Cohomology eigenvalues of equivariant mappings.

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Let X be a topological space which is paracompact Hausdorff and of finite cohomology dimension over a fixed field k. Let G be a compact Lie group acting continuously on X such that there is a finite number of conjugacy classes of isotropy groups G_x, $x \in X$. Conner conjectured in [2] that if $H^*(X; k)$ is acyclic, then $H^*(X/G; k)$ is also acyclic, and he proved the conjecture in case $k = Q$. The conjecture was recently proven in all characteristics by Robert Oliver [8]. The problem of relating $H^*(X/G; k)$ and $H^*(X; k)$ is still largely unsolved even in case X is the unit sphere of a linear representation. In this paper we will consider equivariant mappings $f: X \to X$ and relate the eigenvalues of the induced endomorphisms of $H^*(X/G; k)$ and of $H^*(X; k)$. The result obtained should be seen as a generalization of the Conner conjecture to G-spaces which are not necessarily acyclic.

THEOREM 1. *Let f be an equivariant self-mapping of a G-space X. Then each eigenvalue of the induced endomorphism of $\tilde{H}^*(X/G; k)$ is an eigenvalue of the induced endomorphism of $\tilde{H}^*(X; k)$, provided* $\dim_k H^*(X; k) < \infty$.

More generally we consider the monoid Map (G, X) of all equivariant mappings $X \to X$, and a homorphism from a monoid $\mathcal{F}$ into Map (G, X). Then $H^*(X; k)$ and $H^*(X/G; k)$ become right $\mathcal{F}$-modules. Let M be an abelian group which is a right $\mathcal{F}$-module. *A simple subquotient of the $\mathcal{F}$-module M* is a simple $\mathcal{F}$-module isomorphic to M_2/M_1 where $M_1 \subset M_2 \subset M$ are $\mathcal{F}$-submodules. M may be a module over a field k and $\mathcal{F}$ commuting with k. Even if M is not finitely generated, the following lemma is straightforward.

LEMMA 1. *Let*

$$O \to M' \to M \to M'' \to O$$

be an exact sequence of $\mathcal{F}$-modules. Then a simple $\mathcal{F}$-module is a subquotient of M if and only if it is a subquotient of $M' \oplus M''$.

Our main result then is,

THEOREM 2. *Let X be a G-space and let $\mathcal{F}$ be a monoid of equivariant self-mappings of X. Then every simple subquotient of the $\mathcal{F}$-module $\tilde{H}^*(X/G; k)$ is a simple subquotient of the $\mathcal{F}$-module $\tilde{H}^*(X; k)$. If $Y \subset X$ is a closed subspace invariant under G and under all $f \in \mathcal{F}$, then every simple subquotient of the $\mathcal{F}$-module $H^*(X/G, Y/G; k)$ is a simple subquotient of the $\mathcal{F}$-module $H^*(X, Y; k)$.*

This result may be interpreted in terms of Serre classes of $\mathcal{F}$-modules. Let N be a simple $\mathcal{F}$-module over k. Then by Lemma 1, those $\mathcal{F}$-modules which do not have N as a subquotient from a Serre class, say C_N. Theorem 2 says that if $\tilde{H}^*(X; k)$ belongs to C_N, then so does $\tilde{H}^*(X/G; k)$. It is then a Conner conjecture modulo the Serre class C_N. If we forget equivariant mappings and consider the Serre class of finitely generated abelian groups, we obtain,

THEOREM 3. *Let X be a G-space, and assume that X has finite cohomology dimension over $\mathbf{Z}$. Then if $H^*(X; \mathbf{Z})$ is finitely generated, so is $H^*(X/G; \mathbf{Z})$.*

We use Čech cohomology with closed supports. We use some results on cohomology dimension [9] and the localization theory of Borel-Segal-Hsiang-Quillen [1, 6, 9, 10] without further comments. When G is finite or abelian, the proof of Theorems 1–3 is based on the localization theory. When G is connected simple, the proof is based on the Conner conjecture and on the existence of the spheres of Floyd-Hsiang [3, 5]. We first simplify the group G.

LEMMA 2. (i) *Let $N \subset G$ be a closed normal subgroup such that Theorem 2 holds for actions of N and of G/N. Then Theorem 2 holds for actions of G.*

(ii) *It suffices to prove Theorem 2 when G is either a finite group of prime order, a circle group acting semifreely, or a simple connected Lie group.*

Proof. (i) Let $\mathcal{F}$ be a monoid of equivariant self-mappings of the G-space X. There is a natural homomorphism $\mathcal{F} \to \operatorname{Map}(G/N, X/N)$, and hence every simple subquotient M of the $\mathcal{F}$-module $\tilde{H}^*(X/G; k) = \tilde{H}^*((X/N)/(G/N); k)$ is a subquotient of $\tilde{H}^*(X/N; k)$. Because $\mathcal{F} \subset \operatorname{Map}(G, X) \subset \operatorname{Map}(N, X)$, and Theorem 2 holds for actions of N, the simple module M must be a subquotient of $\tilde{H}^*(X; k)$. Hence Theorem 2 holds for the G-action on X.

(ii) By (i) we may assume that G is a finite group, a circle group, or a connected simple group. If $G = SO(2)$, let $Z \subset G$ be a finite subgroup containing all finite isotropy groups. Then the action of G/Z on X/Z is semifree. By (i), it suffices to prove Theorem 2 for actions of cyclic groups and for semifree circle actions to give a proof for all circle actions. If G is finite, let S be the p-Sylow subgroup of G, where $p = \text{char}(k)$, and where $S = \{1\}$ if $p = 0$. Then by [1] (p. 38), we have $H^*(X/G; k) \subset H^*(X/S; k)$. Therefore, it suffices to prove Theorem 2 for the group S. Because S is solvable, it follows from (i) that we can reduce the problem to finite groups of prime order.

Proof of Theorem 2 *for G connected simple.*

We shall construct a compact G-space Z such that for each closed subgroup H of G the orbit mapping $Z \to Z/H$ induces an isomorphism

$$H^*(Z/H; \mathbf{Z}) \xrightarrow{\simeq} H^*(Z; \mathbf{Z}).$$

Z is a compact G-CW complex in the sense of Matumoto [7], and G has no fixed points in Z. We construct Z by using,

THEOREM (*Floyd-Hsiang* [3, 5]) *Each simple connected compact Lie group G admits a real linear representation without one-dimensional direct summands such that the unit sphere admits an equivariant self-mapping of degree* 0.

Let S be the unit sphere, and $n: S \to S$ an equivariant self-mapping of degree 0. Let $Z = T(n)$ be the mapping torus of n, that is the space obtained from $S \times [0, 1]$ by identifying $(x, 1)$ with $(n(x), 0)$ for $x \in S$. Let $\pi: T(n) \to S^1$ be the projection on the second factor where $S^1 = [0, 1]/\{0, 1\}$. $T(n)$ is a G-CW complex because n is constructed by extending a piecewise linear map of a fundamental domain into the fixed point set of a principal isotropy group, where the simplicial structure is compatible with the orbit type stratification. (This is actually done for an action of some $SO(2r+1)$ on S, and the action is restricted to G by a representation of G of degree $2r+1$. This construction is found in [3, 5] and with more details in [11].) $T(n)$ is a G-space in a natural way such that the fibres $\pi^{-1}(z)$, $z \in S^1$, are canonically G-homeomorphic to S. Since n is nullhomotopic, it follows that π is a homotopy equivalence, and hence that the mapping cone $C(\pi)$ of π is contractible. Since $C(\pi)$ is a finite CW complex, the Conner conjecture, proved by Oliver, implies that $H^*(C(\pi)/H; \mathbf{Z}) = \mathbf{Z}$ for each closed subgroup H of G. Clearly $C(\pi)/H$ is the mapping cone of $T(n)/H \to S^1$, and hence

$$H^*(T(n)/H; \mathbf{Z}) \simeq H^*(S^1; \mathbf{Z}) \simeq H^*(T(n); \mathbf{Z}).$$

The G-CW structure on Z defines a finite cell complex structure on Z/G ([7]).

For each cell c of Z/G, choose $x \in Z$ such that $G(x)$ is in the interior of c, and set $G_c = G_x$. The cellular system (G_c) will be used in the Borel construction. Given two G-spaces X and Z, we consider $Z \times X$ as a G-space with the diagonal (joint) action, and there are projections of orbit spaces,

$$pr_1 : (Z \times X)/G \to Z/G, \qquad pr_2 : (Z \times X)/G \to X/G.$$

The fibres of pr_1 and pr_2 are, for $x \in X$, $z \in Z$,

$$pr_1^{-1}(G(z)) = (G(z) \times X)/G = X/G_z$$

and

$$pr_2^{-1}(G(x)) = (Z \times G(x))/G = Z/G_x.$$

We apply the Leray spectral sequence to the mappings pr_2 and p_2 of the following commutative diagram where the vertical arrows are induced by π.

$$\begin{array}{ccccc} Z/G_x & \to & (Z \times X)/G & \xrightarrow{pr_2} & X/G \\ \downarrow{\scriptstyle \pi} & & \downarrow & & \downarrow{\scriptstyle 1} \\ S^1 & \to & S^1 \times (X/G) & \xrightarrow{p_2} & X/G \end{array}$$

Here pr_2 and p_2 are proper mappings. Since π induces cohomology isomorphisms of the fibres, we have

$$H^*(S^1) \otimes H^*(X/G) \simeq H^*((Z \times X)/G)$$

for any coefficient ring. This clearly is an isomorphism of $\mathcal{F}$-modules. For the mapping

$$pr_1 : (Z \times X)/G \to Z/G$$

we obtain a spectral sequence defined by the skeleton filtration of the cell complex Z/G, with

$$E_1 = C^*_{\text{cell}}(Z/G; \mathcal{H}^*(X/G_c))$$

and converging to $H^*((Z\times X)/G)\simeq H^*(S^1)\otimes H^*(X/G)$. For reduced cohomology, there is the spectral sequence $\tilde{E}$ with $\tilde{E}_1=C^*_{\text{cell}}(Z/G;\tilde{\mathcal{H}}^*(X/G_c;k))$ converging to $H^*(S^1)\otimes\tilde{H}^*(X/G;k)$. This is a spectral sequence of $\mathcal{F}$-modules. A simple subquotient of the $\mathcal{F}$-module $\tilde{H}^*(X/G;k)$ must be a simple subquotient of $\tilde{E}_1$ and hence of some $\tilde{H}^*(X/G_c;k)$. Because Z is without fixed points, $G_c<G$ for each c. By induction on dim G, we may assume that Theorem 2 holds for actions of G_c. Hence each simple subquotient of $\tilde{H}^*(X/G_c;k)$ is a subquotient of $\tilde{H}^*(X;k)$, and this proves Theorem 2 for the given action of G. The proof for a closed pair (X, Y) of G-spaces is similar, using a spectral sequence converging to

$$H^*(S^1)\otimes H^*(X/G, Y/G;k)$$

with

$$E_1=C^*_{\text{cell}}(Z/G;\mathcal{H}^*(X/G_c, Y/G_c;k)).$$

Proof of Theorem 2 for $G=\mathbf{Z}/p$ and $G=S^1$.

By Lemma 2, we may assume that G is acting semifreely. Let X_G be the Borel space of the G-action; it is the total space of a fibre bundle $X\to X_G\to B_G$ where B_G is the classifying space of principal G-bundles. We set $H^*_G(X)=H^*(X_G)$ and refer to [1, 6, 9] for the basic properties of this functor.

PROPOSITION 1. *Let G be a compact Lie group acting semifreely on a space X with fixed point set F. Then there is a long exact Mayer-Vietoris sequence of the form*

$$\cdots\xrightarrow{\delta} H^q(X/G)\to H^q(F)\oplus H^q_G(X)\to H^q_G(F)\xrightarrow{\delta}\cdots$$

Proof. Because the action is semifree and X is paracompact, there is an isomorphism

$$H^*(X/G, F)\to H^*_G(X, F)$$

induced by the projection $\pi: X_G\to X/G$, for any coefficient group. π induces, with its restriction to F_G, a homomorphism of long exact cohomology sequences,

$$(*)\qquad \begin{array}{ccccccc} \cdots\xrightarrow{\delta} & H^*_G(X, F) & \to & H^*_G(X) & \to & H^*_G(F) & \xrightarrow{\delta}\cdots \\ & \simeq\uparrow\pi^* & & \uparrow\pi^* & & \uparrow\pi^* & \\ \cdots\xrightarrow{\delta} & H^*(X/G, F) & \to & H^*(X/G) & \to & H^*(F) & \xrightarrow{\delta}\cdots \end{array}$$

The Mayer-Vietoris sequence is deduced from (*) by a standard argument, see p. 3 of [4]. Let P be a one-point space with its unique G-action. We set $\tilde{H}^*_G(X) = \operatorname{coker}(H^*_G(P) \to H^*_G(X))$. There is then a reduced Mayer-Vietoris sequence if $F \neq \varnothing$,

$$(RMV) \cdots \xrightarrow{\delta} \tilde{H}^*(X/G) \to \tilde{H}^*(F) \oplus \tilde{H}^*_G(X) \to \tilde{H}^*_G(F) \xrightarrow{\delta} \cdots$$

LEMMA 3. *Let $G = \mathbf{Z}/p$ or S^1 be acting semifreely on X with fixed point set $F \neq \varnothing$. Let $\mathcal{F}$ be a monoid of equivariant self-mappings of X. Then every simple subquotient of any of the three $\mathcal{F}$-modules $\tilde{H}^*_G(X; k)$, $\tilde{H}^*_G(F; k)$, and $\tilde{H}^*(F; k)$ is a subquotient of the $\mathcal{F}$-module $\tilde{H}^*(X; k)$.*

Proof. If k is of characteristic p, then $G = \mathbf{Z}/p$ or S^1. Because $\tilde{H}^*_G(F; k) = \tilde{H}^*(F; k) \otimes H^*(B_G; k)$ and the restriction homomorphism $\tilde{H}^*_G(X; k) \to \tilde{H}^*_G(F; k)$ is surjective in high degrees, it follows that every simple subquotient of the $\mathcal{F}$-modules $\tilde{H}^*(F; k)$ and $\tilde{H}^*_G(F; k)$ is a subquotient of $\tilde{H}^*_G(X; k)$. The fibre bundle $X \to X_G \to B_G$ gives a spectral sequence converging to $\tilde{H}^*_G(X; k)$ with

$$E_1 = C^*_{\text{cell}}(B_G; \mathcal{H}^*(X; k)).$$

Hence every simple subquotient of the $\mathcal{F}$-module $\tilde{H}^*_G(X; k)$ is a simple subquotient of the $\mathcal{F}$-module $\tilde{H}^*(X; k)$.

COROLLARY 1. *If $F \neq \varnothing$, then Theorem 2 holds for $G = \mathbf{Z}/p$, S^1.*

Proof. The reduced Mayer-Vietoris sequence (RMV) shows that every simple subquotient of $\tilde{H}^*(X/G; k)$ is a subquotient of $\tilde{H}^*_G(F; k) \oplus \tilde{H}^*_G(X; k) \oplus \tilde{H}^*(F; k)$. By Lemma 3, it is a subquotient of the $\mathcal{F}$-module $\tilde{H}^*(X: k)$.

When $F = \varnothing$, $G = \mathbf{Z}/p$ or S^1 is acting freely, and there is an isomorphism $H^*(X/G; k) \simeq H^*_G(X; k)$. There is the spectral sequence of the fibring $X_G \to B_G$ with

$$E_1 = C^*_{\text{cell}}(B_G; \mathcal{H}^*(X; k)),$$
$$E_2^{ab} = H^a(\mathbf{Z}/p; H^b(X; k)) \quad \text{for} \quad G = \mathbf{Z}/p, \quad \text{and}$$
$$E_2^{ab} = H^a(\mathbf{CP}^\infty) \otimes H^b(X; k) \quad \text{for} \quad G = S^1,$$

and converging to $H^*(X/G; k)$. To prove Theorem 2 in this case, it suffices to show that every simple subquotient of the $\mathcal{F}$-module E_∞/k (where $k \subset E_\infty^{00}$ is the field of coefficients) is a subquotient of $\tilde{H}^*(X; k)$. Clearly, for $r \geq 1$, $b > 0$, every simple subquotient of E_r^{ab} is a subquotient of $H^b(X; k)$. Hence, for $r \geq 2$, every simple subquotient of $d_r(E_r)$ is a subquotient of $H^+(X; k) = \sum_{b>0} H^b(X; k)$. For $a > c$, $c =$ the cohomology dimension of X over k, $E_\infty^{ao} = 0$. It follows that for $a > c$, each simple subquotient of E_2^{ao} is a subquotient of $H^+(X; k)$. As $\mathcal{F}$-modules, $E_2^{a0} \simeq E_2^{a+2o}$ for $a > 0$, and hence the last statement is valid for all $a > 0$. It remains only the module E_∞^{00}/k which is contained in $\tilde{H}^0(X; k)$, and the proof is complete for the case $F = \varnothing$.

The proof of Theorem 2 for a closed pair (X, Y) of G-spaces is quite similar to the proof in the absolute case with $F \neq \varnothing$. There is a Mayer-Vietoris sequence of a semifree group action,

$$\cdots \xrightarrow{\delta} H^*(X/G, Y/G) \to H^*(F, F \cap Y) \oplus H_G^*(X, Y) \to H_G^*(F, F \cap Y) \xrightarrow{\delta} \cdots$$

and there is a spectral sequence with

$$E_1 = C_{\text{cell}}^*(B_G; \mathcal{H}^*(X, Y; k)) \quad \text{converging to} \quad H_G^*(X, Y; k).$$

This completes the proof of Theorem 2.

Next we give a proof of Theorem 3 which states that $H^*(X/G; \mathbf{Z})$ is finitely generated when $H^*(X; \mathbf{Z})$ is finitely generated. A preliminary result is,

PROPOSITION 2. *Let X be a G-space with a closed invariant subspace Y. Assume that X has finite cohomology dimension over a field k. Then if $H^*(X, Y; k)$ is finite dimensional over k, so is $H^*(X/G, Y/G; k)$.*

Proof. The proof is basically the same as the proof of Theorem 2, but with simplifications. Lemma 2 is valid for the present proof. If $G = \mathbf{Z}/p$ or S^1 acting semifreely, the proof is a direct consequence of the Mayer-Vietoris sequence of a semifree group action and the fact that the restriction homomorphism $H_G^*(X, Y; k) \to H_G^*(F, F \cap Y; k)$ is an isomorphism in high degrees. The exact sequence

$$\cdots \xrightarrow{\delta} H^*(X/G; Y/G; k) \to H^*(F, F \cap Y; k) \oplus H_G^*(X, Y; k)$$
$$\longrightarrow H_G^*(F, F \cap Y; k) \xrightarrow{\delta} \cdots$$

then implies that $H^*(X/G; Y/G; k) \to H^*(F, F \cap Y; k)$ has finite dimensional kernel and cokernel. But $\dim_k H^*(F, F \cap Y; k) \leq \dim_k H^*(X, Y; k) < \infty$, and it follows that $\dim_k H^*(X/G, Y/G; k) < \infty$. In case G is connected simple, we use the spectral sequence of the first part of the proof of Theorem 2 with

$$E_1 = C^*_{\text{cell}}(Z/G; \mathcal{H}^*(X/G_c, Y/G_c; k))$$

and converging to $H^*(S^1) \otimes H^*(X/G; Y/G; k)$. By induction on $\dim G$, we may assume that $\dim_k H^*(X/G_c, Y/G_c; k) < \infty$ for each cell c of Z/G. Since Z/G is a finite cell complex, it follows that $\dim_k E_1 < \infty$, and hence that $\dim_k H^*(X/G, Y/G; k) < \infty$. This completes the proof of Proposition 2.

THEOREM 3'. *Assume that a compact Lie group G is acting on a space X which is paracompact Hausdorff and has finite cohomology dimension (over $\mathbf{Z}$). Assume that there is a finite number of conjugacy classes of isotropy groups. Let Y be a closed invariant subspace. Then if $H^*(X, Y; \mathbf{Z})$ is finitely generated, so is $H^*(X/G, Y/G; \mathbf{Z})$.*

Proof. Again, the proof is basically the same as that of Theorem 2, with some changes for finite G. Let G be finite. Let $q: (X, Y) \to (X/G, Y/G)$ be the orbit mapping, and let $t: H^*(X, Y; \mathbf{Z}) \to H^*(X/G, Y/G; \mathbf{Z})$ be the transfer mapping ([1] p. 38). Then tq^* is multiplication by $m = |G|$ in $H^*(X/G, Y/G; \mathbf{Z})$, and hence, $\operatorname{coker}(tq^*) \subset H^*(X/G, Y/G; \mathbf{Z}/m)$. Since tq^* factors through the finitely generated group $H^*(X, Y; \mathbf{Z})$, it suffices to show that $H^*(X/G, Y/G; \mathbf{Z}/m)$ is finitely generated. This is the case because, by Proposition 2, $H^*(X/G, Y/G; \mathbf{Z}/p)$ is finitely generated for each prime p. Now let G be a circle group. We may assume that G is acting semifreely, in which case the localization theory for circle actions is valid for cohomology with arbitrary coefficient group. Hence the argument in the proof of Proposition 2 is valid with integral coefficients. To prove Theorem 3' for general G, we may assume that G is connected, and that the theorem holds for all H with $\dim H < \dim G$, and hence that G is a connected simple group. Using the spectral sequence converging to $H^*(S^1) \otimes H^*(X/G, Y/G; \mathbf{Z})$, with $E_1 = C^*_{\text{cell}}(Z/G; \mathcal{H}^*(X/G_c, Y/G_c; \mathbf{Z}))$ where $\dim G_c < \dim G$, it follows that $H^*(X/G, Y/G; \mathbf{Z})$ is finitely generated.

Example. There is a pair (X, Y) of G-spaces and an equivariant mapping $f: (X, Y) \to (X, Y)$ such that a certain eigenvalue $\neq 1$ is of multiplicity one in $H^*(X/G, Y/G; k)$, and of multiplicity at least two in $H^*(X, Y; k)$. Let V be the linear space of all real n by n symmetric matrices of trace 0, and let X be the unitsphere in V. The group $SO(n)$ acts on X by conjugation with principal isotropy group $H \simeq (\mathbf{Z}/2)^{n-1}$. Let Y be the subspace consisting of all $x \in X$ such that G_x is not principal, equivalently such that $\dim G_x > 0$. In the author's paper

[11] there is constructed equivariant mappings $f_s: X \to X$ for $0<2s<n$, $n\geq 3$ of degrees $\deg f_s = 1-\binom{m}{s}$ where $2m\leq n\leq 2m+1$. Those mappings generalize the mapping f_m of Floyd-Hsiang, which is of degree 0 when $n=2m+1$. The mapping f_s' in the orbit space $\Delta = X/G$ is a self mapping of the orientable manifold-with-boundary Δ which is a simplex of dimension $n-2$. In $H^*(\Delta, \partial\Delta; \mathbf{Z}) = \mathbf{Z}$ f_s' induces multiplication by $\deg f_s'$, and by Theorem (2.1) of [11], $\deg f_s' = \deg f_s = 1-\binom{m}{s}$. It follows that in $\tilde{H}^*(\partial\Delta; \mathbf{Z}) \simeq \mathbf{Z}$, f_s' induces multiplication by $1-\binom{m}{s}$. Because $\partial\Delta = Y/G$, Theorem 2 implies that, for each field k, $1-\binom{m}{s}$ is an eigenvalue of $(f_s \mid Y)^*$ in $\tilde{H}^*(Y; k)$. From the exact sequence

$$0 \to \tilde{H}^*(Y; k) \xrightarrow{\delta} H^*(X, Y; k) \to \tilde{H}^*(X; k) \to 0$$

it follows that the eigenvalue $1-\binom{m}{s}$ has multiplicity at least two in $H^*(X, Y; k)$, while it has multiplicity one in $H^*(X/G, Y/G; k) \simeq H^*(\Delta, \partial\Delta; k) \simeq k$.

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