lasts shorter than in any other country — eleven or twelve years.) A Dutch teacher will not readily adopt a course like the one of van Albada who takes his line. Mrs. van Hiele's preparatory course is rather long. It takes more than one year or even two years before the pupil reaches the deductive level. Nevertheless every step in this course is consciously and deliberately directed towards that goal. The other preparatory courses are much more straight-forward. All of them show very interesting details.

14. In this final paragraph I would like to say a few words about the impact psychological and pedagogical research may

have on geometrical instruction in the initiating phase. The role of psychology and general pedagogics is often misunderstood. It is as imperfectly understood as that of mathematics by people who do not know what mathematics are. Mathematics is an important tool. We cannot dispense with mathematics if we are building, for example, an airplane. But this does not mean that a mathematician can tell you how to build an airplane. As little as this can a psychologist tell you how to teach, or a general pedagogue, how to teach mathematics. It is true that gestalt psychology has influenced lower education, and I hope it will influence more and more mathematical instruction. But this influence will be restricted to some general principles. All will admit that Piaget's research is highly interesting. But it is quite another thing to apply his results to teaching mathematics, firstly because Piaget's mathematical background has been rather weak, but mainly because Piaget's approach hardly reflects the teaching situation in the classroom, but the rather unusual laboratory situation of the psychologist. Mathematical teaching theory can be furthered by mathematical teachers who are able mathematicians and able educators.

In the Netherlands report you can find a brief account of the theoretical work of Mr. and Mrs. van Hiele. I shall not try to summarize that summary. I will only draw your critical attention to their theory of the discontinuities in the learning process, the so-called thinking levels. These levels form a hierarchy which reminds of that of systems in logistical analysis. The relation between one level and the next higher one is analogous to that between a system and a meta-system. At every level the subject matter is a certain field that will be organized on this level. The devices of organizing on a certain level will form the field, and therefore the subject matter, on the subsequent higher level. Perception space is the field on level 0. Rhomb is a first level notion, equality of line segments and symmetry are on the second level, a logical relation like implication belongs to the third level, logical thinking itself becomes a subject matter on the fourth level. Under this aspect the van Hieles have analyzed their teaching experiences and particularly the above mentioned experiment of Mrs. van Hiele.

ANNEXE AU RAPPORT SUR L'ÉTUDE COMPARÉE DES MÉTHODES D'INITIATION A LA GÉOMÉTRIE

présenté au Congrès international des Mathématiciens par la Commission internationale de l'Enseignement mathématique

par W. Servais

(Reçu le 28 janvier 1959.)

Le rapport précédent a de grands mérites. Non seulement il organise en une synthèse ordonnée les contributions de dix pays, mais il contient aussi les vues personnelles de l'auteur, M. le professeur H. Freudenthal.

Par sa richesse, sa diversité et sa profondeur, ce rapport amènera le lecteur à remettre en question tel ou tel aspect de l'initiation à la géométrie.

Comme les mathématiques gardent leur jeunesse parce qu'elles sont sans cesse repensées, il convient que leur enseignement soit sans cesse repensé pour garder la vivacité de l'esprit.

Sans doute le plus grand mérite du rapport ci-dessus est de servir de catalyseur aux réflexions.

L'auteur, ayant eu connaissance de notes écrites au fil de la lecture de son rapport, a tenu à ce que soient formulées certaines remarques.

I. Tout d'abord, lorsque l'on parle d'initiation à la géométrie, on pense assez naturellement à la manière de présenter un premier contact pour le jeune débutant.

Cependant, si on considère l'enseignement de la géométrie moins comme un moyen de « transvaser » des connaissances adultes dans des têtes d'enfants, mais comme l'entreprise active de former progressivement la pensée géométrique, il doit y avoir des initiations successives réparties tout au long des études.

Il y aura ainsi une initiation aux figures, une initiation à les reconnaître et à les décrire, à déplacer les figures, à effectuer des transformations du plan et de l'espace et, de même, une initiation à définir les figures, à déduire des propriétés d'autres propriétés connues, à organiser la déduction.

Enfin, l'initiation au statut de l'édifice déductif et au rôle des transformations et des groupes.

Dans cette manière de voir les choses, il ne faut pas séparer trop nettement l'initiation propédeutique de l'exposé déductif, ni abandonner trop vite le recours au concret parce que l'on croit que l'élève a pris pied dans l'abstrait, ni vouloir faire trop tôt un exposé satisfaisant pleinement le mathématicien.

II. L'enseignement géométrique actif a pour objet de constituer d'abord un «back-ground» de structures mentales. Lorsque celles-ci sont stabilisées et deviennent conscientes, le moment est venu de bâtir sur elles.

Au début de l'initiation, il n'est pas suffisant de s'en remettre à l'expérience géométrique spontanée: celle-ci n'est ni assez ferme, ni assez consciente. A chaque degré d'initiation, la même nécessité d'une structuration active préalable se pose.

Pour vaincre les difficultés, il faut d'abord s'y débattre. Il est donc vain de vouloir sauter un apprentissage mental par une présentation jugée plus directe et plus claire. Souvent, si on recule l'âge où certaines difficultés sont abordées afin de réduire celles-ci, on retrouve les mêmes difficultés à l'âge ultérieur.

La notion abstraite de maturité intellectuelle doit être revue et nuancée. Evidemment, il ne s'agit pas d'affirmer que l'on peut enseigner tout à n'importe qui et à n'importe quel moment. Mais la possibilité d'enseigner une matière est certes liée à l'expérience mentale des élèves, acquise à la faveur des situations pédagogiques progressives dans lesquelles ils ont été placés.

L'inventaire de thèmes fournissant des situations stimulant la curiosité et la recherche est fondamental. En particulier, la question des problèmes *réels* (pour l'élève) est cruciale.

L'enfant se pose-t-il spontanément des problèmes géométriques ? Lesquels ? L'habitude de se poser des problèmes s'acquiert-elle, et comment ?

Lorsque le débutant est conduit à soulever des problèmes, quels sont pour lui les vrais problèmes capables de déclencher la mise en action de ses moyens intellectuels ? Il y a toute une investigation à faire au sujet de l'intérêt relatif des enfants pour certains problèmes « adultes » ou pour des problèmes de jeux, de montages ou de curiosités scientifiques.

De même, on peut se demander si le besoin de preuves déductives est naturel, spontané ou s'il résulte d'une éducation.

Enfin, dans ce qui précède, on a parlé de l'élève en général. Tous les élèves concrets sont singuliers. Ne faut-il pas que l'éducation géométrique soit assez variée dans ses méthodes et ses moyens pour présenter des aspects accessibles à des mentalités différentes ?

A notre époque, il faudra rechercher les modes d'enseignement pour concilier dans une certaine mesure une éducation mathématique de masse et une formation sélective des élites. N'y aurait-il pas lieu de délimiter une géométrie de base étudiée par tous et débarrassée des développements réservés à l'étude individuelle ?

III. Une question traitée, à bon droit, assez largement dans le rapport est le rôle des transformations géométriques et, singulièrement, l'influence du « programme d'Erlangen » (1872).

Les essais de bâtir une géométrie élémentaire scolaire sont contemporains du « programme de Klein ».

En France, la tentative la première en date est celle de Ch. MÉRAY, dans ses *Nouveaux éléments de géométrie* (1re édition, 1874). L'ouvrage original, touffu, surchargé de néologismes, est assez impropre à l'enseignement. Les *Eléments de géométrie* de Carlo BOURLET (1908), procèdent de ceux de Méray et sont très allégés.

On y trouve:

Postulat de Méray: deux translations rectilignes successives peuvent être remplacées par une seule (page 21).

Définition: deux droites, situées dans un même plan, sont dites parallèles lorsque l'une se déduit de l'autre par une translation (page 24).

Principe: Deux droites parallèles qui ont un point commun coïncident (page 25).

Dans l'esprit du programme d'Erlangen est écrit le *Cours de géométrie* plus récent de R. ESTEVE et H. MITAULT (1936).

Le programme d'Erlangen, qui peut organiser de façon remarquable un cours de géométrie avancé (étude des transformations et géométrie analytique, emploi de la géométrie en physique), doit faire l'objet d'une initiation progressive.

A la faveur de celle-ci, on considère d'abord une transformation d'une figure en une autre, puis la transformation étendue au plan ou à l'espace tout entier. On étudie ensuite les produits de transformations. Les élèves comprennent vite la transformation d'une figure en une autre; ils acceptent plus lentement l'idée de la transformation opérant sur le plan ou sur l'espace tout entier.

La raison, d'ordre psychologique, semble bien la difficulté à saisir ce qui se passe à l'infini. D'une manière générale, les élèves s'acclimatent malaisément à l'infini géométrique. S'ils répètent avec aisance que la droite n'a pas d'épaisseur et est illimitée, cela n'est bien souvent qu'un exercice verbal vide de pensée personnelle. D'ailleurs, ils accepteront sans sourciller la définition redondante et naïve:

> Deux droites parallèles sont des droites situées dans un même plan et qui ne se rencontrent pas, *aussi loin qu'on les prolonge.*

En fait, la géométrie des débutants est essentiellement locale. C'est pourquoi ils ont besoin de figures bornées, qu'ils considèrent avec prédilection. Pour de nombreux élèves, une droite, un angle ne sont pas des figures, tandis qu'un polygone ou un cercle sont des figures pour tous.

Lors de l'étude des droites parallèles, on peut proposer aux élèves le postulat d'Euclide sous sa forme originale:

> Dans un plan, lorsque deux droites forment d'un même côté d'une sécante des angles intérieurs dont la somme est inférieure à deux droits, les deux droites se rencontrent de ce côté de la sécante.

On peut aussi donner le postulat de l'unicité de la parallèle:

> Dans un plan, par un point extérieur à une droite, passe une seule parallèle à celle-ci.

Si on demande alors la préférence des élèves pour l'un de ces postulats, les élèves plus évolués choisissent en général le dernier, tandis que les élèves plus faibles trouvent le premier plus clair (sans doute parce qu'il n'engage pas l'infini).

Parmi les déplacements de *figures*, le plus aisé pour les débutants est la translation et le plus difficile, la rotation.

La compréhension d'une transformation présente d'ailleurs plusieurs stades. Ainsi pour la symétrie axiale, on a les étapes suivantes:

1° reconnaître qu'une figure a un axe de symétrie (examiner l'effet de la position de l'axe de symétrie relativement à la position du plan de symétrie de la tête);
2° reconnaître que deux figures se correspondent dans une symétrie axiale;
3° concevoir la définition d'une symétrie axiale du plan;
4° démontrer à l'aide de symétrie axiale;
5° faire des produits de symétrie axiale.

Ces produits de symétries, nécessaires pour engendrer le groupe des déplacements, sont une notion très élaborée à laquelle on ne parvient pas tout de suite. Quels que soient l'intérêt et la puissance des transformations géométriques pour une étude plus avancée, ce sont des notions qui demandent à être présentées progressivement.

Quant au programme d'Erlangen proprement dit, sa portée n'apparaît que par degrés. Par exemple, on peut faire remarquer, comme le faisaient déjà en 1865 Rouché et Comberousse dans leur *Traité de Géométrie élémentaire*, que les propriétés relatives au parallélisme sont susceptibles d'un exposé autonome antérieur à l'étude de la perpendicularité. La notion de transformation affine, présentée, par exemple, à partir des ombres produites par un faisceau de lumières parallèles, servira ensuite à expliquer cette autonomie.

Mais la propédeutique des transformations peut commencer très tôt. N'est-il pas frappant que les tout jeunes élèves qui apprennent la géométrie, lorsqu'ils copient la figure considérée dans une question, tracent toujours, et vraiment d'instinct, une figure semblable ? Ils ont la ferme conviction que toute l'étude s'appliquera à cette figure. Comment ne pas voir qu'il y a là, en germe, l'idée maîtresse: la géométrie euclidienne est l'en-semble des propriétés invariantes pour le groupe des similitudes !

En conclusion, au lieu de considérer l'enseignement des mathématiques comme la transmission d'une discipline toute faite, il conviendra de plus en plus de le considérer sous l'angle de techniques d'apprentissage. Pour améliorer ce dernier èt le hâter, il faudra étudier quelles sont les situations les plus favorables à susciter l'activité spontanée, mentale et concrète de l'étudiant, activité créatrice des structures mathématiques.

TÄTIGKEITSBERICHT DER INTERNATIONALEN MATHEMATISCHEN UNTERRICHTSKOMMISSION

erstattet vom derzeitigen Präsidenten Heinrich BEHNKE auf der Generalversammlung der IMU, St. Andrews (Schottland) August 1958.

(Reçu le 24 avril 1959.)

Die internationale mathematische Unterrichtskommission (IMUK, französisch: CIEM, englisch: ICMI) ist jetzt gerade 50 Jahre alt. Sie wurde in Rom 1908 gelegentlich eines internationalen Mathematikerkongresses gegründet. Felix KLEIN (Göttingen), David E. SMITH (New York), George GREENHILL (London) und Henri FEHR (Genf) waren ihre Gründer. Henri FEHR war für eine sehr lange Zeitspanne ihr Sekretär und gab seitdem das offizielle Organ der IMUK — das *L'Enseignement* — heraus. 1952 wurde FEHR Ehrenpräsident der IMUK. Doch mussten wir schon 3 Jahre später sein Ableben beklagen.

Während dieses halben Jahrhunderts schwankte die Aktivität der IMUK ganz beträglich. Sie war besonders stark in den Jahren nach ihrer Gründung 1908-1914. Von allen grossen internationalen Mathematikerkongressen sah Cambridge (1912) die grösste IMUK Delegation mit einem Vortragsprogramm, das an Umfang dem der Sektion 2 (Analysis) glich. Damals war die Verbindung zwischen Universität und Gymnasium in den europäischen Länder durchweg stärker als heute und weit mehr Gymnasiallehrer nahmen an der neuen wissenschaftlichen Entwicklung teil. Die Themen der IMUK aber umfassten damals insbesondere die viel umkämpfte Einführung der Differential- und Integralrechnung sowie des Erlanger Programms in den Schulunterricht.

Der erste Weltkrieg unterbrach die Arbeiten der IMUK. 1916 lief die grosse 1. Serie von Publikationen der IMUK aus. Henri FEHR war dann der erste und zugleich der einzige unter den Gründern, der die Tätigkeit der IMUK nach dem Kriege wieder in Gang zu bringen trachtete. Doch blieb er ziemlich alleine. Die politischen Bedingungen der zwanziger und dreissiger Jahre waren für die gemeinsame internationale Arbeit nicht günstig. Und die Verhandlungen der Kommission für den mathematischen Unterricht sind weit mehr von diesen Bedingungen abhängig als Konferenzen über Mathematik.

Die Programme der grossen internationalen Mathematiker Tagungen von Zürich (1932), Oslo (1936) und Cambridge, USA (1950) brachten so auch sehr wenig auf dem Gebiete der IMUK. Doch inzwischen ist die Aktivität der IMUK wieder sehr angewachsen. Schon der Kongress in Amsterdam (1954) umfasste viele Vorträge über den mathematischen Unterricht. Die glänzende Entwicklung der modernen Mathematik, insbesondere der Algebra, beginnt sich im Schulunterricht auszuwirken und so einen neuen Stoff für die IMUK zu liefern.

Im einzelnen ist über die Tätigkeit der IMUK folgendes zu berichten:

1955 wurde eine Konferenz in Genf in memorian Henri Fehr abgehalten. Sie stand unter der Überschrift: „Die Spannungen in den verschiedenen Ländern zwischen dem Unterricht an den Gymnasien (secondary schools) und den Universitäten". Dann folgte 1956 in Münster (Westf.) eine Konferenz, die mit einer deutschen Veranstaltung zur Pflege des Zusammenhanges von Universität und höherer Schule verbunden war. 1957 hatte die belgische Unterkommission zu einer Zusammenkunft in Brüssel eingeladen, die betitelt war: „Der geometrische Unterricht an den höheren Schulen (secondary schools)".

Für den grossen internationalen Kongress in Edinburgh hat die IMUK 3 Themen präpariert: 1. Mathematischer Unterricht für Schüler bis zum Alter von 15 Jahren. 2. Die wissenschaftlichen Grundlagen für den mathematischen Unterricht in höheren Schulen (secondary schools). 3. Vergleichende Studien zum geometrischen Anfangsunterricht. 33 Berichte der nationalen Unterrichtskommissionen sind dazu angemeldet. Die Generalberichterstatter sind Howard Fehr für Thema 1, H. Freudenthal für Thema 3 und H. Behnke für Thema 2. Die nationalen Berichte sind soweit wie möglich in den Generalreferaten verarbeitet. Das trifft vor allem für Thema 1 und 3 zu.

An gedruckten Schriften werden auf dem Kongress vorgelegt:

1. Von der niederländischen Unterkommission durch H. FREUDENTHAL: „Methods of initiation into geometry".

2. Von der deutschen Unterkommission (herausgegeben von F. DRENCKHAHN): „Der mathematische Unterricht für die 6—15jährige Jugend in der Bundesrepublik Deutschland."

3. Von der deutschen Unterkommission (herausgegeben von H. BEHNKE): „Grundzüge der Mathematik für Lehrer an höheren Schulen, Band 1, Algebra".

Ausserdem ist für den Kongress eine Ausstellung von Schulbüchern vorbereitet (so wie es auch eine im Amsterdam gab). Auf dieser Ausstellung werden 2000 Bücher von 17 Nationen gezeigt. Die mühselige und kostspielige Vorbereitung dafür ist wieder von Frankreich geleistet worden. Der französischen Regierung und dem verantwortlichen Organisator M. DOLMAZON vom Institut pédagogique de France spricht die IMUK dafür ihren ganz besonderen Dank aus. Die 2000 Bücher bleiben im Institut pédagogique de France und stehen den Unterkommissionen für zukünftige Ausstellungen zur Verfügung.

Schaut man rückblickend auf die Tätigkeit der IMUK während der letzten 4 Jahre, so fällt auf, dass alle Konferenzen in Europa stattfanden. Regelmässiger Teilnehmer von ausserhalb Europas war die Unterkommission der USA. In Brüssel war auch Australien vertreten. Es gab auch einigen Kontakt mit einzelnen anderen aussereuropaïschen Ländern.

In jedem Semester kamen einzelne Vertreter von nationalen Unterkommissionen nach Münster zur Aussprache mit dem Berichterstatter. Besonders befriedigend waren die Besprechungen mit Prof. STRASZEWICZ aus Warschau und einigen japanischen Kollegen.

Es kann aber nicht übersehen werden, dass es sehr schwierig für die IMUK war, Kontakt mit asiatischen, afrikanischen und südamerikanischen Ländern aufzunehmen. Umso höher ist die Arbeit von Marshall STONE (Chicago) zu schätzen, der auf eigene Initiative mit manchen dieser Länder in Verbindung trat und an Ort und Stelle ihren mathematischen Unterricht kennen lernte. Er hat einen sehr umfangreichen Bericht über den mathematischen Unterricht in einigen unterentwickelten Ländern veröffentlicht.

Von einigen Ländern, deren Schulwesen im Aufbau ist, kamen auch Anfragen an das Executivkomitee wegen der Gestaltung des mathematischen Unterrichts. Doch waren wir nicht imstande, auf solche Fragen zu antworten. Sollte, wie für Sozialversicherungen beim Bureau du travail in Genf, bei der IMUK eine Auskunftsstelle für den Ausbau des mathematischen Unterrichts in fernen Ländern errichtet werden, so gehörte dazu vor allem ein Amt mit hauptamtlichen Kräften. So etwas könnte nur von den U.N. unterhalten werden.

Mathematischer Unterricht ist nicht Mathematik. Die Möglichkeiten eines fruchtbaren Unterrichtes hängen nicht alleine vom ausgewählten Gebiete der Mathematik und seiner Darstellung ab. Der Erfolg des Unterrichtes ist ebenso vom ausgewählten Schülerkreis wie auch vom Erziehungssystem abhängig, das seinerseits wieder durch den historischen und politischen Hintergrund gegeben ist. So kommt es, dass die Idee völlig utopisch ist, Lehrbücher der Mathematik schreiben zu wollen, die in der ganzen Welt gebraucht werden können. (Ausgenommen ist natürlich immer die rein wissenschaftliche Darstellung für Studenten im Sinne europäischer Universitäten). Es ist auch unmöglich von Europa aus Ratschläge zu geben, wie der mathematische Unterricht in Südasien oder etwa Südamerika zweckmässig aufzubauen wäre. Ich bedaure diese Begrenzung der Wirkungsmöglichkeit der IMUK — aber sie ist eine Tatsache. Um sich Klarheit über einen zweckmässigen mathematischen Unterricht zu verschaffen, wäre es gut, wenn Vertreter der interessierten Länder eine ausführliche Informationsreise durch Europa machten. Solche Besucher sollten sich nicht zu lange an einer der bekannten Universitäten aufhalten, sondern danach trachten, Schulen aller Art kennen zu lernen, sowie an Prüfungen teilnehmen und die hohen Aufsichtsbeamten bei ihren Visitationen begleiten. Ich habe meinerseits mich sehr darum bemüht, solche Reisen zu organisieren. Doch die sprachlichen Schwierigkeiten, die notwendigerweise längere Dauer solcher Reisen und die entstehende Kosten bereiten der Verwirklichung dieser Pläne erhebliche Widerstände. Die Kosten, die innerhalb der europäischen Länder entstehen, würden von den Gastländern getragen. Aber die Reisekosten sind derart hoch, dass nur die Vereinigten Staaten von Amerika in der Lage sind, sie zu tragen.

So ist es nicht leicht, die Aktivität der IMUK auf globaler Basis zu heben. Ganz anders ist die Lage zwischen den Ländern, die nicht durch grosse Entfernungen getrennt sind und die zugleich einander ähnelnde Schulsysteme haben. Ich habe schon erwähnt, dass die rege Tätigkeit der IMUK in Europa in den letzen 4 Jahren sich in einer Atmosphäre ganz ungewöhnlicher Herzlichkeit auswirken konnte. So etwas könnte vielleicht auch in anderen Teilen der Erde verwirklicht werden. Deshalb schlage ich vor, dass zwischen dem globalen Executiv-Komitee der IMUK und ihren nationalen Unterkommissionen auf freiwilliger Basis regionale Gruppen gebildet werden sollen. Die Statuten der IMUK sollten dementsprechend ergänzt werden. Die Sitzungen des globalen Executiv-Komitees sollen dann von den Vertretern der regionalen Gruppen besucht werden. Nur so ist eine Arbeit der IMUK möglich, die wirklich global ist.

Es ist schliesslich noch zu erwähnen, dass die IMUK eine Unterkommission gebildet hat, um die Gründe für die weit verbreitete Knappheit an Mathematiklehren zu untersuchen. Dieses Komitee umfasst die Herren:

1. Kurepa (Zagreb);
2. van Dantzig (Amsterdam);
3. Servais (Brüssel);
4. Howard Fehr (New York).

Mehrere Konferenzen der Unterkommissionen haben stattgefunden (und ein Bericht ist erstattet, der im *L'Enseignement*, Bd. 4, p. 220 abgedruckt ist).

Die IMUK spricht ihren besonderen Dank dem Herausgeber des *L'Enseignement*, Herrn Prof. Karamata, aus für die Veröffentlichungen aller ihrer Rundschreiben. Es war ein Vergnügen, mit Prof. Karamata zu arbeiten. Ebenso gilt der Dank des Vorstandes den Herren Professoren Hopf und Eckmann vom Präsidium der IMU für die stets entgegenkommende Behandlung der Angelegenheiten unserer Kommission.

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