

A generalization of a two triangle inequality

Autor(en): **Tsintsifas, G.**

Objektyp: **Article**

Zeitschrift: **Elemente der Mathematik**

Band (Jahr): **42 (1987)**

Heft 6

PDF erstellt am: **20.09.2024**

Persistenter Link: <https://doi.org/10.5169/seals-40043>

Nutzungsbedingungen

Die ETH-Bibliothek ist Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Inhalten der Zeitschriften. Die Rechte liegen in der Regel bei den Herausgebern.

Die auf der Plattform e-periodica veröffentlichten Dokumente stehen für nicht-kommerzielle Zwecke in Lehre und Forschung sowie für die private Nutzung frei zur Verfügung. Einzelne Dateien oder Ausdrucke aus diesem Angebot können zusammen mit diesen Nutzungsbedingungen und den korrekten Herkunftsbezeichnungen weitergegeben werden.

Das Veröffentlichen von Bildern in Print- und Online-Publikationen ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. Die systematische Speicherung von Teilen des elektronischen Angebots auf anderen Servern bedarf ebenfalls des schriftlichen Einverständnisses der Rechteinhaber.

Haftungsausschluss

Alle Angaben erfolgen ohne Gewähr für Vollständigkeit oder Richtigkeit. Es wird keine Haftung übernommen für Schäden durch die Verwendung von Informationen aus diesem Online-Angebot oder durch das Fehlen von Informationen. Dies gilt auch für Inhalte Dritter, die über dieses Angebot zugänglich sind.

LITERATURVERZEICHNIS

- 1 L. Bieberbach: Theorie der geometrischen Konstruktionen. Birkhäuser, Basel 1952.
- 2 G. Fischer, R. Sacher: Einführung in die Algebra. Teubner, Stuttgart 1978.
- 3 S. Lang: Algebra. Addison-Wesley, 1967.
- 4 H. Lebesgue: Leçons sur les constructions géométriques. Gauthier-Villars, Paris 1950.
- 5 O. Perron: Algebra II. Göschen, Berlin 1951.
- 6 A. Schinzel: On linear dependence of roots. Acta Arithmetica (1975) 28, 161–175.

© 1987 Birkhäuser Verlag, Basel

0013-6018/87/060000-00\$1.50+0.20/0

A generalization of a two triangle inequality

1. Introduction

By coupling an inequality of Bottema [1] together with one of Pedoe [1], O. Bottema and M. S. Klamkin obtained the two chain inequality

$$a'x + b'y + c'z \cong \left[\frac{P}{2} + 8FF' \right]^{1/2} \cong 4\sqrt{FF'},$$

see [2], where $P = \sum a'^2(b^2 + c^2 - a^2)$, x, y, z the distances of an interior point M of the triangle ABC from the vertices A, B, C , a, b, c and a', b', c' the sides of the triangles ABC and $A'B'C'$ and F, F' their area respectively.

In this note the author will generalize the part

$$a'x + b'y + c'z \cong 4\sqrt{FF'}$$

of the above inequality for two simplices $s^{(n)} = (A_1 A_2 \dots A_{n+1})$ and $s'^{(n)} = (A'_1 A'_2 \dots A'_{n+1})$.

2. Notations

We denote by $V(W)$ the volume of the simplex W , $s^{(n)} = (A_1 A_2 \dots A_{n+1})$, $s'^{(n)} = (A'_1 A'_2 \dots A'_{n+1})$ two simplices of E^n . The facets

$$(A_1 A_2 \dots A_{i-1} A_{i+1} \dots A_{n+1}), (A'_1 A'_2 \dots A'_{i-1} A'_{i+1} \dots A'_{n+1})$$

will be denoted by $s_i^{(n-1)}$, $s'_i{}^{(n-1)}$ respectively.

Suppose that M is an interior point of the simplex $s^{(n)}$ with distances $A_i M = x_i$. We put:

$$D = \sum_{i=1}^{n+1} x_i V(s'_i{}^{(n-1)}).$$

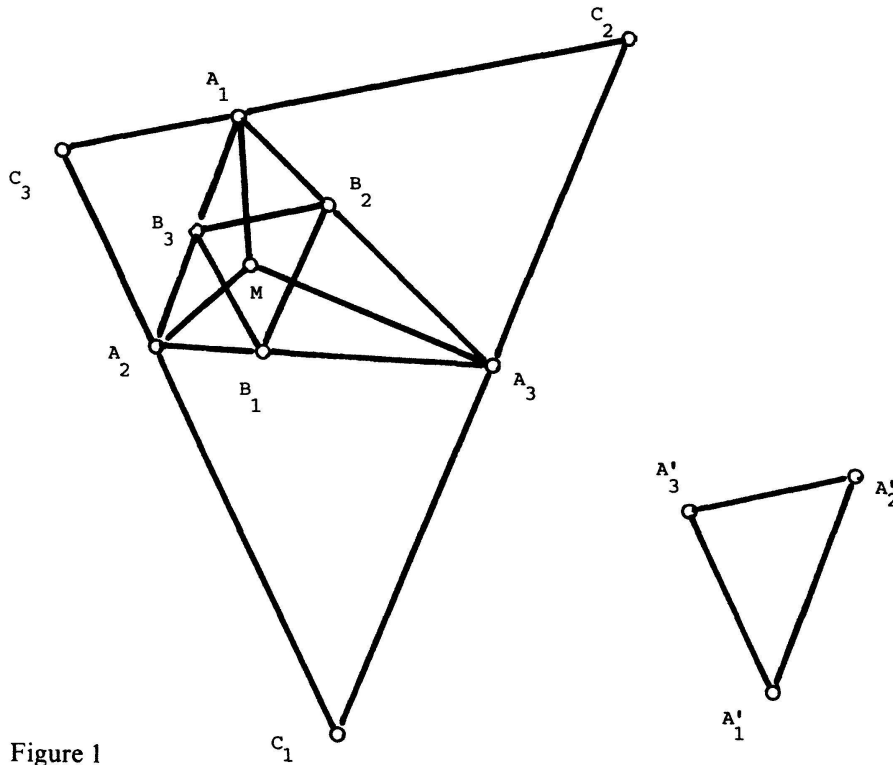


Figure 1

The quantity D depends on the labeling of the vertex set but for every case we can prove:

3. Theorem

For two simplices $s^{(n)}$ and $s'^{(n)}$ holds:

$$D^2 \geq n^4 \cdot V^{\frac{n}{2}}(s^{(n)}) \cdot V^{\frac{2n-2}{2}}(s'^{(n)}) .$$

Proof: We assume that the simplex $p^{(n)} = (B_1 B_2 \dots B_n)$ is similar to $s'^{(n)}$ and inscribed in $s^{(n)}$, so that $B_i \in s_i^{(n-1)}$. We also consider the simplex $q^{(n)} = (C_1 C_2 \dots C_{n+1})$ similar to $s'^{(n)}$ and circumscribed to $s^{(n)}$ so that $A_i \in q_i^{(n-1)}$, where $q_i^{(n-1)} = (C_1 C_2 \dots C_{i-1} C_{i+1} \dots C_{n+1})$. It is known that:

$$V^n(s^{(n)}) \geq V^{n-1}(p^{(n)}) \cdot V(q^{(n)}) , \tag{1}$$

see [3].

Suppose $W' = kW$ is the homothetical image of W with ratio k . We put:

$$p^{(n)} = \lambda s'^{(n)} \quad \text{and} \quad q^{(n)} = \mu s'^{(n)} . \tag{2}$$

We can easily see that: $x_i \cdot V(p_i^{(n-1)}) \cong nV(A_i B_1 B_2 \dots B_{i-1} B_{i+1} \dots B_{n+1} M)$ and $x_i \cdot V(q_i^{(n-1)}) \cong nV(C_1 C_2 \dots C_{i-1} C_{i+1} \dots C_{n+1} M)$. From the above follows

$$\sum_{i=1}^{n+1} x_i V(p_i^{(n-1)}) \cong nV(s^{(n)}), \tag{3}$$

$$\sum_{i=1}^{n+1} x_i V(q_i^{(n-1)}) \cong nV(q^n), \tag{4}$$

where $p_i^{(n-1)} = (B_1 B_2 \dots B_{i-1} B_{i+1} \dots B_{n+1})$.

See the figure, above, for the elementary case $n = 2$. From (2) follows,

$$V(p_i^{(n-1)}) = \lambda^{n-1} V(s'^{(n-1)}), \tag{5a}$$

$$V(q_i^{(n-1)}) = \mu^{n-1} V(s'^{(n-1)}) \quad \text{and} \tag{5b}$$

$$V(q^{(n)}) = \mu^n V(s'^{(n)}), \tag{5c}$$

see (2).

Therefore from (3), (4), (5) we have:

$$\lambda^{n-1} D \cong nV(s^{(n)}), \quad \mu^{n-1} D \cong n\mu^n V(s'^{(n)}) \tag{6}$$

or,

$$\frac{\lambda^{n-1}}{\mu} D^2 \cong n^2 V(s^{(n)}) V(s'^{(n)}). \tag{7}$$

Using (1), (5c) we take:

$$\frac{V^n(s^{(n)})}{V^2(q^{(n)})} \cong \frac{\lambda^{n(n-1)} V^{n-1}(s'^{(n)})}{\mu^n V(s'^{(n)})} = \left(\frac{\lambda^{n-1}}{\mu}\right)^n V^{n-2}(s'^{(n)})$$

or,

$$V^{n-2}(s^{(n)}) \left(\frac{V(s^{(n)})}{V(q^{(n)})}\right)^2 \cong \left(\frac{\lambda^{n-1}}{\mu}\right)^n V^{n-2}(s'^{(n)}). \tag{8}$$

But it is known that if $V(q^{(n)})$ is minimum then A_i is the centroid of $q_i^{(n-1)}$, see [4] or [5], therefore

$$\frac{1}{n^n} \cong \frac{V(s^{(n)})}{V(q^{(n)})}. \tag{9}$$

From (8), (9) follows:

$$\frac{V^{n-2}(s^{(n)})}{n^{2n}} \cong \left(\frac{\lambda^{n-1}}{\mu}\right)^n V^{n-2}(s'^{(n)})$$

or,

$$\frac{1}{n^2} \frac{V^{\frac{n-2}{n}}(s^{(n)})}{V^{\frac{n-2}{n}}(s'(n))} \geq \frac{\lambda^{n-1}}{\mu}. \tag{10}$$

Therefore from (7) and (10) we obtain

$$D^2 \geq n^4 V^{\frac{2}{n}}(s^{(n)}) V^{\frac{2n-2}{n}}(s'(n)).$$

The author is grateful to the referee for his helpful suggestions.

G. Tsintsifas, Thessaloniki, Greece

REFERENCES

- 1 O. Bottema, R. Z. Djordjević, R. R. Janić, D. S. Mitrinović, and P. M. Vasić: Geometric inequalities. Wolters-Noordhoff, Groningen 1969.
- 2 O. Bottema and M. S. Klamkin: Joint triangle inequalities. Simon Stevin, wis-en Natuur’Kundig Tijdschrift 48^e Jeergang (1974–1975), Afleviving I, II (Juli–October 1974).
- 3 G. D. Chakerian: Minimum area of circumscribed polygons. Elemente der Mathematik, Vol. 28, Heft 5, 1973.
- 4 M. M. Day: Polygons circumscribed about closed convex sets. Trans. Amer. Math. Soc. 62, pp. 315–319, 1957.
- 5 G. D. Chakerian and L. H. Lange: Geometric extremum Problems. Math. Magazine 44, N° 2, 1971.

A tournament result deduced from harems

There is a large class of difficult problems of the type: “does there exist a graph with n vertices having prescribed degrees d_1, \dots, d_n ?” Restricting the problem to particular types of graphs can lead to some very neat characterisations. For example, it is a straightforward exercise to show that a tree exists on $n (\geq 2)$ vertices with degrees d_1, \dots, d_n if and only if the d_1, \dots, d_n are positive integers with

$$d_1 + \dots + d_n = 2(n - 1).$$

We shall now restrict attention to ‘tournaments’. A *tournament* is a directed graph in which each pair of distinct vertices is joined precisely once (in one direction or the other). Alternatively it can be thought of as a competition of a set of players in which each pair plays once resulting in a win for one of the players. Before proceeding, note that, for example, there exists a tournament of 4 players in which their numbers of wins are 1, 1, 2 and 2 (e.g. A beats B, B beats D, C beats A, C beats B, D beats A and