

Leafwise hyperbolicity; a correction.

Autor(en): **Cantwell, John / Conlon, Lawrence**

Objekttyp: **Corrections**

Zeitschrift: **Commentarii Mathematici Helvetici**

Band (Jahr): **66 (1991)**

PDF erstellt am: **20.09.2024**

Nutzungsbedingungen

Die ETH-Bibliothek ist Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Inhalten der Zeitschriften. Die Rechte liegen in der Regel bei den Herausgebern.

Die auf der Plattform e-periodica veröffentlichten Dokumente stehen für nicht-kommerzielle Zwecke in Lehre und Forschung sowie für die private Nutzung frei zur Verfügung. Einzelne Dateien oder Ausdrucke aus diesem Angebot können zusammen mit diesen Nutzungsbedingungen und den korrekten Herkunftsbezeichnungen weitergegeben werden.

Das Veröffentlichen von Bildern in Print- und Online-Publikationen ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. Die systematische Speicherung von Teilen des elektronischen Angebots auf anderen Servern bedarf ebenfalls des schriftlichen Einverständnisses der Rechteinhaber.

Haftungsausschluss

Alle Angaben erfolgen ohne Gewähr für Vollständigkeit oder Richtigkeit. Es wird keine Haftung übernommen für Schäden durch die Verwendung von Informationen aus diesem Online-Angebot oder durch das Fehlen von Informationen. Dies gilt auch für Inhalte Dritter, die über dieses Angebot zugänglich sind.

Leafwise hyperbolicity; a correction

JOHN CANTWELL* AND LAWRENCE CONLON**

In [1], a proof of the following theorem was proposed.

THEOREM 1. *Let (M, \mathcal{F}) be a C^2 -foliated manifold of codimension 1, transversely orientable and such that M is compact, every leaf is proper, and \mathcal{F} is tangent to ∂M . If no leaf of \mathcal{F} is a torus or a sphere, then there is a Riemannian metric on M relative to which each leaf of \mathcal{F} has constant curvature -1 .*

This theorem is correct, but there was an erroneous step in the proof, namely [1, Lemma (2.2)]. We are grateful to S. Matsumoto and N. Tsuchiya for pointing this out to us.

We fix the hypotheses of Theorem 1. A metric g with the property in that theorem will be called leafwise hyperbolic.

Let $M_0 \subseteq M_1 \subseteq \cdots \subseteq M_k \subseteq M_{k+1} \subseteq \cdots$ denote the level filtration [2]. Each M_k is a compact, nonempty, \mathcal{F} -saturated set, the leaves in $M_k \setminus M_{k-1}$ being the leaves of \mathcal{F} at level k . When all leaves are proper, it has become customary to use the term “depth” rather than “level”. Since all leaves are proper and the foliation is of class C^2 , every leaf of \mathcal{F} has finite depth, hence $M = \bigcup_{k=0}^{\infty} M_k$.

PROPOSITION 1. *Let M_k denote the union of leaves at depths at most k . Then there is a nest $W_0 \subseteq W_1 \subseteq \cdots \subseteq W_k \subseteq W_{k+1} \subseteq \cdots \subseteq M$, where W_k is an open neighborhood of M_k , and there is a Riemannian metric g_k on M such that $g_k \upharpoonright W_k$ is leafwise hyperbolic for $\mathcal{F} \upharpoonright W_k$, $\forall k \geq 0$.*

Theorem 1 follows. Indeed, $\{W_k\}_{k=0}^{\infty}$ is an open, nested cover of the compact manifold M , hence passing to a finite subcover yields a value of k for which $W_k = M$. It remains, then, to prove Proposition 1.

We fix a smooth, 1-dimensional foliation \mathcal{F}^\perp , everywhere transverse to \mathcal{F} . Projections along the leaves of \mathcal{F}^\perp can be used to define local diffeomorphisms between leaves of \mathcal{F} .

* Research partially supported by N.S.F. Contract DMS-8900127

** Research partially supported by N.S.F. Contract DMS-8822462

If $U \subseteq M$ is an open, connected, \mathcal{F} -saturated set, we use the notations $\hat{U}, \hat{i}: \hat{U} \rightarrow M$, $\hat{\mathcal{F}} = \hat{i}^{-1}(\mathcal{F})$, and $\hat{\mathcal{F}}^\perp = \hat{i}^{-1}(\mathcal{F}^\perp)$ from [1], [2], *et al.*, for the completion of U , its natural immersion into M , and the induced foliations of \hat{U} , respectively. Recall that U and \hat{U} are called *foliated products* if \hat{U} is diffeomorphic to $L \times [0, 1]$ in such a way that the leaves of $\hat{\mathcal{F}}^\perp$ are the $[0, 1]$ -fibers. Recall that, if U is a foliated product, then $\hat{i}(\partial\hat{U})$ is either a single leaf or a pair of leaves of \mathcal{F} .

DEFINITION 1. A closed subset $X \subset M$ that is a finite union of leaves of \mathcal{F} will be called a *skeleton* if each component of $M \setminus X$ is a foliated product. If k is the highest depth of the leaves in X , the skeleton has depth k . We will say that X (of depth k) is a *full skeleton* if, for each component U of $M \setminus X$, at least one of the following holds.

- (1) Every leaf L of $\hat{\mathcal{F}}$ has image $\hat{i}(L)$ at the same depth $k_0 \leq k$.
- (2) If $L \subset \partial\hat{U}$ is a boundary leaf, then $\hat{i}(L)$ is a leaf at depth k .

If X is a skeleton, it was proven in [1, (1.2)] that there is an open neighborhood $W \supset X$ and a Riemannian metric g on M such that $g|_W$ is leafwise hyperbolic for $\mathcal{F}|_W$. Furthermore, projection along the leaves of \mathcal{F}^\perp defines local isometries between the leaves of $\mathcal{F}|_W$. Finally, $\hat{U} \setminus \hat{i}^{-1}(W)$ is compact, for each component U of $M \setminus X$.

LEMMA 1. *If there is a full skeleton X of depth N , then there is a neighborhood $W_N \supset M_N$ and a Riemannian metric g_N on M which is leafwise hyperbolic on W_N .*

Proof. Let U be a component of $M \setminus X$. There are two cases, corresponding to possibilities (1) and (2) of Definition 1.

(1) In this case, the proof of [1, Lemma (2.1)] shows how to extend the metric smoothly over all of U so as to make the curvature of the leaves of $\mathcal{F}|_U$ constantly -1 . Indeed, the metric was already appropriately defined on all but a compact submanifold $A \times [0, 1] \subset \hat{U}$ and \mathcal{F} induces the product foliation on this submanifold. A deformation argument, using the Teichmüller space of A , created the extension. (The error in [1] was to claim that, even in the second case, where the foliation of $A \times [0, 1]$ was not a product, the above metric on the product could be “tilted” to give a hyperbolic metric along the leaves.)

(2) We assume that the situation in (1) does not also occur. In this case, the argument is actually easier. Since M_N is compact [2, (4.6)], $\hat{i}^{-1}(M_N) \cap \hat{U} = L \times C$, where $C \subset [0, 1]$ is a closed subset containing $\{0, 1\}$. Since $U \setminus M_N \neq \emptyset$, $[0, 1] \setminus C$ has at least one component (a, b) . Let $a < a' < b' < b$. The metric g is already defined on $W \cap \hat{i}(\hat{U})$ in such a way that projections along \mathcal{F}^\perp are local isometries between leaves. Using the projections $p^+ : L \times (b', 1] \rightarrow L \times \{1\}$ and $p^- : L \times [0, a') \rightarrow$

$L \times \{0\}$, one lifts this metric smoothly to $L \times [0, a') \cup L \times (b', 1]$. This metric agrees with g wherever both are defined.

Finite repetition of this argument, as U ranges over the components of $M \setminus X$, completes the proof. \square

LEMMA 2. *For some integer $N \geq 0$, there exists a full skeleton of depth N .*

Proof. As in [1, (1.1)], one constructs a skeleton X . Let N be the depth of X . If X is not full, consider a component U of $M \setminus X$ with boundary component(s) at depth $k < N$. If every leaf of $\mathcal{F} \upharpoonright U$ is at depth k , there is nothing to do. Otherwise, there is a leaf $L \subset U$ at depth $k + 1 \leq N$. It is elementary that $X' = X \cup L$ is again a skeleton of depth N . If X' is not full repeat the process for X' . Finite repetition will ultimately produce a full skeleton of depth N . \square

For $0 \leq k \leq N$, we set $W_k = W_N$ and $g_k = g_N$. We also set $X = X_N$.

Each component U_i of $M \setminus X_N$ that has not been engulfed by W_N must contain a leaf L_i at depth $N + 1$. Throwing these finitely many leaves in with X_N provides a full skeleton X_{N+1} of depth $N + 1$. An application of Lemma 1 produces W_{N+1} and g_{N+1} as desired. It is not hard to see that W_{N+1} can be chosen to engulf W_N . Proceeding in this way, we construct the nest of open sets and the metrics as in Proposition 1.

REMARK. Projection along the leaves of \mathcal{F}^\perp does not always define local isometries between the leaves of \mathcal{F} . In the pieces $A \times [0, 1]$, where the metric is extended by a deformation in Teichmüller space, these projections will not be isometric. If it were possible to avoid introducing these regions, it would follow that the leafwise hyperbolic metric for \mathcal{F} is a bundlelike metric for \mathcal{F}^\perp , hence that the leaves of \mathcal{F} are totally geodesic in this metric. But totally geodesic foliations of compact 3-manifolds by surfaces are relatively rare.

REFERENCES

- [1] CANTWELL J. and CONLON L. *Leafwise hyperbolicity of proper foliations*. Comment. Math. Helv., 64 (1989) 329–337.
- [2] CANTWELL J. and CONLON L. *Poincaré-Bendixson theory for leaves of codimension one*. Trans. Amer. Math. Soc., 265 (1981) 181–209.

*St. Louis University
Department of Mathematics
Ritter Hall
St. Louis, MO 63103, USA*

*Washington University
Department of Mathematics
Campus Box 1146
St. Louis, MO 63130-4899, USA*

Received October 20, 1990