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# Differential Creep, Shrinkage and Stress Redistribution in Composite Prestressed Concrete Beams

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In the preliminary publication, a study is contained on differential shrinkage (1). In addition to the theories by Birkeland, Evans and Branson (Ref.4-6 of paper (1)) that of Evans and Parker (2) should be mentioned. All these theories are based on the same assumptions i.e. that the free shrinkage of the prestressed and non-prestressed components, acting at the respective centroids, are known and that the resulting strain distribution is straight. Thus, if the free shrinkage and/or creep strains of the prestressed component is  $\epsilon_{pf}$  and that of the non-prestressed component is  $\epsilon_{nf}$ , it is possible to obtain the position of the resulting strain distribution from the difference of these two free strains  $\Delta\epsilon = \epsilon_{nf} - \epsilon_{pf}$ , as seen from Fig.1 in which the strain distributions due to (i) the free and (ii) the resultant strains are plotted. It is shown in this figure that the actual resulting strain at the centroid of the prestressed component is  $\epsilon_{pf} + \Delta\epsilon_p$  and that at the centroid of the non-prestressed component amounts to  $\epsilon_{nf} + \Delta\epsilon_n$ , where  $\Delta\epsilon_p = \Delta\epsilon/K_1$  and  $\Delta\epsilon_n = \Delta\epsilon/K_2$ . The two constants  $K_1$  and  $K_2$  depend solely on the properties of the sections and E-values and amount to:

$$K_1 = A_{pc}/n_o \cdot A_n + (A_p \cdot e_o^2)/(I_p + n_o \cdot I_n) \text{ and } K_2 = A_{pc}/A_p + (n_o \cdot A_n \cdot e_o^2)/(I_p + n_o \cdot I_n)$$

In these equations  $A_p$ ,  $A_n$  and  $A_c$  are the respective cross sectional areas of the prestressed, non-prestressed and composite sections;  $I_p$  and  $I_n$  are the corresponding I-values and  $n_o = E_{cn}/E_{pn}$  is the ratio of  $E_c$ -values of the two components, whereas  $e_o$  is the vertical distance between the two centroids. The remaining strains can be computed from the geometrical conditions when the strains at the two centroids are known. This relationship has been published in (3) but was used already in paper (4) at the IABSE Congress Stockholm 1960. Fig.2, taken from this paper, shows comparative results of three different cross section. It is seen that only with a cross section according to example No.1

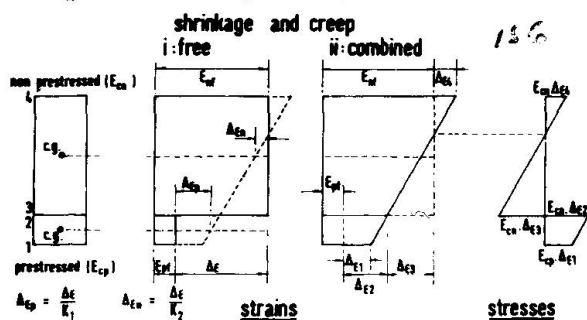


Fig.1

Diagrams of strains and stresses due to differential shrinkage and creep.

similar results are obtained to those, presented by the authors in paper (1), resulting in additional tensile stresses due to differential shrinkage at the outer tensile face of the section. With examples 2 and 3, compressive stresses

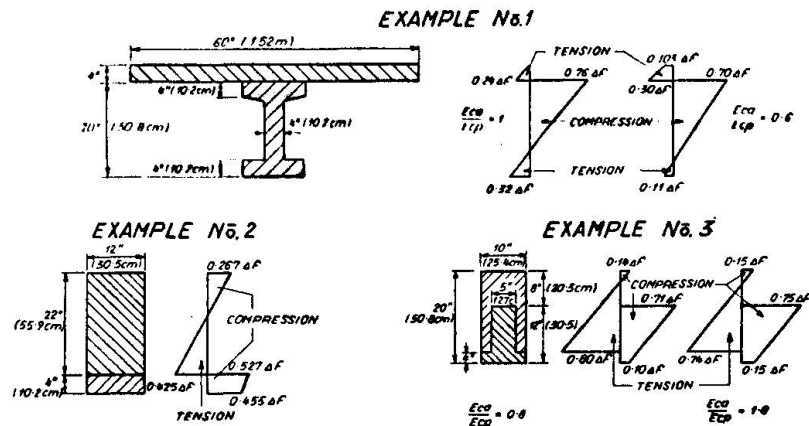


Fig.2. Examples: Stresses due to differential shrinkage, as shown at the IABSE Congress Stockholm, 1960.

are induced by differential shrinkage, whereas tensile stresses occur at the lower face of the non-prestressed component. This results in a stress redistribution owing to the greater stress difference at the adjoining faces of the prestressed and non-prestressed components, as discussed in paper (4). In this case cracks became visible at a lower nominal tensile stress at the outer tensile face than is the case in similar homogeneous sections.

In order to clarify the stress redistribution and to ascertain the magnitude of differential shrinkage and creep in the prestressed and non-prestressed components, tests were carried out at DUKE University, North Carolina, U.S.A. in 1967/68 which are to be described in detail elsewhere (5). Here, only some important results are illustrated. Fig.3 shows particulars of the specimens. It was endeavoured to study the extreme cases at which either mainly creep or shrinkage takes place ('C' and 'S' beams); the former was obtained when the added concrete was cast upon the prestressed concrete on release of the prestress after the plank had been moist cured to avoid shrinkage. For the 'S' beams, relating to differential shrinkage, the added concrete was placed much later, after a substantial part of shrinkage and creep of the prestressed component had already taken place. The third case, relating to stress redistribution ('R' beams), was investigated in such a way that the two components were separately

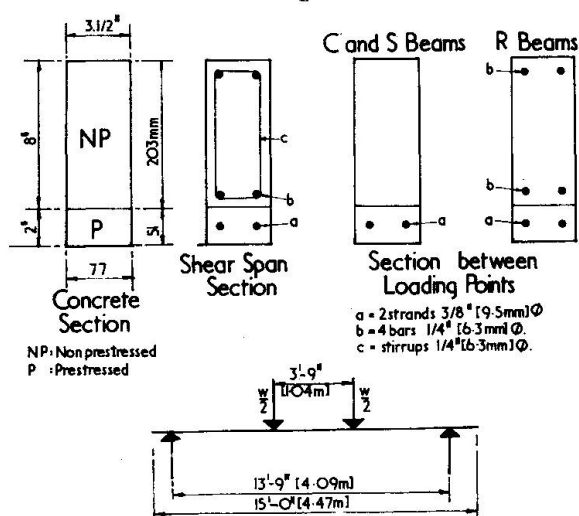


Fig.3. Specimens at tests DUKE University.

precast and then glued together which allowed strain measurements. Stirrups were provided in the shear spans of the non-prestressed component only and the reinforcement 'b' was limited to the shear spans in the 'C' and 'S' beams, but provided along the entire length in the 'R' beams.

Some of the results are shown in figure 4. Generally, with the 'C' beams the precompression was greatly reduced by the creep in the prestressed plank (which was further increased by differential creep), whereas with the 'S' beams the shrinkage stresses in the added concrete and the compressive stresses in the prestressed component are increased. The loads at which visible

Specimen	Nominal Differential Stresses	Self Load [W=0]	Microcracking		Visible cracking
			N.P. Concrete	P. Concrete	
C1 [creep only]			W=4500 		W=5600 
S1 [shrinkage only]			W=3500 		W=5500 
R3 [glued 12 days before testing]			W=3500 	W=5000 	W=6500 

Fig.4. Some Results, DUKE University Tests 1967/68.  
(Stresses in psi. (lbf/in.<sup>2</sup>); Loads in lbf).

cracks occurred were approximately the same for the 'C' and 'S' beams, although the theoretical stress conditions were completely different in both cases. With the 'S' beams there were relatively high nominal concrete stresses in the non-prestressed component already under self load, when microcracks developed, as can be seen from the photograph Fig.5. This was obtained from a photoelastic coating, using a method, as described in paper (6). With the 'C' beams microcracks occurred at a later stage.

In the 'R' beams microcracks developed first in the non-prestressed concrete at the same load at which they had already become visible in the prestressed plank of the 'S' beams. Microcracking in the prestressed concrete occurred at a load slightly less than that at which they had become visible with the 'C' and 'S' beams. However, the load at which the cracks became visible in the 'R' beam was appreciably higher than those in the 'C' and 'S' beams. These studies have shown that redistribution of stresses may cause visible cracking at relatively low nominal tensile stresses at the outer face, if there are very high nominal tensile stresses in the non-prestressed component.



Fig.5. Microcracking in 'S' beam under self load.

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