Nonlinear analysis of concrete structures by the imposed deformations method: comparison with experimental results

Nonlinear Analysis of Concrete Structures by the Imposed Deformations Method. Comparison with experimental results.

Analyse non-linéaire de structures en béton par la méthode des déformations imposées. Comparaison avec des résultats expérimentaux.

Nichlineare Berechnung von Betonkonstruktionen mit der Methode aufgezwungener Verformungen.

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SUMMARY
This paper presents a method to analyse structures with nonlinear materials, using the superposition of linear problems in which the actions can be of any kind: imposed deformations, loads, displacements. In the same way, the characteristics of the numerical resolution process are exposed and finally an application example is presented in which the results obtained by other analytical methods are compared with the ones obtained experimentally.

RÉSUMÉ
Dans cet exposé on présente une méthode d'analyse de structures avec des matériaux non-linéaires en utilisant la superposition de problèmes linéaires dans lesquels les actions peuvent être de n'importe quel type: déformations imposées, charges, déplacements. Cette méthode peut s'utiliser manuellement ou avec un computer, si bien ici on utilise l'ordinateur. En dernier lieu, on présente un exemple d'application avec lequel on compare les résultats obtenus par d'autres méthodes et avec des résultats expérimentaux.

ZUSAMMENFASSUNG
In diesem Berich wird eine Methode für die Berechnung von Konstruktionen mit nichtlinearen Materialien gegeben. Wir benutzen die Überlagerung von aufgezwungenen Verformungen, Lasten, Verschiebungen. Wir beschreiben auch die Merkmale der numerischen Lösungsmethode und geben ein Anwendungsbeispiel, in dem wir die erzielten Ergebnisse mit denen anderer analytischer Methoden und mit Experimenten vergleichen.

1. INTRODUCTION

The concrete nonlinear behaviour and its influence in hyperstatic structures made out of prismatic pieces is known since the beginning of this century, although its adoption by the different national normatives was made later. The research on this subject can be made by any of the existing methods which go from the so called "exact methods" until the "approximated methods". Those in the first group demand the simultaneous fulfilment of the different conditions which form the structural analysis such as: equilibrium, compatibility and material conditions. Those included in the second group normally imply the partial fulfillment of the previous conditions, but they have the advantage of giving, in general, simple solutions more or less precise regarding the exact solution.

The nonlinear analysis (by the material) of structures can be made, from our point of view, according to compact analysis or to separative analysis:

1.- In the compact analysis, the material nonlinear characteristics are introduced in each section, changing its mechanical properties (i.e. area, inertia, modulus of elasticity, etc...) This fact sticks out the mechanical treatment of the structure with an equilibrium preponderance.

2.- In the separative analysis, the nonlinear traits of the material are introduced in each section (or zone) as a virtual action of geometrical kind (strain or imposed rotation) (1). This fact means a geometrical treatment of the structure with compatibility preponderance.

In each of these two ways of analysis, there are methods of direct resolution or through iterations. This case implies generally the utilization of computers, hypothesis of great precision, etc...

Inside the general framework of nonlinear methods of analysis, which we have just established, the method set out below fits into the exact methods treating the problem by a separative analysis in which we can work with linear problems superposing them, and obtaining the right solution by iterations.

2. PROPOSED ANALYSIS METHOD

2.1 Calculation Bases

The material behaviour is presented on a section level by a nonlinear diagram Moment-curvature (M-C) which means a section by section treatment and therefore, we do not work with concentrate rotations in plastic hinges . (Diagram M-θ) (1), (2).

The diagram shape M-C is not intrinsic to the method of analysis, and we can use any of the diagrams adopted by the different authors (G. MACCHI (1), A.L. BAKER (2)), as: curve, bilinear, trilinear, etc... Fig. 1. In case of using multilinear

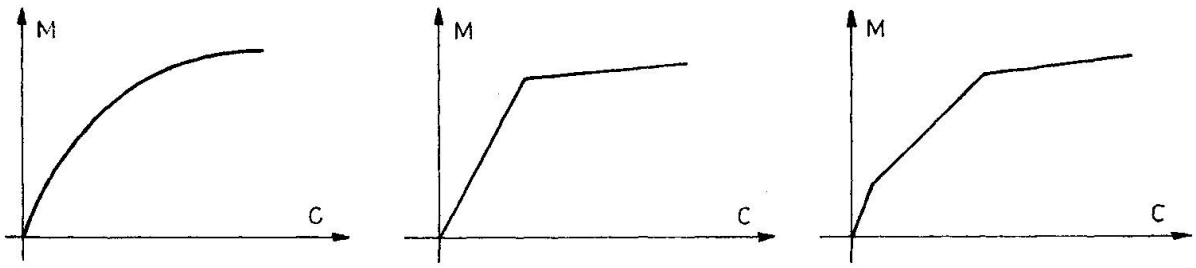

Fig. 1 Moment-Curvature diagrams.

diagrams the slope of any of the straight lines must not be null, because we will have problems in the numerical resolution.

The M-C diagrams have been obtained numerically according to G. MACCHI and E. SIVIERO (3) where the tension stiffening is considered as a mean to achieve a greater precision when we analyse the structure in service situation.

The method allows to analyze the structure behaviour in front of any action: loads, strains (increases and gradients of temperature, shrinkage, creep, etc...) and displacements (bearing settlements, etc...).

As data issue for the calculation steps in which the method is detached, we have: the actions working in the structure (they can be any of those mentioned before) and the Moment-curvature diagrams of each section Fig. 2.b.

In this case we have not considered the strains caused by shear or axial forces although the proposed method can consider these strains as another virtual action.

2.2 Calculation steps

In the first step, we take the actions working in the structure and we make a linear calculation of it, taking the stiffness to bending K_o =EI (Fig. 2.a). This way we obtain the effects (moment, shear, axial force, etc...) to which each section is submitted.

With the structure linear calculation in this step, we have got a balanced and compatible solution, but it does not fulfill with the real characteristics of the material. In a generical section, the solution obtained by the calculation is represented by the point A (Fig. 2.a) while the fulfillment of the material conditions would require to be in the point B (Fig. 2.b).

In the second step the curvature increases (ΔC_1), of each section between the real curvature and the one obtained in the calculation of the first step (C_A), are not generally compatible. If the structure is isostatic, these strains mean an increase of strain, but no forces are shown; however in a hyperstatic structure besides the strains increase some hyperstatic effects are shown.

The calculation in this second step is made by a linear calculation with the initial traits of the structure (bending stiffness Ko) taking imposed deformations

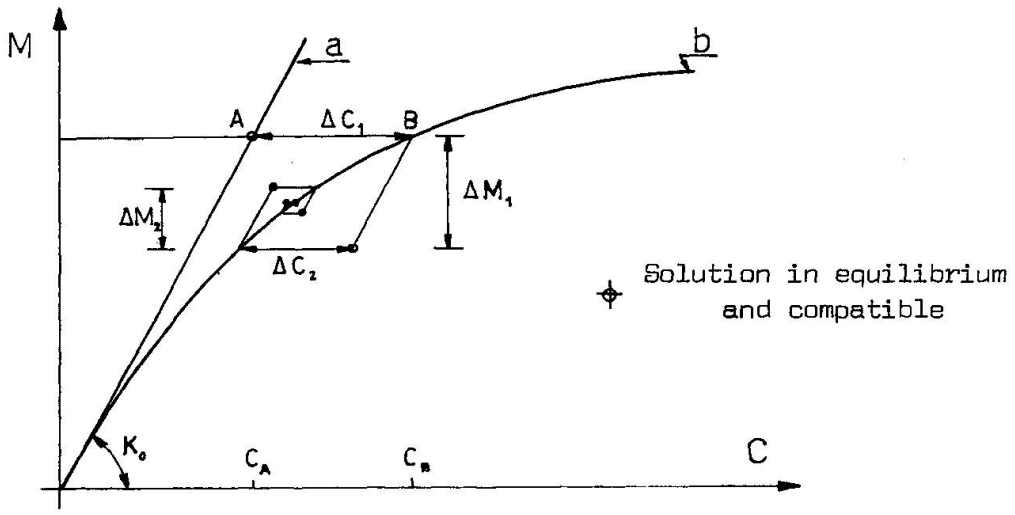

Fig. 2 Sectional study of the proposed analysis method.

(Curvature increases ΔC_1) as virtual actions in each section. With these calculations we get the hyperstatic load effects which make compatible the curvature system; in a section this hyperstatic moment is given by ΔM_1. Fig. 2. Structural analysis procedure taking imposed deformations (curvatures system) as actions can be seen in ref. (4).

Since we work below the dominion of small strains in each of these two calculation steps, either the superposition of the deflection system (y) or the superposition of the curvature system ($c \simeq y''$) are valid, and so the superposition principle can be applied in the analysis in first order. Consequently we superpose to the forces obtained in the first step those deduced in the second one, getting in a section the point 1. This point is on a parallel to the initial stiffness K_0 since we work with this stiffness in the two linear problems.

The point 1 represents a balanced and compatible solution to the problem, although it does not fulfill the real characteristics of the material. This is why it would be necessary to introduce, as action, the strain system given by the different curvatures increases ΔC_2 of each section.

The determination of the hyperstatic forces system (ΔM_2) caused by this curvature system (ΔC_2) is made in the same way to the one mentioned previously. Thus, this procedure implies an iterating process in which we pivot over the material real conditions for each iteration. We get the final solution when the forces system (ΔM_i) vanishes. In this moment (point S) we have a balanced and compatible solution which fulfills the material traits.

As we have seen this process does not impose any conditions about which must be the situation the structure is found. This is why is valid either for service situations or for situations next to failure. In the same way this process is valid in front of any kind of action (strain, loads, movements) working in the structure which is considered in the first calculation step, while in the second step we take the curvature increases ΔC as another (virtual) action.

2.3 Numerical resolution

In order to reflect the material nonlinear behaviour through the diagram M-C in each section, an accurate definition is needed for all the sections of the different elements. In the numerical resolution each element is divided in n parts being determined in every of them a diagram M-C representing all the sections of it. The number of parts n depends on the kind of structural elements (constant or veriable inertia, reinforcement distribution) and it often changes between 1 and 5, being in the practice the most common n=3.

Every one of the linear problems representing both calculation steps can be approached either by compatibility or equilibrium and its resolution can be achieved by any of the existing calculation methods, which means that the process of the nonlinear analysis exposed here by, has a great flexibility according to the resolution system.

Since the initial characteristics (stiffness or flexibility matrix) of the structure are kept, in order to simplify the calculations, it is interesting to systematize them. We have to use, for that, the same method in the two steps and this calculation, in its turn, must systematize at the maximum the operations to be done.

To obtain the numerical solution a convergence criterion defining the end of the iterations must be fixed. Its number depends on different factors: precision defining the final solution, solution being the issue for the analysis (first step) numerical process used, etc.

As follows we study the incidence of every one.

a.- The precisions is influenced in one hand by the adopted convergence criterion. In this case it is established according to a certain percentatge of the initial effects in some definite sections e.g.:

$$\Delta M_i \leq \alpha \cdot M_0$$

where

ΔM_i = Increase of moments in the fixed sections for two consecutives iterations.
α = Fixed percentatge according to the criterion (values below 5%)
M_0 = Bending moment in the fixed sections obtained in the first step, and the other hand, the number of sections in which the mentioned criterion is verified, in this case, we have taken the sections at the edge of each bar.

b.- In a hyperstatic structure the moment law deduced from the first calculation step depends on the stiffness K_o adopted. Thus, if this stiffness is near the ultimate value, the number of iterations is going to be small, in the opposite case this number will increase.

c.- The numerical process can follow the described theorical process which graph is made in Fig. 2, or can reckon among the results obtained in the first calcula

tion step and those of the previous iteration to the one being reckoned. With this procedure the convergence proves to be faster.

In the examples presented in (4), (5), considering between 2% and 4%, and establishing the convergence criterion in both edge sections of every bar, the number of iterations needed to reach the numerical solution, were 2 or 3 for service situations and something higher (5 or 6) for situations near failure.

Finally the method of analysis said above, can be applied manually (for small grade of hyperstaticity structures, e.g. a bridge) or by computer (in high grade of hyperstaticity structures e.g., frames).

3. EXAMPLE : COMPARISON WITH EXPERIMENTAL RESULTS.

We have shown until this point the validity of this method from a theoretical point of view. An enlargement of the subject as well as its comparison with analytical methods can be seen in the ref. (4). To conclude we are going to show its practical validity by the comparison with experimental results.

As experimental test we will take one of the test sequences made by A. MATTOCK (6) on a continuous beam of two spans (Fig. 3) with a concentrated load at the middle of one of them. The choice of this test brings us an additional advantage since it allows us to compare at the same time the results with those achieved by other analytical methods, which have been explained by A. GRELAT (7) and A. APARICIO (8).

The Moment-curvature diagrams (M-C) of the sections were obtained numerically by computer and then an adjustement of trilinear diagrams to them was made. Nevertheless and as we have said, the kind of diagram M-C is not intrinsic to the proposed method. The use of multilinear diagrams (trilinear in this case) has the advantage on one hand of getting and accurate adjustement regarding the one got numerically, and on the other hand it allows to obtain a simple expression of the curvature increases (ΔC) taken as virtual actions in the second calculation step. This fact is to the advantage of calculations systematization mentioned previously.

To obtain the diagram MC concrete contribution between cracks (tension stiffening) has been considered. This way we obtain a better precision in the structure behaviour in front of service load, which according to A. GRELAT can be evaluated in this case up to 20%. (7)

The results obtained by the different methods are presented in Fig. 4 by a Moment-load diagram corresponding to the sections where the load is applied (section B) and the support one (section C).

In the proposed analysis method the value to a load P_u=70,4 KN has been considered as ultimate moment since the result obtained experimentally P_u = 74,2 KN. Fig. 4 can be due to a hardening of the steel stress-strain diagram after the

yielding point.

The results exposed in the mentioned diagram state the accurate precision of the proposed method, as well as the other analytical methods regarding the experimental results either in failure situations or in service situations (in the last the consideration of the tension-stiffening has an special influence).

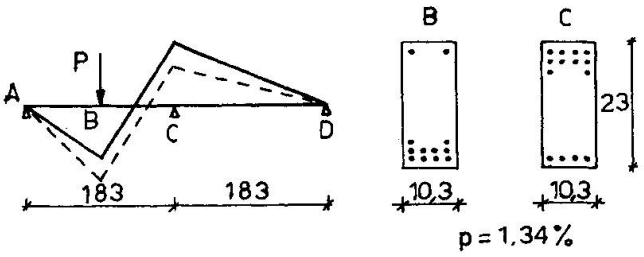

Fig. 3 Analysed structure.

Thus, if we accept the accurate precision of the analytic methods the argument will be centred in the advantages and disadvantages of each one. We would like to review those advantages and disadvantages as a way of concluding, regarding the method we are proposing.

4. CONCLUSIONS

In the field of linear structures this method allows to work with any kind of action (load, strain, movements) and specially with imposed deformations due to increases and gradients of temperature (5), shrinkage, creep, etc...

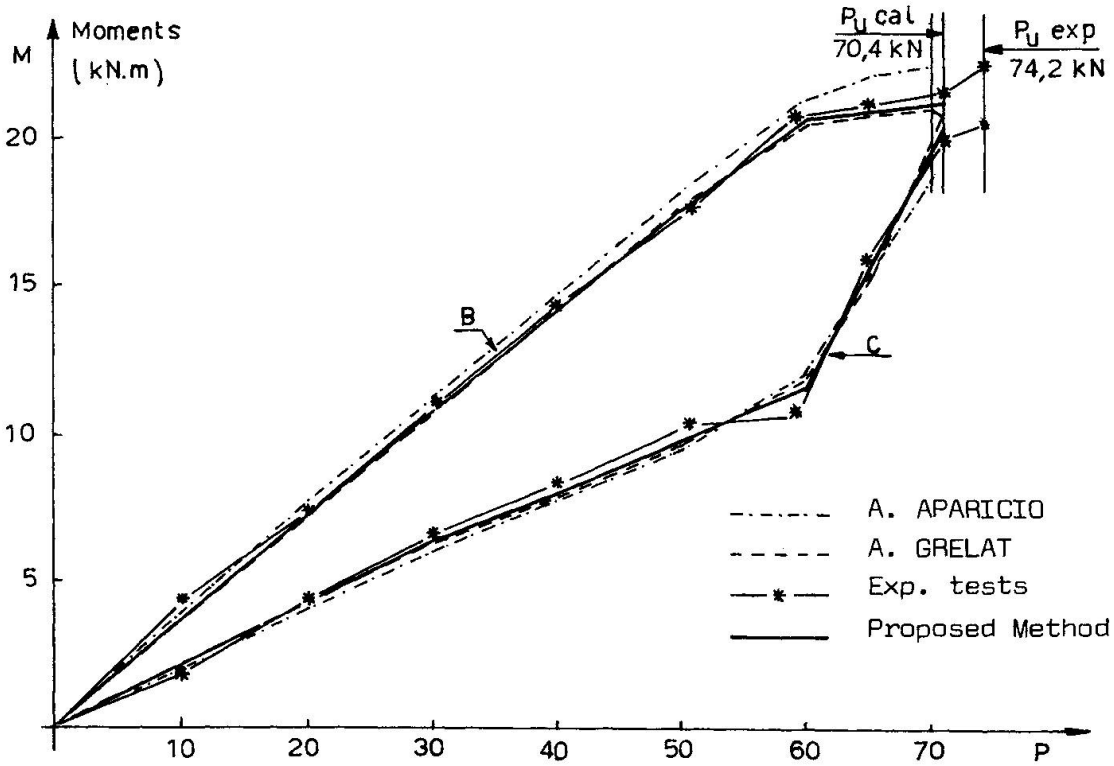

Fig. 4 Bending Moment evolution in a continuous beam under increasing load.

The method belongs to the group of "exact methods" since the conditions of equilibrium, compatibility and material constitutives are fulfilled. The comparison with experimental results confirms the accuracy with a great precision of the numerical solution.

The practical application of the method has a great independence regarding the concrete procedure of resolution of both steps, since it can be approached by compatibility methods and by equilibrium methods.

We always work with linear problems with only one stiffness or flexibility matrix. This is the reason why they can be superposed validly. In the method resolution by computer this trait allows to take advantage of common blocks, decreasing the execution time and the capacity of memory needed.

Finally, this method has a great field of application since it allows to analyze any kind of linear structure: Bridges (5), continuous beams, multistory frames (4), etc.

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