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# Analysis of Continuous Box Girder Bridges Without Interior Diaphragms

Etude sur les ponts à poutres continues en caisson sans cloison transversale

Analyse von Brücken mit durchlaufenden Kastenträgern ohne Querwände

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### Introduction

The increasing use of box girder sections in the construction of highway bridges and elevated motorways is evidenced by the number of box girder bridges constructed in the past decade and the number of recent publications on box girders. In using box girder sections to span long bridges, the provision of intermediate supports becomes necessary. Such continuous box girders may or may not have interior diaphragms. While the use of interior diaphragms have definite functional advantage, other requirements may dictate the use of box girders without interior diaphragms. For instance, the Nusle Bridge in Czechoslovakia, Křístek (1970), is made up of continuous single-cell box girders with spans of almost 400 feet. Its upper flange carries the highway while the underground goes through the box cell and therefore interior diaphragms are not feasible.

This paper presents a method of analysing continuous multi-cell box girder bridges without interior diaphragms as shown in Fig. 1. The box girder is assumed to be simply supported at its two transverse edges and the intermediate supports can be placed arbitrarily along the span. As shown in Fig. 1, the intermediate support reaction may be treated as line reaction with uniform intensity along the dimension of the support in the longitudinal direction if it is located at an edge formed by the intersection of plate elements or as uniformly distributed reaction acting over the finite area of the support if it is

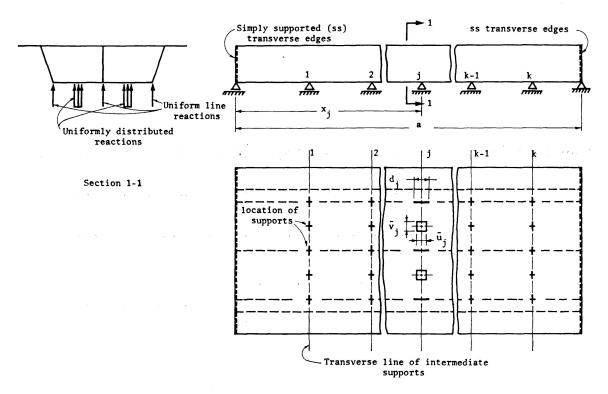


Fig. 1. Multi-cell box girder with intermediate supports.

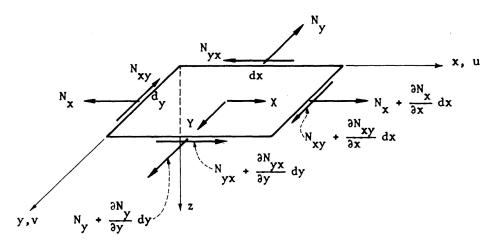
located elsewhere in the plate element. It is further assumed that the intermediate supports offer no bending resistance.

The proposed method of analysis is based on the folded plate theory presented elsewhere. The structure is therefore considered to be made up of elastic, homogeneous and isotropic plate elements with simple supports at the transverse edges. The theory utilizes the plane stress elasticity theory which is formulated in terms of Airy's stress function and the classical thin plate bending theory to determine the membrane and the bending actions of the individual plate elements. The theory has also been applied to the analysis of simply supported box girders without intermediate supports, Pulmano (1972).

In a similar manner to the analysis of frames using the flexibility method, the solution to the problem entails the following steps: (1) the vertical displacements of the primary structure due to the applied loads at the location of the intermediate supports are determined; the primary structure being the structure without the intermediate supports; (2) the flexibility influence coefficients at these locations are calculated; (3) using the results of steps (1) and (2), the magnitude of the intermediate support reactions are determined from the compatibility conditions that there are no vertical displacements of the structure at the supports; and (4) with the intermediate support reactions known, the magnitude of the stress resultants and displacement components of the actual structure are obtained by superposition.

## Theory

The detailed treatment of the folded plate theory used in the analysis has been presented elsewhere, Lee, Pulmano and Lin (1965), Pulmano and Lee (1965). For the sake of completeness and brevity of presentation, only pertinent equations and expressions are given.



a) Membrane stress resultants

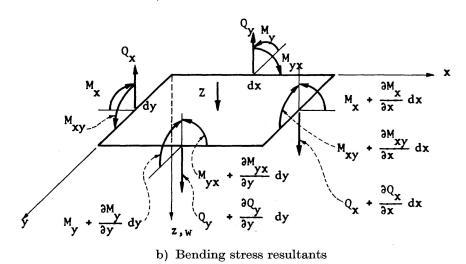


Fig. 2. Positive directions of stress resultants, displacement components and load components.

An element of the middle surface of the plate as shown in Fig. 2, shows the positive directions of stress resultants, the load components X, Y and Z and the displacement components u, v and w. The orientation of the plane of an individual plate is defined by the angle  $\theta$ , which is measured clockwise from the horizontal plane to the xy-plane which contains the plane of the middle surface of the plate in question.

For the case where the load component, X, is taken equal to zero, the equations governing the membrane and the bending actions of the plate are given respectively by

$$\nabla^4 \phi = \frac{\partial^2}{\partial x^2} (\int Y \, dy) - \mu \, \frac{\partial Y}{\partial y} \tag{1}$$

and

$$\nabla^4 w = \frac{Z}{D},\tag{2}$$

in which

$$D = \frac{E h^3}{12 (1 - \mu^2)}. (3)$$

E, the modulus of elasticity,  $\mu$ , the Poisson's ratio, h, the thickness of the plate and  $\phi$ , the Airy's stress function which is related to the membrane stress resultants as follows:

$$N_x = \frac{\partial^2 \phi}{\partial y^2},\tag{4}$$

$$N_y = \frac{\partial^2 \phi}{\partial x^2},\tag{5}$$

$$N_{xy} = -\frac{\partial^2 \phi}{\partial x \, \partial y}.\tag{6}$$

In view of the boundary conditions at the transverse edges of the plate, that is

$$N_x = M_x = w = v = 0$$
 (at  $x = 0$  and  $x = a$ ) (7)

a, being the length of the plate, the complete solution of Eqs. (1) and (2), are given respectively by

$$\phi = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \phi_{mn} \cos \beta y \sin \alpha x + L \sum_{m=1}^{\infty} \{A_{1m} e^{\alpha y} + A_{2m} e^{-\alpha y} + A_{3m} \alpha y e^{\alpha y} + A_{4m} \alpha y e^{-\alpha y}\} \sin \alpha x$$
(8)

and

$$w = \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} W_{mn} \cos \beta \, y \sin \alpha \, x + K \sum_{m=1}^{\infty} \{ B_{1m} \, e^{\alpha y} + B_{2m} \, e^{-\alpha y} + B_{3m} \, \alpha \, y \, e^{\alpha y} + B_{4m} \, \alpha \, y \, e^{-\alpha y} \} \sin \alpha \, x \,, \tag{9}$$

in which  $A_{1m} 
ldots A_{4m}, B_{1m} 
ldots B_{4m}$  are constants of integration,  $\alpha = \frac{m \pi}{a}$ ,  $\beta = \frac{n \pi}{b}$ , b, the width of the plate and K, L are dimensional factors. In Eqs. (8) and (9), the Fourier coefficients  $\phi_{mn}$  and  $W_{mn}$  are defined as follows:

$$\phi_{mn} = \frac{\alpha^2 - \mu \, \beta^2}{\beta \, (\alpha^2 + \beta^2)^2} \, Y_{mn}, \tag{10}$$

in which 
$$Y_{mn} = \frac{4}{ab} \int_{0}^{a} \int_{0}^{b} Y(x, y) \sin \alpha x \sin \beta y \, dx \, dy \tag{11}$$

and 
$$W_{mn} = \frac{1}{D} \frac{Z_{mn}}{(\alpha^2 + \beta^2)^2},$$
 (12)

in which 
$$Z_{mn} = \frac{4}{ab} \int_{0}^{a} \int_{0}^{b} Z(x, y) \cos \beta y \sin \alpha x \, dx \, dy$$
  $(m, n = 1, 2, 3...),$  (13)

$$Z_{m0} = \frac{2}{a} \int_{0}^{a} Z(x) \sin \alpha x \, dx \qquad (n = 0, m = 1, 2, 3...). \tag{14}$$

The resulting expressions for the in-plane displacement components, u and v, which can be obtained utilizing the force-displacement relations, are

$$u = \frac{1}{E h} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left\{ \phi_{mn} \left( \frac{\beta^2 - \mu \alpha^2}{\alpha} \right) + \frac{\mu}{\alpha \beta} Y_{mn} \right\} \cos \beta y \cos \alpha x$$

$$- \frac{L}{E h} \sum_{m=1}^{\infty} \alpha \left\{ A_{1m} (1 + \mu) e^{\alpha y} + A_{2m} (1 + \mu) e^{-\alpha y} + A_{3m} (2 + \alpha y + \mu \alpha y) e^{\alpha y} + A_{4m} (-2 + \alpha y + \mu \alpha y) e^{-\alpha y} \right\} \cos \alpha x$$
(15)

and 
$$v = \frac{1}{Eh} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left\{ \frac{\phi_{mn}}{\alpha^2 \beta} \left[ \alpha^2 \left( \mu \beta^2 - \alpha^2 \right) + (\alpha^2 + \beta^2)^2 \right] - \frac{\mu}{\alpha^2} Y_{mn} \right\} \sin \beta y \sin \alpha x$$

$$- \frac{L}{Eh} \sum_{m=1}^{\infty} \alpha \left\{ A_{1m} \left( 1 + \mu \right) e^{\alpha y} - A_{2m} \left( 1 + \mu \right) e^{-\alpha y} \right. \tag{16}$$

$$+ A_{3m} \left[ \left( \mu - 1 \right) + \alpha y \left( 1 + \mu \right) \right] e^{\alpha y} + A_{4m} \left[ \left( \mu - 1 \right) - \alpha y \left( 1 + \mu \right) \right] e^{-\alpha y} \right\} \sin \alpha x.$$

For later use, the expressions of the stress resultants are given in Appendix 1.

# **Boundary Conditions**

The boundary conditions at the longitudinal edges of the plate are treated as follows:

a) At a free edge, 
$$N_{xy} = N_y = R_y = M_y = 0$$
. (17)

b) At an edge lying in a vertical plane of symmetry,

$$N_{xy} = 0, (18a)$$

$$N_y \sin \theta + R_y \cos \theta = 0, \qquad (18b)$$

$$v\cos\theta - w\sin\theta = 0, \tag{18c}$$

$$\frac{\partial w}{\partial y} = 0. {(18d)}$$

c) At an edge lying in a vertical plane of antisymmetry,

$$N_y = M_y = w = u = 0. (19)$$

d) At a continuous edge which is formed by r number of plates, there are four equilibrium conditions and 4(r-1) compatibility conditions.

## Reactions of Intermediate Supports

The reactions of intermediate supports may be treated in two ways, namely:

a) Reaction as Uniform Line Load. The reaction of an intermediate support at j located at a longitudinal edge as shown in Fig. 1, is assumed as line reaction of uniform intensity throughout the support dimension in the longitudinal direction. The magnitude of the uniform line reaction,  $V_j$ , is defined by

$$V_i = 0$$
  $0 \le x < (x_i - \frac{1}{2}d_i),$  (20a)

$$V_{j} = \frac{R_{j}}{d_{j}} \qquad (x_{j} - \frac{1}{2} d_{j}) < x < (x_{j} + \frac{1}{2} d_{j}), \qquad (20 b)$$

$$V_j = 0$$
  $(x_j + \frac{1}{2} d_j) < x \le a,$  (20c)

in which  $R_j$  is the total support reaction at j and  $d_j$  the support dimension in the longitudinal direction. The vertical line reaction,  $V_j$ , maybe expanded into a series

$$V_j = \sum_{m=1}^{\infty} V_{mj} \sin \alpha x, \qquad (21)$$

where

$$V_{mj} = \frac{4 R_j}{m \pi d_j} \sin \alpha x_j \sin \frac{1}{2} \alpha d_j.$$
 (22)

For k number of intermediate supports at a longitudinal edge, the total vertical line reaction is

$$V = \sum_{j=1}^{k} V_{j} = \sum_{m=1}^{\infty} \left\{ \sum_{j=1}^{k} \left( \frac{4 R_{j}}{m \pi d_{j}} \sin \alpha x_{j} \sin \frac{1}{2} \alpha d_{j} \right) \right\} \sin \alpha x.$$
 (23)

b) Reaction as Uniformly Distributed Load. In the case where the intermediate support at j is placed at a location other than the longitudinal edge, the support reaction is assumed to be uniformly distributed over the area of the support. The expression for  $Z_{mn}$  in Eq. (12) is given by

$$(Z_{mn})_{j} = \frac{16 R_{j}}{m n \pi^{2} \overline{u}_{j} \overline{v}_{j}} \sin \alpha \xi_{j} \sin \beta \eta_{j} \sin \frac{1}{2} \alpha \overline{u}_{j} \sin \frac{1}{2} \beta \overline{v}_{j}, \qquad (24)$$

in which  $\overline{u}_j$  and  $\overline{v}_j$  are the cross-sectional dimensions of the support at j in the longitudinal and transverse directions, respectively and  $\xi_j$ ,  $\eta_j$  are the coordinates of the support in the x, y directions from the origin.

The magnitude of the support reaction,  $R_j$ , can be determined from the compatibility conditions that the vertical displacements at the center of the supports vanish, that is

$$\sum_{j=1}^{k} a_{ij} R_j + \Delta_i = 0 \qquad (i = 1, 2, \dots k),$$
 (25)

where  $a_{ij}$  is the flexibility influence coefficients defined as the vertical displacement at i due to  $R_j = 1$  and  $\Delta_i$  denotes the vertical displacement of the primary structure at i due to the applied loads.

## Illustrative Example

To illustrate the application of the proposed solution, a single-cell box girder is analysed. The girder, assumed to be simply supported at its transverse edges, has one transverse line of intermediate supports located at the midspan which contains two supports placed symmetrically about the centreline of the box girder section as shown in Fig. 3. The box girder, subjected to

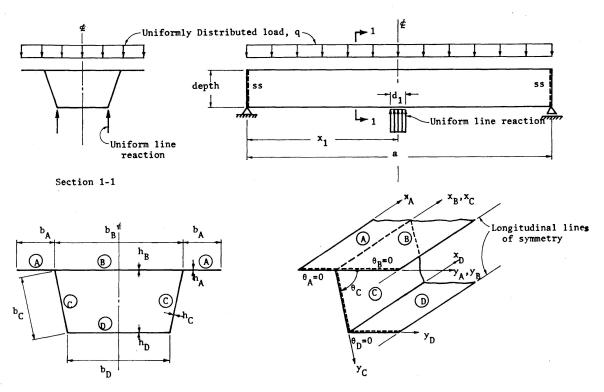


Fig. 3. Illustrative example - two-span single-cell box girder.

a uniformly distributed load of intensity q per unit area acting throughout the top deck, is analysed considering the values of the following parameters:

$$\mu = 0.2, \qquad \frac{x_1}{a} = 0.5,$$

$$\theta_A = \theta_B = \theta_D = 0, \qquad \theta_C = \arctan(0.2),$$

$$\frac{a}{h_A} = \frac{a}{h_B} = \frac{1200}{7}, \qquad \frac{d_1}{a} = 0.01,$$

$$\frac{a}{h_C} = 100, \qquad \frac{a}{h_D} = 200,$$

$$\frac{b_A}{a} = 0.03, \qquad \frac{b_B}{a} = 0.1,$$

$$\frac{b_C}{a} = 0.05099, \qquad \frac{b_D}{a} = 0.08.$$

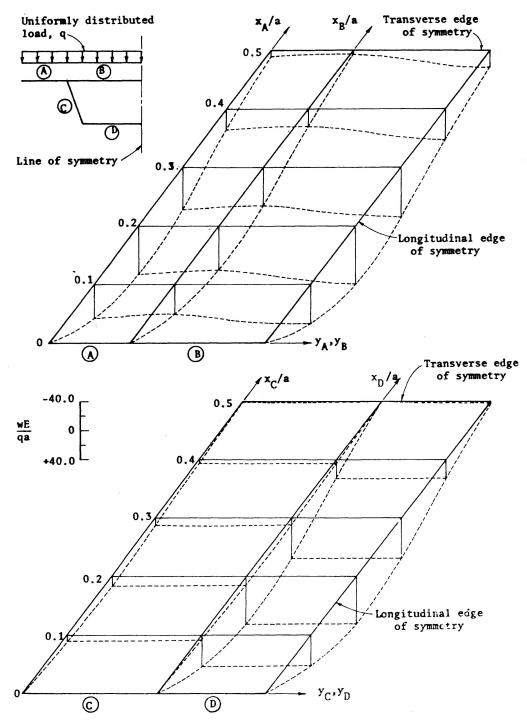


Fig. 4. Distribution of displacement component, w.

The modulus of elasticity, E, is assumed to be constant for all the plate elements. The dimensional factors L in Eq. (8) and K in Eq. (9) are taken equal to  $q\,a^3$  and  $(q\,a^4)/D$ , respectively. By symmetry, only one half of the structure need by analysed. Since the two intermediate supports are located at the longitudinal edges, that is at the intersection of plates C and D, the support reactions are assumed to be uniform line reactions throughout the longitudinal

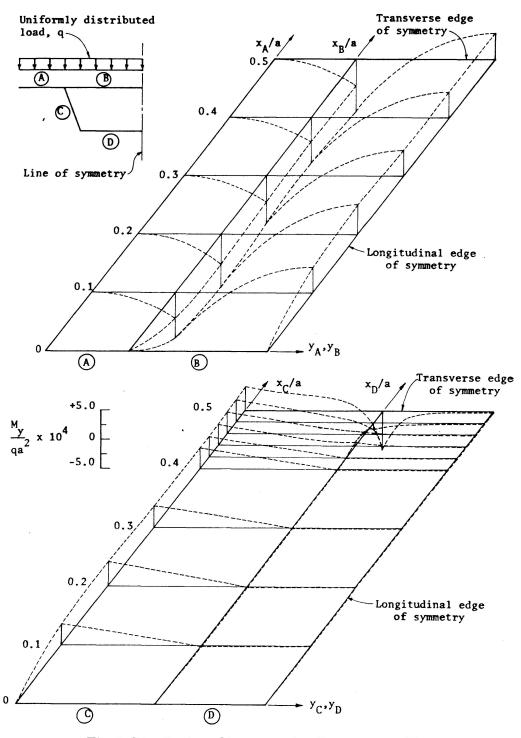


Fig. 5. Distribution of transverse bending moment,  $M_y$ .

dimension of the supports. Thus, substitution of the expressions of the displacement components, the stress resultants and the intermediate support reactions into the appropriate boundary conditions leads to a system of thirty two linear simultaneous equations from which the constants of integration are determined for each harmonic of the Fourier expansion.

The distribution of the displacement component, w and the stress resultants

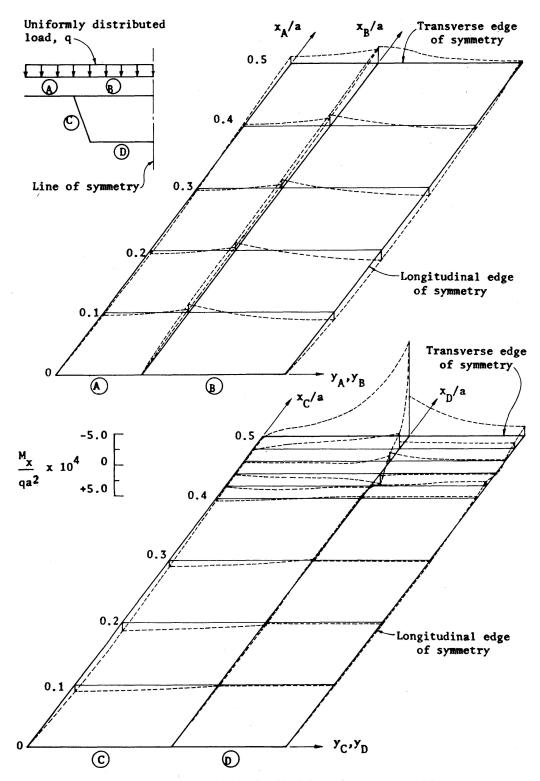


Fig. 6. Distribution of longitudinal bending moment,  $M_x$ .

 $M_y$ ,  $M_x$ ,  $M_{xy}$ ,  $N_y$ ,  $N_x$  and  $N_{xy}$  are shown in Figs. 4 to 10. The values of these quantities are calculated to the number of terms so that the absolute value of the last term is not greater than a half of one percent of the absolute value of the current partial sum computed. In view of the high stress gradients in

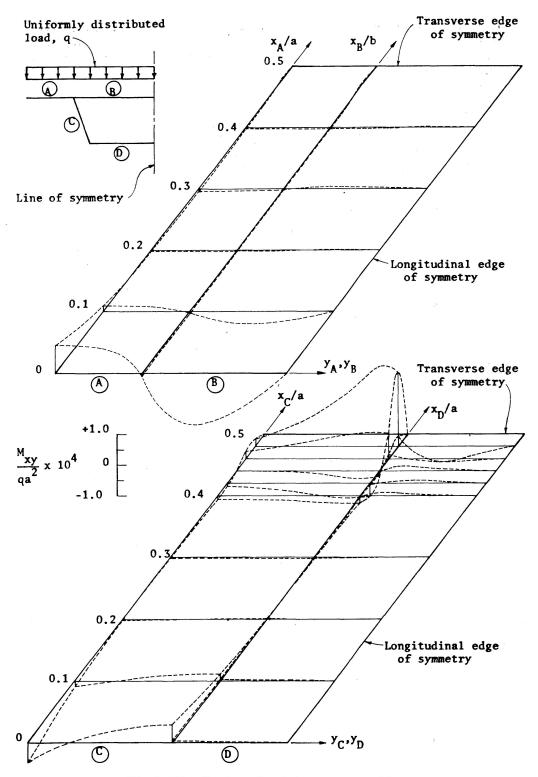


Fig. 7. Distribution of twisting moment,  $M_{xy}$ .

the vicinity of the intermediate supports, the stress resultants in plate elements C and D were evaluated at closely spaced locations using an interval of 0.02 from x/a = 0.4 to 0.5 and an interval in the y-direction equal to one-tenth of the plate width. It is found that the convergence of the stress resultants and

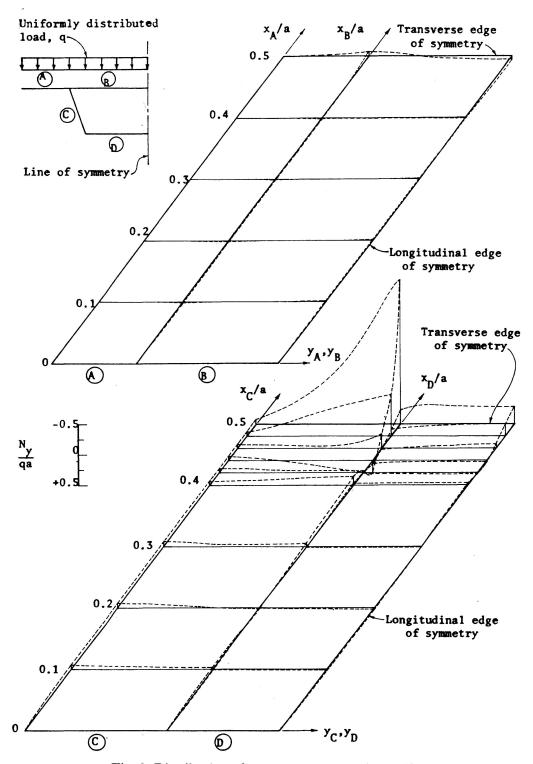


Fig. 8. Distribution of transverse normal force,  $N_y$ .

displacements at points near the edges, especially near the intermediate supports, is slower than for quantities elsewhere.

In regard to the magnitude of the reactions of intermediate supports the two supports, located along the same transverse line at the midspan, carry about 52.8 per cent of the total load acting on the top deck of the box girder.

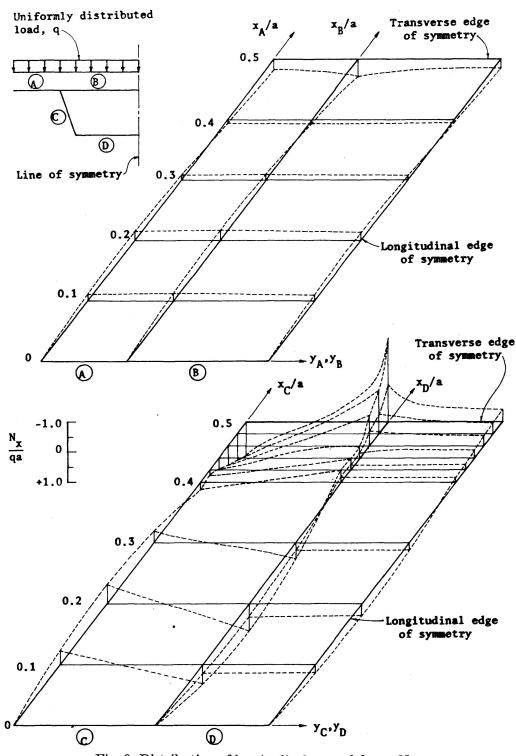


Fig. 9. Distribution of longitudinal normal force,  $N_x$ .

It is interesting to note that for a two-span continuous beam of uniform section under a uniformly distributed load, the interior support carries 62.5 percent of the total load when only bending deformation is considered.

In the computer program, double precision is used throughout in order to control round-off errors using the IBM 360/50 computer.

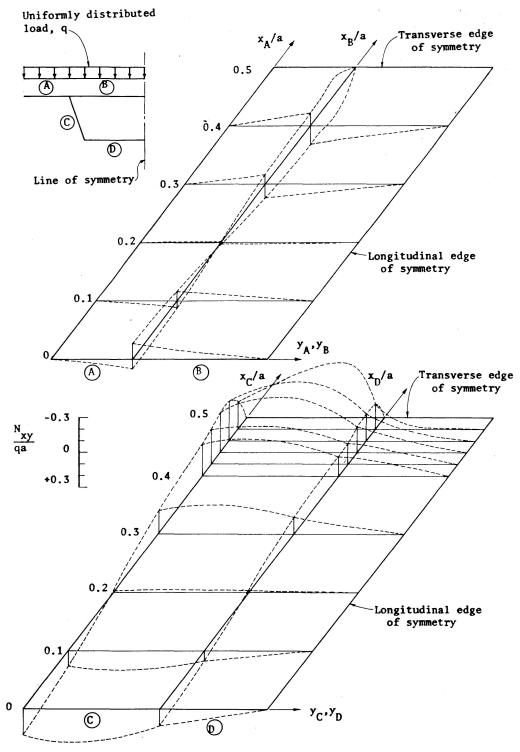


Fig. 10. Distribution of membrane shearing force,  $N_{xy}$ .

#### **Conclusions**

An analytical approach to the solution of continuous box girders without interior diaphragms is presented. The box girder, assumed to be simply supported at its transverse edges, may have the intermediate supports arbitrarily located along the span of the structure. Although the illustrative example chosen is a single-cell box girder with one transverse line of intermediate supports at right angle to the box girder, located at the midspan, the method is also applicable to multi-cell box girders with any number of transverse lines of intermediate supports, either orthogonal or skew with respect to the longitudinal direction of the structure.

The proposed method can also be used to give approximate solution to box girders with interior idaphragms resting on intermediate supports or to box girder sections with stiffening cross-beams.

## Appendix I. Stress Resultants

$$\begin{split} N_{x} &= -\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \varPhi_{mn} \, \beta^{2} \sin \alpha \, x \cos \beta \, y + L \sum_{m=1}^{\infty} \alpha^{2} \{ A_{1_{m}} e^{\alpha y} + A_{2_{m}} e^{-\alpha y} \\ &+ A_{3_{m}} \, e^{\alpha y} \, (\alpha \, y + 2) + A_{4_{m}} \, e^{-\alpha y} \, (\alpha \, y - 2) \} \sin \alpha \, x \,, \end{split} \tag{A.1}$$

$$\begin{split} N_y &= -\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \varPhi_{mn} \alpha^2 \sin \alpha x \cos \beta y \\ &- L \sum_{m=1}^{\infty} \alpha^2 \{ A_{1_m} e^{\alpha y} + A_{2_m} e^{-\alpha y} + A_{3_m} \alpha y e^{\alpha y} + A_{4_m} \alpha y e^{-\alpha y} \} \sin \alpha x \\ &+ \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} Y_{mn} \left( \frac{1}{\beta} \right) \sin \alpha x \cos \beta y \,, \end{split} \tag{A.2}$$

$$\begin{split} N_{xy} &= -\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \Phi_{mn} \, \alpha \, \beta \cos \alpha \, x \sin \beta \, y - L \sum_{m=1}^{\infty} \alpha^2 \{ A_{1_m} e^{\alpha y} - A_{2_m} e^{-\alpha y} \\ &+ A_{3_m} \, e^{\alpha y} \, (\alpha \, y + 1) + A_{4_m} \, e^{-\alpha y} \, (-\alpha \, y + 1) \} \cos \alpha \, x \,, \end{split} \tag{A.3}$$

$$\begin{split} M_{x} &= -\frac{DK}{a^{2}} \left[ \sum_{m=1}^{\infty} m^{2} \pi^{2} \{ B_{1_{m}} e^{\alpha y} (\mu - 1) + B_{2_{m}} e^{-\alpha y} (\mu - 1) \right. \\ &- B_{3_{m}} [2 \mu + \alpha y (\mu - 1)] e^{\alpha y} + B_{4_{m}} [-2 \mu + \alpha y (\mu - 1)] e^{-\alpha y} \} \sin \alpha x \\ &- \sum_{m=1}^{\infty} \frac{2}{m^{3} \pi^{3}} (1 - \cos m \pi) \sin \alpha x \right], \end{split} \tag{A.4}$$

$$M_{y} = -\frac{DK}{a^{2}} \left[ \sum_{m=1}^{\infty} m^{2} \pi^{2} \left\{ B_{1_{m}} e^{\alpha y} (1 - \mu) + B_{2_{m}} e^{-\alpha y} (1 - \mu) + B_{3_{m}} e^{\alpha y} \left[ 2 + \alpha y (1 - \mu) \right] + B_{4_{m}} e^{-\alpha y} \left[ -2 + \alpha y (1 - \mu) \right] \right\} \sin \alpha x$$

$$-2 \mu \sum_{m=1}^{\infty} \frac{1}{m^{3} \pi^{3}} (1 - \cos m \pi) \sin \alpha x \right],$$
(A.5)

 $W_{mn}$ ,  $Y_{mn}$ ,  $Z_{mn}$ 

x, y, z

$$\begin{split} M_{xy} &= -(1-\mu)\frac{DK}{a^2} \sum_{m=1}^{\infty} m^2 \pi^2 \{B_{1_m} e^{\alpha y} - B_{2_m} e^{-\alpha y} \\ &+ B_{3_m} (\alpha y + 1) e^{\alpha y} + B_{4_m} (-\alpha y + 1) e^{-\alpha y} \cos \alpha x \,, \end{split} \tag{A.6}$$

$$\begin{split} R_{y} &= -\frac{DK}{a^{3}} \sum_{m=1}^{\infty} m^{3} \pi^{3} \{B_{1_{m}} e^{\alpha y} (\mu - 1) + B_{2_{m}} e^{-\alpha y} (-\mu + 1) \\ &+ B_{3_{m}} e^{\alpha y} \left[\alpha y (-1 + \mu) + (1 + \mu)\right] \\ &+ B_{4_{m}} e^{-\alpha y} \left[-\alpha y (-1 + \mu) + (1 + \mu)\right] \} \sin \alpha x \,. \end{split} \tag{A.7}$$

# Appendix II. Notation

a, b, h	longitudinal (span), transverse (width) and normal (thick-
	ness) dimensions of a plate, respectively.
$a_{ij}$	flexibility influence coefficients.
$A_{1m}, A_{2m}, A_{3m}, A_{4m}$	constants of integration.
	constants of integration.
D	flexural rigidity of plate = $\frac{E h^3}{12 (1 - \mu^2)}$ .
$d_{j}$	longitudinal dimension of intermediate line support at
	transverse line $j$ .
$\boldsymbol{\mathit{E}}$	modulus of elasticity.
k	number of transverse lines of intermediate supports.
K, L	dimensional factors.
m, n	integers defining the harmonic of the Fourier series in the
	x- and $y$ -directions, respectively.
$M_x, M_y$	longitudinal and transverse bending moments per unit
	length, respectively.
$M_{xy} = M_{yx}$	twisting moment per unit length.
$N_x$ , $N_y$	longitudinal and transverse normal forces per unit length, respectively.
q	intensity of uniformly distributed load per unit area of
•	horizontal projection.
$R_i$	total reaction of support at $j$ .
$R_x$ , $R_y$	longitudinal and transverse edge reactions per unit length,
	respectively.
u, v, w	displacement components in the $x$ -, $y$ - and $z$ -directions,
	respectively.
$\overline{u}_{j},\overline{v}_{j}$	cross-sectional dimensions of intermediate support at
	transverse line $j$ .

coordinate axes in the longitudinal, transverse and normal

Fourier coefficients.

directions, respectively.

$x_i$	distance of the transverse line of intermediate supports
•	at $j$ in the longitudinal direction from the origin.
$V_{m j}$	intensity of the uniform line reaction of support at $j$ .
$\dot{V}$	total reaction of intermediate supports located at the
	same longitudinal edge.
X, Y, Z	load components in the $x$ -, $y$ - and $z$ -directions, respectively.
α	$\frac{m \pi}{m}$ .
<u> </u>	$\frac{m}{a}$ .
β	$\frac{n \pi}{b}$ .
$\mu$	Poisson's ratio.
$oldsymbol{\phi_{mn}}$	Fourier coefficients.
$\phi$	Airy's stress function.
heta	angle measured clockwise from the horizontal plane to
	the xy-plane which defines the plane of the middle sur-
	face of the plate.
$\Delta_i$	vertical displacement of the primary structure at $i$ due
	to applied loads.

#### References

- 1. Křístek, V. (1970): Tappered Box Girders of Deformable Cross-Section. Journal of the Structural Division, ASCE, Vol. 96, No. ST 8, Proc. Paper 7489, August.
- 2. Lee, S. L., Pulmano, V. A. and Lin, T. J. (1965): Ribless Folded Plates. Journal of the Structural Division, ASCE, Vol. 91, No. ST 1, Proc. Paper 4238, February.
- 3. Pulmano, V. A. and Lee, S. L. (1965): Prismatic Shells with Intermediate Columns. Journal of the Structural Division, Vol. 91, No. ST 6, Proc. Paper 4578, December.
- 4. Pulmano, V. A. (1972): Analysis of Simply Supported Box Girder Bridges. UNICIV Report No. R-98, School of Civil Engineering, The University of New South Wales, Kensington, November.

## **Summary**

This report presents an analysis of a continuous multi-cell box girder without interior diaphragms. The girder, assumed to be simply supported at its transverse edges, may have any number of transverse lines of intermediate supports which can be placed arbitrarily along the span of the structure. These transverse lines can be either perpendicular or skew with respect to the longitudinal direction of the structure. The analysis utilizes the folded plate theory which is formulated by using the plane stress elasticity theory and the classical thin plate bending theory to determine the membrane and the bending actions of the individual plates of the box section. To illustrate the proposed method of analysis, a two-span single-cell box girder subjected to a uniformly distributed load acting over its entire deck, is analysed.

#### Résumé

Le sujet de ce rapport est une étude sur les poutres continues à caissons multiples sans cloisons transversales. La poutre, qu'on suppose sur appuis simples à ses extrémités, peut avoir un nombre quelconque d'appuis intermédiaires situés en n'importe quel point de la structure. Les appuis se trouvent sur une ligne qui peut être perpendiculaire ou biaise par rapport à l'axe de l'ouvrage. Cette étude se base sur la théorie des structures plissées qu'on formule en employant la théorie des membranes et la théorie classique de la flexion des plaques minces pour déterminer les effets de membrane et de flexion de chaque élément du caisson. Afin d'illustrer la méthode ci-dessus, on étudie une poutre en caisson simple à deux portées soumise à une charge uniformément répartie sur tout le tablier.

## Zusammenfassung

Gegenstand des vorliegenden Beitrages ist eine Untersuchung über durchlaufende mehrzellige Kastenträger ohne Querwände. Der Träger, der als einfach gestützt an seinen Enden angenommen wird, kann eine beliebige Anzahl
von Zwischenstützen an irgendeinem Punkt des Bauwerkes aufweisen. Diese
Auflager können entweder senkrecht oder schräg in bezug auf die Achse der
Brücke liegen. Die Analyse bedient sich der Theorie gefalteter Platten, die
unter Anwendung der ebenen Elastizitätstheorie und der klassischen Theorie
dünner Platten formuliert wird, um die Wirkung der Membran und Biegung
jedes Elementes des Kastenquerschnittes zu bestimmen. Zur Illustration der
obigen Analysenmethode wird ein einzelliger Kastenträger unter Einwirkung
einer gleichmässig auf die ganze Fahrbahn verteilten Last untersucht.