Auxiliary cable continuous suspension bridge

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Auxiliary Cable Continuous Suspension Bridge

Ponts suspendus à câbles auxiliaires

Hilfskabel für durchlaufende Hängebrücken

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1. Introduction

New combined highway and railway routes connecting two major island in Japan are presently in the process of planning. The project, which involves the construction of several long suspension bridges, has been investigated for the past decade. There was some doubt as to the serviceability of such bridges for railway passage. When a railway train runs across, the stiffening trusses of suspension bridges deflect considerably, causing markedly discontinuous gradients in railway tracks and highway pavement surfaces at main towers and abutments. The elongation and contraction of ends of stiffening trusses produced by the running of a train or temperature change also become extremely great. In regard to the two-hinged suspension bridge with the spans of $300+1100+300$ m comprising the main portion of a series of bridges now being designed in the plan, the discontinuous gradient changes and elongation amounts have already been calculated. The results indicate that the maximum value of discontinuous gradient changes at a main tower will be $45.4\,^{0}/_{00}$ and the maximum elongation 117 cm. Such conditions would greatly impair the running safety and stability of railway trains and automobiles when they pass such parts of the bridge. For this reason, a study is being made of methods to distribute the discontinuous gradient change and elongation to a number of locations by using special devices at the joints of stiffening trusses with main towers and abutments of the abovementioned bridge. However, the structure of such a device is considerably complicated. Another countermeasure answering the requirement is to connect the stiffening trusses with each other at the main towers. If the stiffening trusses of a suspension bridge are made continuous, the gradients at main towers will no longer be noncontinuous and it will become unnecessary to consider elongation and contraction at these points. However, there will be extremely large negative bending moments acting on the stiffening trusses at the supports at the main towers in this case. The deflection change of stiffening trusses in the vicinities of the main towers will be fairly large and there will be vertical curvature in railway tracks and highway pavement surfaces. Consequently, the serviceability for running trains and automobiles will not be greatly improved.

The auxiliary cable continuous suspension bridge proposed here is of a structure as shown in Fig. 1 b in which auxiliary cables are branched out from the main cables near the main towers so that the stiffening trusses are suspended by these auxiliary cables as well as the main cables, and the stiffening trusses are not supported on the bearings at main towers. In the vicinities of main towers, instead of being simply supported as in ordinary continuous suspension bridges, the stiffening trusses are supported dispersedly and elastically by hangers in the sections having auxiliary cables. The large negative bending moments acting on continuous stiffening trusses at main towers are exceedingly lightened by this arrangement. Since the stiffening trusses are deflected at main towers, the vertical curvature in railway tracks and highway pavement surfaces will be greatly alleviated. Therefore, the serviceability of running trains and automobiles will be prominently improved.

Generally speaking, although multi-span auxiliary cable continuous suspension bridges and nonsymmetrical-span auxiliary cable continuous suspension bridges are possible, the present discussion will be restricted to the symmetrical three-span type which is thought to have a wide range of application and the essential nature of the auxiliary cable construction will be investigated. That is, a theory for analyzing the above type of continuous suspension bridge will be developed and computed results will be compared with experimental results. Also, the theory will be applied to the analysis of an actual continuous suspension bridge having spans of $300+1100+300$ m, and the results will be discussed.

2. Notations

The notations used are indicated in Fig. 2. In order to derive a theoretical formula, the suspension bridge is divided into the five sections shown in Fig. 2. When discriminating values for these sections, the subscript i $(i=1,2,3,4,5)$ is added to l, f, x, y, s, w, H_w, H, M and η, while in order to discriminate values of M, η and θ at branching points of cables, the subscript $i\,0$ $(i=1,2,3,4)$ is added to result in the notations M_{i0}, η_{i0} and θ_{i0}.

l	=	span length ($l_4 = l_1, l_5 = l_2$).
l_{21}, l_{22}	=	span lengths between main tower and branching point of cables.
λ	=	spacing between hanger.

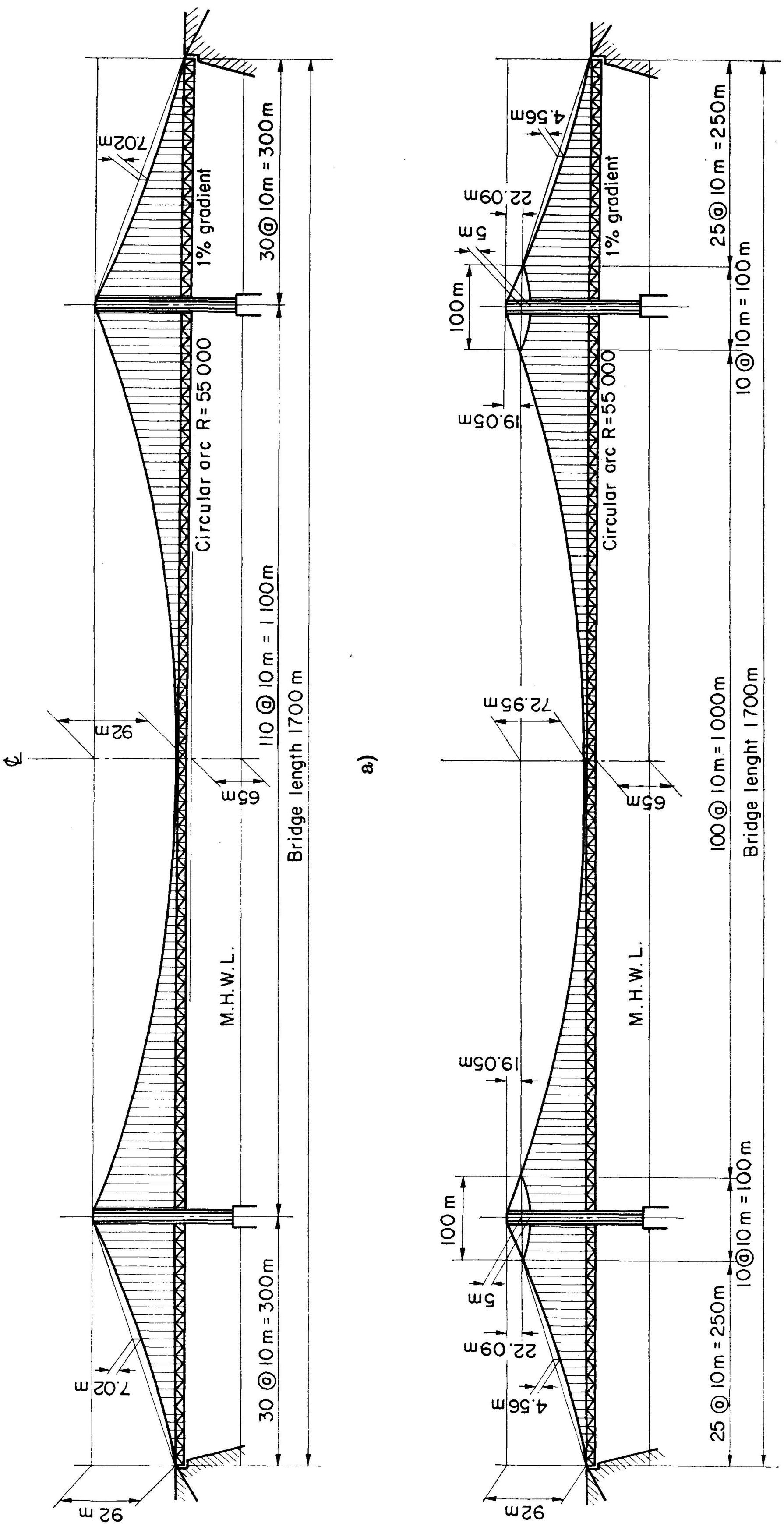

Fig. 1. General drawing of continuous suspension bridge of 300 + 1100 + 300 span.

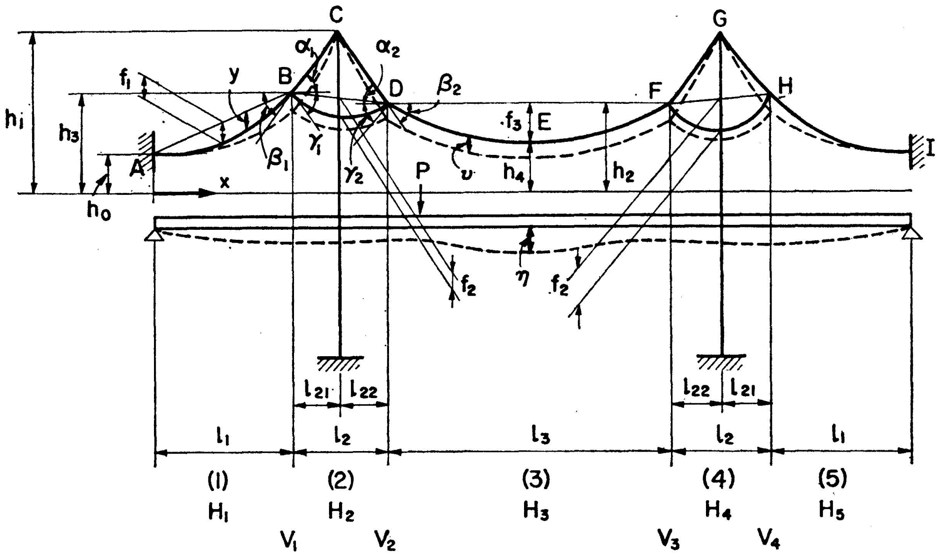

Fig. 2. Notations.

f = cable sag.

h_0, h_1, h_2, h_3, h_4 = heights of branching point of cables.

h = length of hanger.

$\alpha_1, \beta_1, \gamma_1, \alpha_2, \beta_2, \gamma_2$ = slope angles of cable tangent at branching point of cables.

x = abscissa.

y = cable ordinate.

s = length of cable arc.

E_s, E_c, E_n = elastic moduli of stiffening truss, cable and hanger respectively.

γ = coefficient of expansion.

A_c, A_c', A_n = sectional areas of main and auxiliary cable, inclined cable and hanger respectively.

I_s = moment of inertia of stiffening truss.

w = dead load.

$p(x)$ = distributed live load.

P = concentrated live load at location of hanger.

Δt = temperature change.

H_w, H = horizontal components of cable force due to dead load and live load plus temperature change.

V = tensile forces of hanger at branching point of cables.

S = tensile force of hanger.

M = bending moment of stiffening truss due to live load and temperature change.

η = vertical deflection of stiffening truss.

θ = slop angle of vertical deflection of stiffening truss.

v = vertical deflection of cable.

3. Dead-Load Geometry of Cable

It is assumed that cables are in complete equilibrium under dead load and stiffening trusses are neither stressed nor deflected. From the equilibrium condition

$$H_{w1} = H_{w5} = \frac{w_1}{8}\frac{l_1^2}{f_1}, \tag{1}$$

$$H_{w2} = H_{w4} = \frac{w_2}{8}\frac{l_2^2}{f_2}, \tag{2}$$

$$H_{w3} = \frac{w_3}{8}\frac{l_3^2}{f_3}. \tag{3}$$

From the equilibrium of horizontal component of cable forces

$$H_{w1} = H_{w3}. \tag{4}$$

From Eqs. (1), (3) and (4)

$$\frac{w_1 l_1^2}{f_1} = \frac{w_3 l_3^2}{f_3} = \frac{w_3 l_3^2}{h_2 - h_4}. \tag{5}$$

The tangents of slope angle of the cable at branching points B and H in Fig. 2 are expressed as follows:

$$\begin{aligned} \tan\alpha_1 &= \frac{h_1 - h_3}{l_{21}}, \\ \tan\beta_1 &= \frac{4 f_1}{l_1} + \frac{h_3 - h_0}{l_1}, \\ \tan\gamma_1 &= \frac{4 f_2}{l_2} + \frac{h_3 - h_2}{l_2}. \end{aligned} \tag{6}$$

From the equilibrium conditions of cable forces at the branching points of cables

$$H_{w1}\tan\beta_1 + H_{w2}\tan\gamma_1 = (H_{w1} - H_{w2})\tan\alpha_1. \tag{7}$$

Substituting Eqs. (1) $\sim$ (6) into Eq. (7)

$$\frac{w_1 l_1}{2} + \frac{w_2 l_2}{2} + \frac{w_1 l_1}{8 f_1}(h_3 - h_0) + \frac{w_2 l_2}{8 f_2}(h_3 - h_2) = \left(\frac{w_1 l_1^2}{8 f_1} - \frac{w_2 l_2^2}{8 f_2}\right)\frac{h_1 - h_3}{l_{21}}. \tag{8}$$

Similarly, for branching points D and F in Fig. 2

$$\begin{aligned} \tan\alpha_2 &= \frac{h_1 - h_2}{l_{22}}, \\ \tan\beta_2 &= \frac{4 f_3}{l_3}, \\ \tan\gamma_2 &= \frac{4 f_2}{l_2} - \frac{h_3 - h_2}{l_2} \end{aligned} \tag{9}$$

and

$$H_{w3}\tan\beta_2 + H_{w2}\tan\gamma_2 = (H_{w3} - H_{w2})\tan\alpha_2. \tag{10}$$

From these expressions

$$\frac{w_2 l_2}{2} + \frac{w_3 l_3}{2} - \frac{w_2 l_2}{8 f_2}(h_3 - h_2) = \left(\frac{w_3 l_3^2}{8 f_3} - \frac{w_2 l_2^2}{8 f_2}\right)\frac{h_1 - h_2}{l_{22}}. \tag{11}$$

Eqs. (5), (8) and (11) are constraint conditions among the parameters which determine the geometry of cables under dead load. For the special case when the dead load w_i is constant and $h_0 = h_4 = 0$, we find from Eqs. (5), (8) and (11)

$$\begin{aligned} \frac{h_3}{h_2} = \kappa &= \frac{4\{(l_3 l_{22} - l_1 l_{21}) + l_2 (l_{22} - l_{21})\} + l_3^2}{l_3^2 (l_{21} + l_1)} l_1, \\ l_2 (\kappa l_{22} + l_{21}) h_2^2 - \{4 (l_2 + l_3) & f_2 l_{22} + l_3^2 f_2 + h_1 l_2^2\} h_2 + l_3^2 f_2 h_1 = 0, \end{aligned} \tag{12}$$

where h_1 is assumed to be a known value.

If the spans of a suspension bridge and the heights of the main towers are given, the heights of the branching points of the main and auxiliary cables can thus be determined from Eq. (12).

4. Theory

Possible theories for the auxiliary cable continuous suspension bridge would be the elastic theory and the deflection theory as with ordinary suspension bridges. The former is a theory that analyzes the stresses and deformations of suspension bridges ignoring deformations of cables. However, when large live loads such as of trains act on an auxiliary cable continuous suspension bridge, there are cases when a compressive force acts on the auxiliary cables and hangers. Therefore, it is necessary to increase dead load and provide sufficiently large tensile forces to the auxiliary cables and hangers before loading. For this reason, the span length feasible for auxiliary cable continuous suspension bridges will be long. For suspension bridges of long span the deflection theory which takes into account the influence of cable deformation will generally give more accurate results.

The classical differential equation for the deflection theory is valid for auxiliary cable continuous suspension bridges. That is, for stiffening trusses suspended from cables in equilibrium under dead load

$$E_s I_s \frac{d^4 \eta}{dx^4} - (H_w + H)\frac{d^2 \eta}{dx^2} = p(x) + H\frac{d^2 y}{dx^2}. \tag{13}$$

In order to solve this equation, the stiffening trusses are divided into five sections at the four branching points of cables. From the equilibrium conditions the horizontal components H_1, H_3 and H_5 of cable forces in the main cables are equal each other. If $H_w + H$ is assumed to be constant at these sections, the equation will be linear.

The differential equation for the deflection theory linearized in this manner can be solved by various methods. The theoretical calculations in the following chapters were performed both by an analytical method and by a finite difference method. In the former method, the solution is obtained analytically by expressing the deflection of stiffening trusses as a force function. In the latter method, the stiffening trusses are assumed to be a continuous beam supported by many hangers and the differential equation is replaced with a series of finite difference equations using the three moment theorem. The latter method has the advantage of enabling analysis considering the influence of elongation of hangers. In auxiliary cable continuous suspension bridges, elongations of hangers have great influences, especially those of hangers at branching points of main and auxiliary cables, so this is a more compatible method.

The finite difference equations are derived on the assumption that cable portions between hangers are straight and that hangers do not sustain horizontal component of forces. Namely, if the spacing between hangers is λ, load at location of hanger j is P_j, length and tensile force of hanger are h_j and S_j, ordinate and deflection of cable are y_j and v_j, and bending moment and deflection of stiffening trusses are M_j and η_j respectively, the finite difference equations are expressed in the following forms:

$$\frac{M_{j-1}}{I_{s\,j-1,j}}+2M_j\left(\frac{1}{I_{s\,j-1,j}}+\frac{1}{I_{s\,j,j+1}}\right)+\frac{M_{j+1}}{I_{s\,j,j+1}}=-\frac{6E_s}{\lambda^2}(\eta_{j-1}-2\eta_j+\eta_{j+1}), \quad (14)$$

$$S_j=\frac{1}{\lambda}(M_{j-1}-2M_j+M_{j+1})+P_j, \quad (15)$$

$$S_j=-\frac{H}{\lambda}(y_{j-1}-2y_j+y_{j+1})-\frac{H_w+H}{\lambda}(v_{j-1}-2v_j+v_{j+1}), \quad (16)$$

$$\eta_j-v_j=\frac{S_j h_j}{A_n E_n}+h_j\gamma\Delta t. \quad (17)$$

Rearranging these Eqs. (14) $\sim$ (16) to obtain the form of a differential equation, an equation in which η of the second term on the left side of Eq. (13) is replaced with v is obtained. Consequently, the effect of elongation of hangers is introduced into the differential equation of the deflection theory by Eq. (17).

Next, in order to derive an equation on the elongation and contraction of cables, the change in the projected horizontal length of cables δi in section i is obtained. This is expressed as shown below by integrating the deformations of minute portions of the cables in section i.

$$\delta_i=\frac{H_i}{E_c A_c}\int_0^{l_i}\left(\frac{ds_i}{dx_i}\right)^3 dx_i-\left\{\frac{dy_i}{dx_i}\eta_i\Big|_0^{l_i}\right\}+\frac{d^2y_i}{dx_i^2}\int_0^{l_i}\eta_i dx_i+\gamma\Delta t\int_0^{l_i}\left(\frac{ds_i}{dx_i}\right)^2 dx_i. \quad (18)$$

The equation on deformation of cables is derived from the following two conditions.

1. The total change in all projected horizontal lengths of main and auxiliary cables is zero.

2. In sections 2 and 4, the changes in projected horizontal length of auxiliary cables and inclined cables are equal.

These conditions can be expressed as follows:

$$\sum_{i=1}^{5} \delta_i = 0, \tag{19}$$

$$\delta_2 = \frac{(l_{21}\sec^3\alpha_1 + l_{22}\sec^3\alpha_2)}{E_c A_c'}(H_1 - H_2) - \eta_{10}\tan\alpha_1 - \eta_{20}\tan\alpha_2, \tag{20}$$

$$\delta_4 = \frac{(l_{21}\sec^3\alpha_1 + l_{22}\sec^3\alpha_2)}{E_c A_c'}(H_1 - H_4) - \eta_{30}\tan\alpha_2 - \eta_{40}\tan\alpha_1. \tag{21}$$

Eqs. (13), (19), (20) and (21) constitute a set of simultaneous equations which can be solved for η, H_1, H_2 and H_4, and the solution of the deflection theory is obtained. In auxiliary cable continuous suspension bridges, the peculiar hanger tension V is produced at the branching points of main and auxiliary cables due to live loads. Therefore, it is necessary for Eq. (13) to be solved taking the influence of V into consideration. This hanger force V is obtained from the equilibrium conditions of forces of cables and hanger at branching points as follows:

$$\boldsymbol{V} = \boldsymbol{T}\boldsymbol{H} + \boldsymbol{D}\boldsymbol{H}_w. \tag{22}$$

where
$$\boldsymbol{V} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix},$$

$$\boldsymbol{T} = \begin{bmatrix} \tan\alpha_1 - \tan\beta_1 & -(\tan\alpha_1 + \tan\gamma_1) & 0 \\ \tan\alpha_2 - \tan\beta_2 & -(\tan\alpha_2 + \tan\gamma_2) & 0 \\ \tan\alpha_2 - \tan\beta_2 & 0 & -(\tan\alpha_2 + \tan\gamma_2) \\ \tan\alpha_1 - \tan\beta_1 & 0 & -(\tan\alpha_1 + \tan\gamma_1) \end{bmatrix},$$

$$\boldsymbol{H} = \begin{bmatrix} H_1 \\ H_2 \\ H_4 \end{bmatrix},$$

$$\boldsymbol{D} = \begin{bmatrix} \sec^2\alpha_1 \dfrac{\eta_{10}}{l_{21}} + \theta_{10} \\ \sec^2\alpha_2 \dfrac{\eta_{20}}{l_{22}} - \theta_{20} \\ \sec^2\alpha_2 \dfrac{\eta_{30}}{l_{22}} + \theta_{30} \\ \sec^2\alpha_1 \dfrac{\eta_{40}}{l_{21}} - \theta_{40} \end{bmatrix}, \qquad \boldsymbol{H}_w = [H_{w1} - H_{w2}].$$

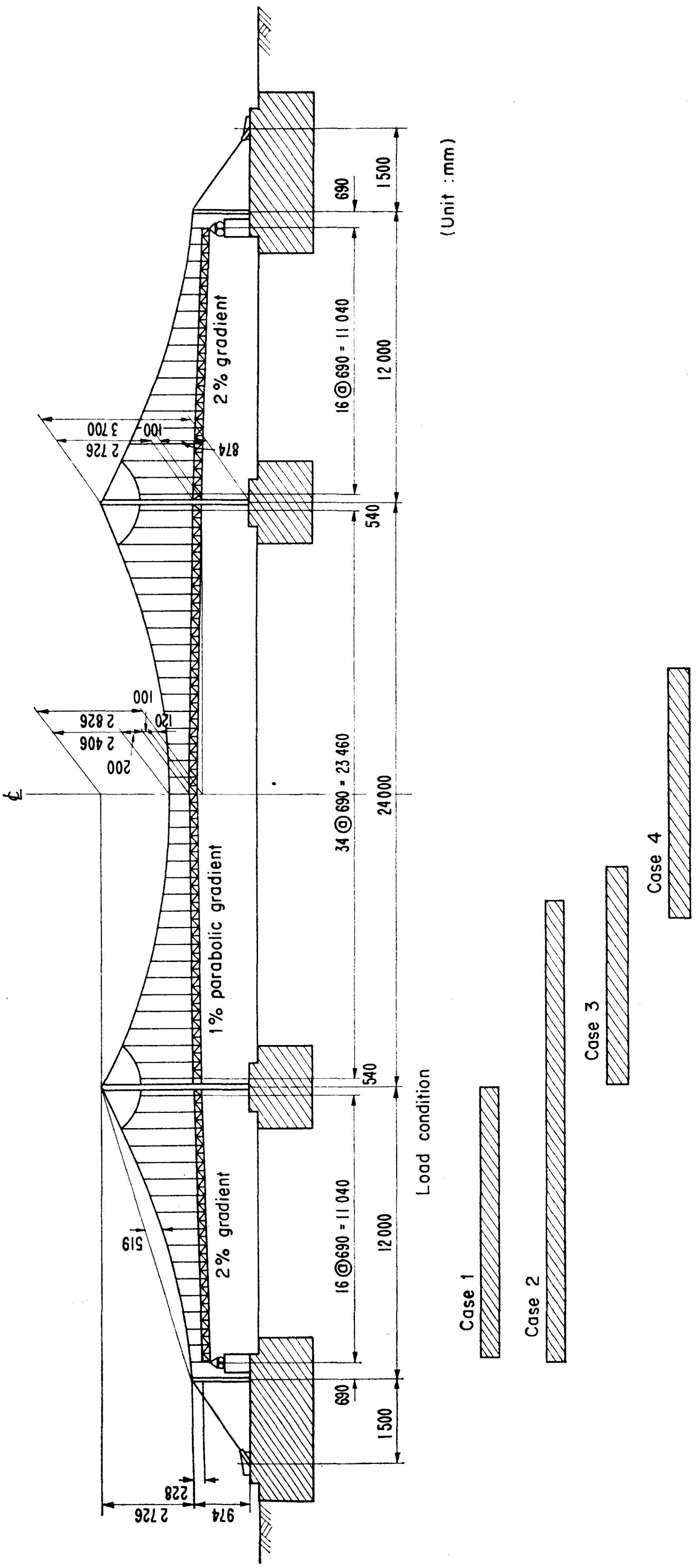

Fig. 3. General drawing of model suspension bridge.

Eqs. (18) $\sim$ (22) indicate the relationships with deflections η of stiffening trusses, but in case of obtaining solutions using the finite difference method, cable deflection v in place of η and cable deflection angle $\frac{dv}{dx}$ in place of θ are employed.

5. Model Experiments

In order to examine the theory of the auxiliary cable continuous suspension bridge derived in the preceding chapter, static loading tests and vibration experiments were carried out using a model suspension bridge with a total length of 48 m. The outline of the experiments and their results are described below.

a) Models

The model suspension bridge used in the experiments is shown in Fig. 3. It had continuous stiffening trusses with the spans of $12+24+12$ m. The stiffening trusses were Warren-type trusses having the height of 10 cm and a uniform cross section composed of steel channels of $19\times 12\times 1.5$ mm.

The models were of five kinds as follows:

Type 1 was an ordinary continuous suspension bridge model as shown Fig. 4.

Type 2 was an auxiliary cable continuous suspension bridge model having hangers at the branching points of main and auxiliary cables as shown in Fig. 5. The hanger is arranged to resist directly the peculiar vertical force V produced at branching points.

Type 3 $\sim$ Type 5 were three different auxiliary cable continuous suspension bridge models having no hangers at branching points as shown in Fig. 6 $\sim$ Fig. 8. When no hangers are jointed at branching points, the vertical force is dispersed on hangers near the branching point. The purpose of the experiments using the models Type 3 $\sim$ Type 5 was to determine a method for analyzing structures having no hangers at branching points.

b) Test Program

Flexural strains in upper and lower chords of the stiffening trusses and their deflections were measured statically by applying four varieties of loads (Case 1 $\sim$ Case 4) as illustrated in Fig. 3. Deflections and horizontal displacements at the branching points of main and auxiliary cables were also measured.

The models of Type 1, Type 2 and Type 3 were vibrated vertically after the statical loading tests and dynamic deflections in each case were recorded into an electrical recorder. These vibration experiments were carried out on the conditions of no loading and the loading of Case 3 shown in Fig. 3.

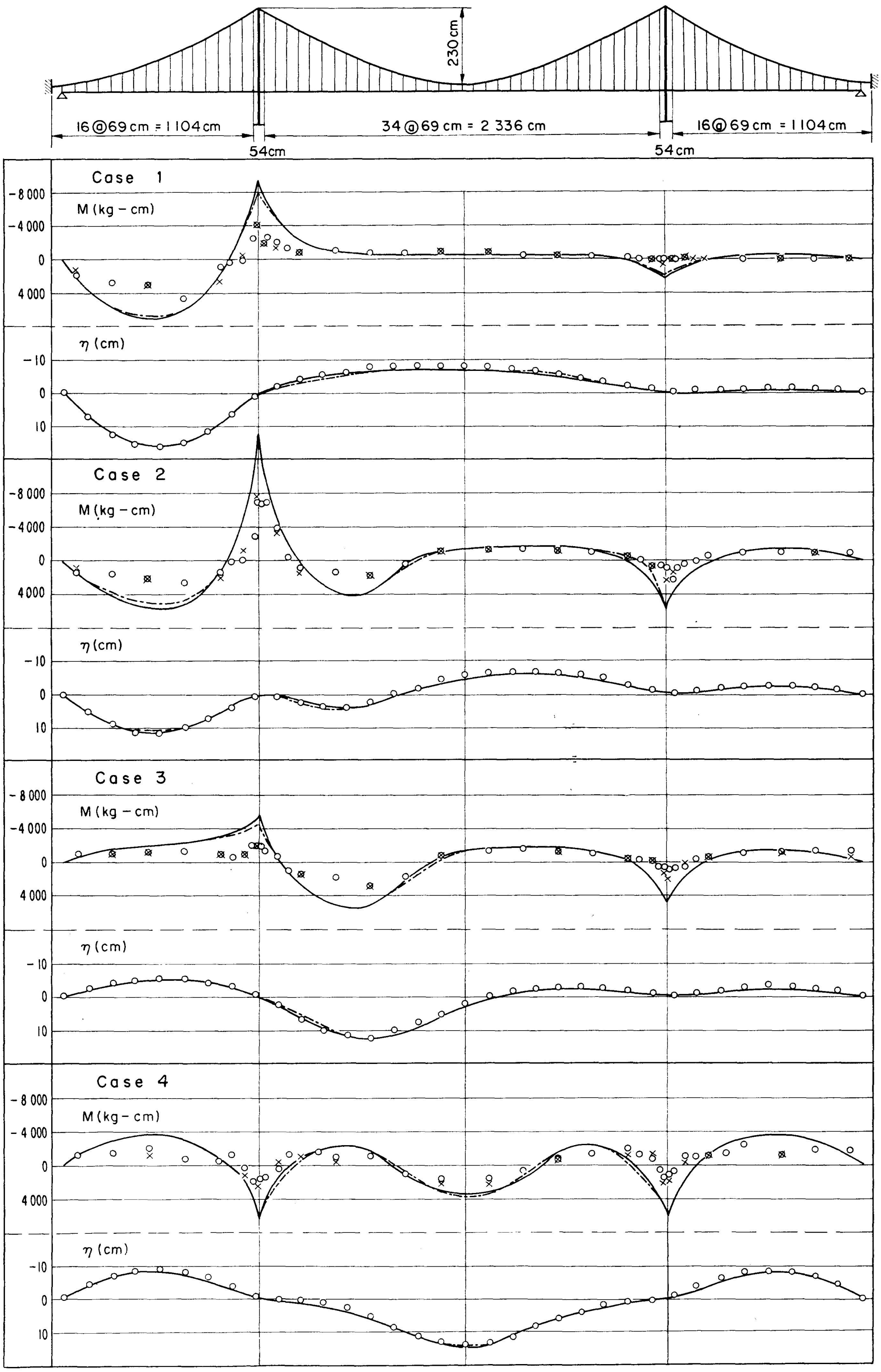

Fig. 4. Model of type 1 and test results.

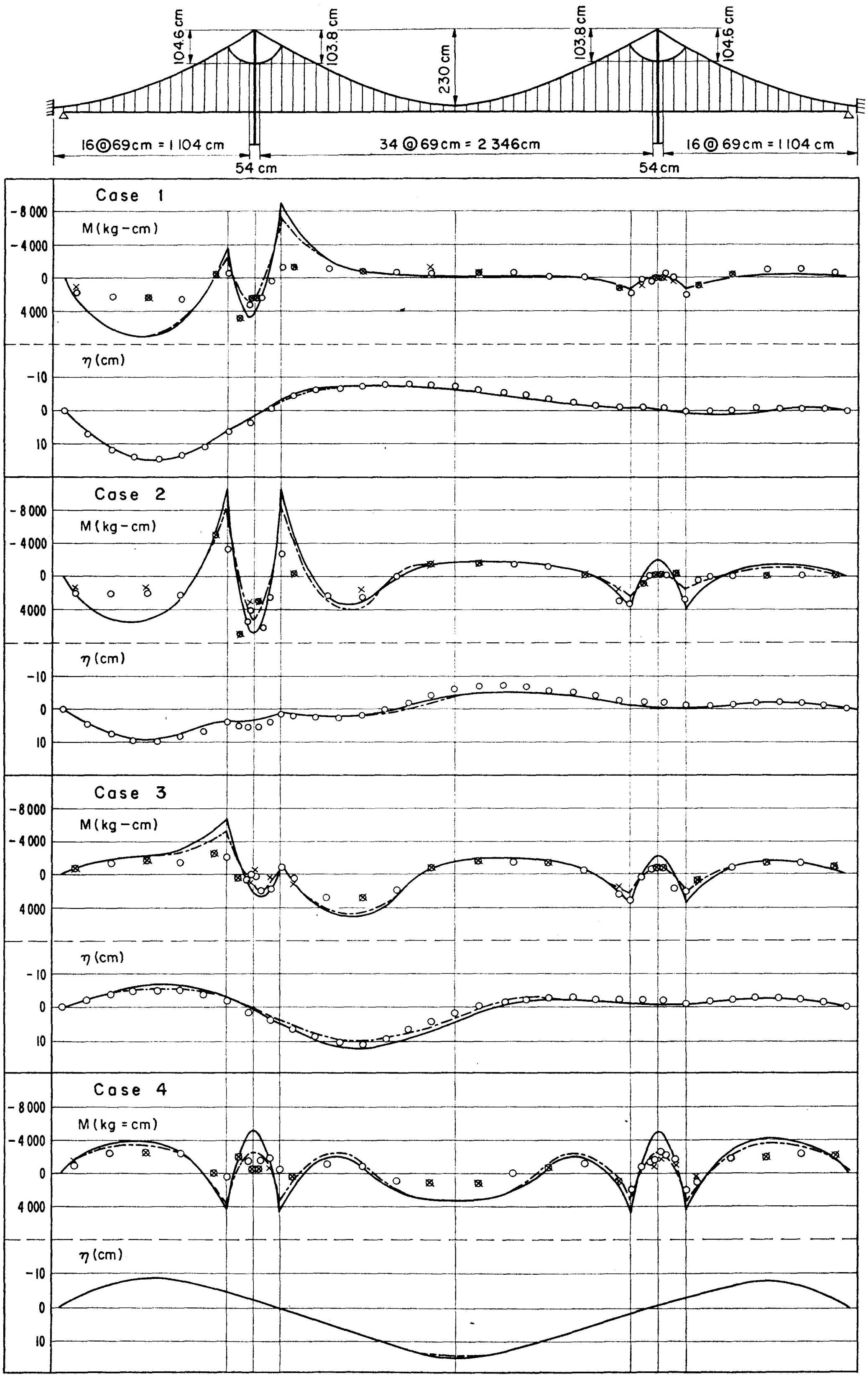

Fig. 5. Model of type 2 and test results.

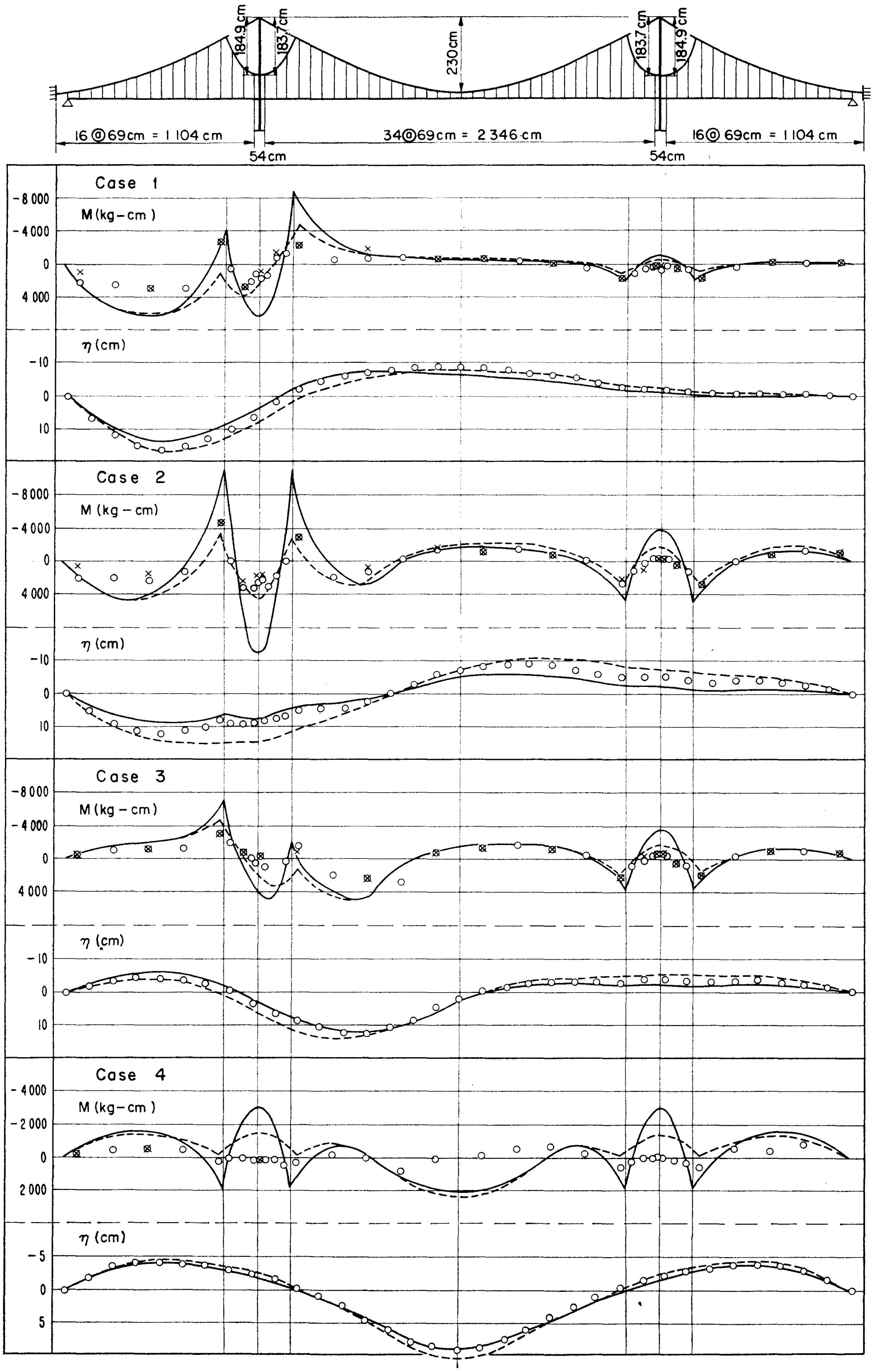

Fig. 6. Model of type 3 and test results.

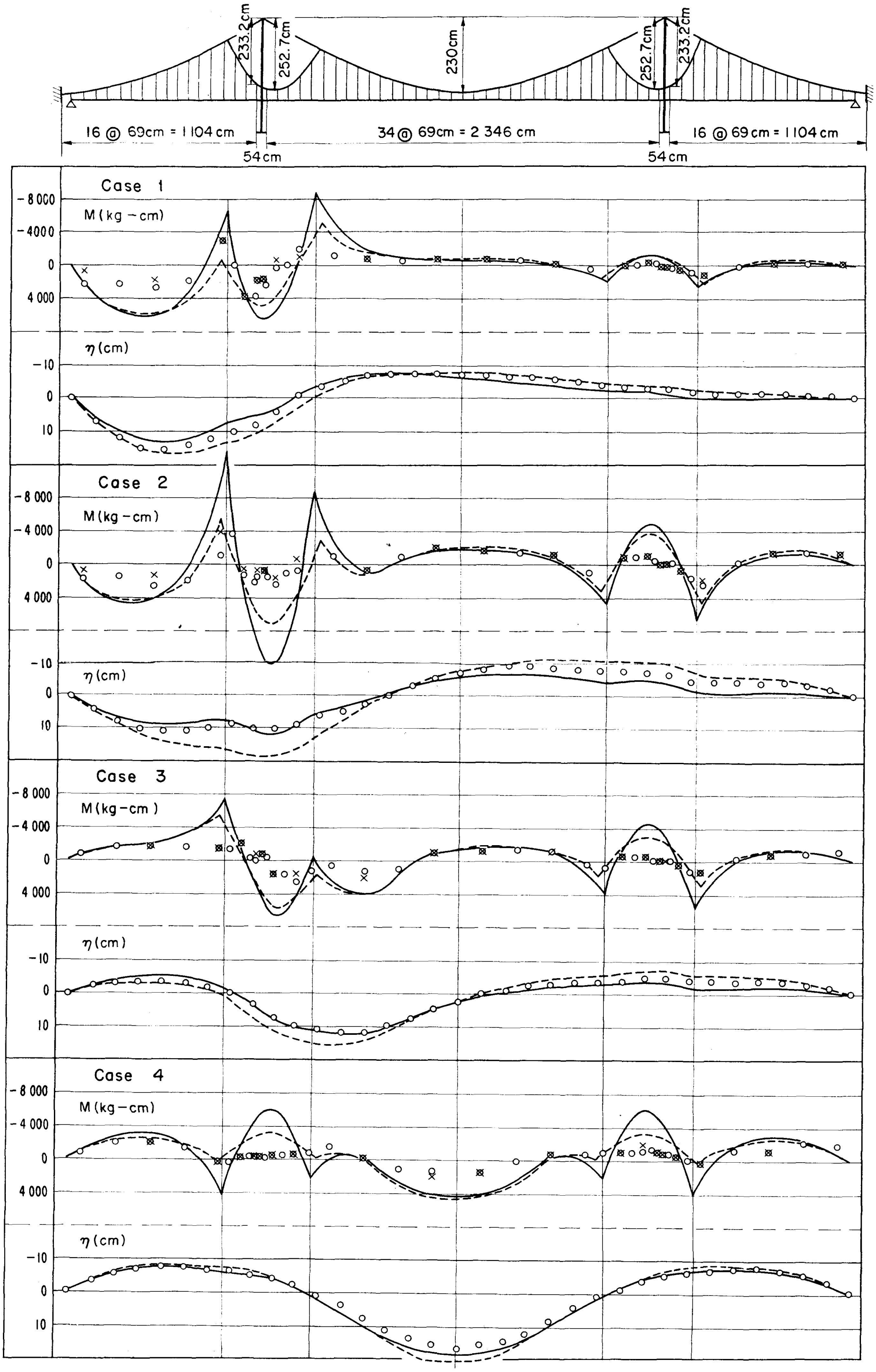

Fig. 7. Model of type 4 and test results.

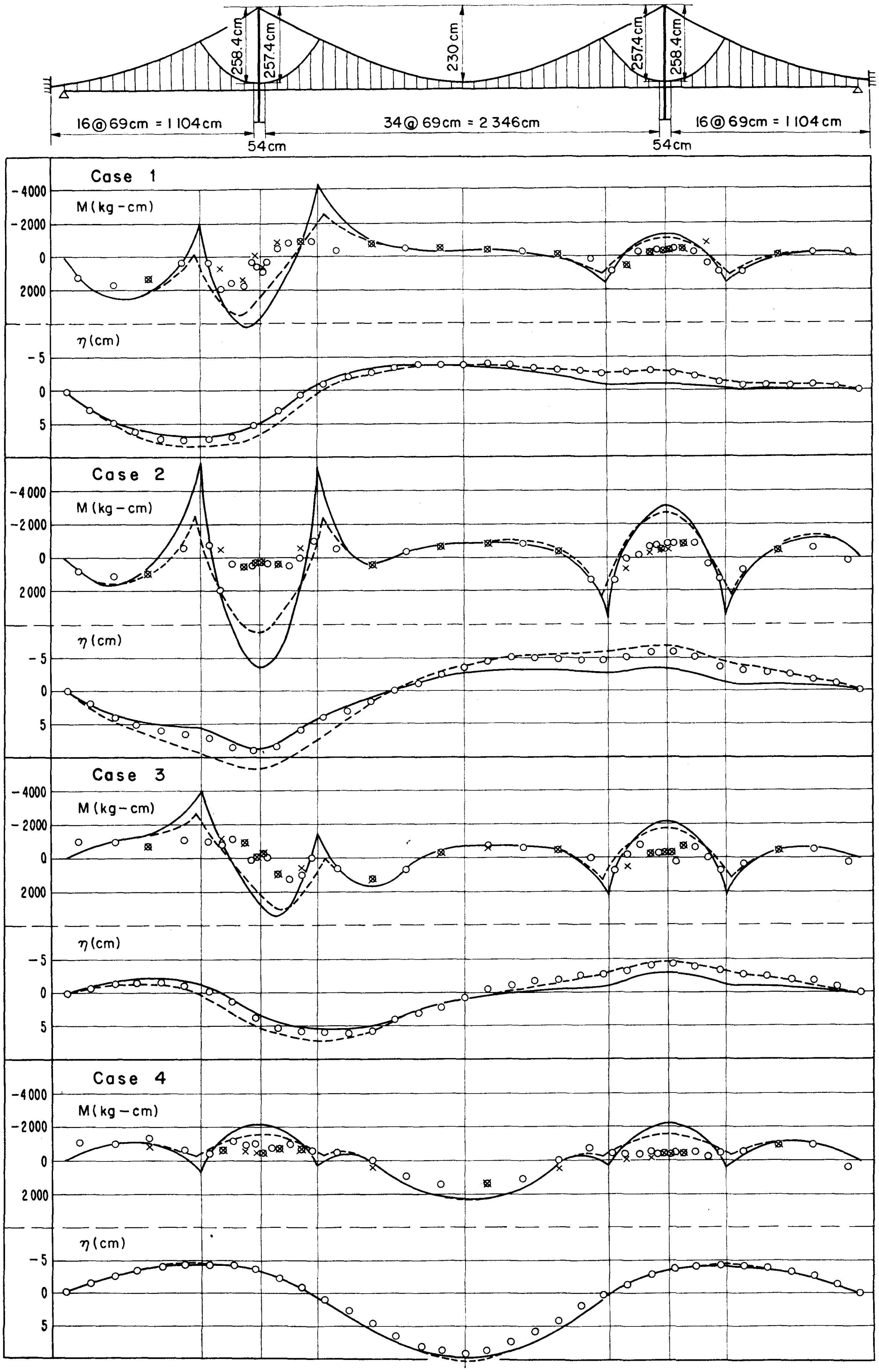

Fig. 8. Model of type 5 and test results.

c) *Results of Static Loading Tests*

Fig. 4 $\sim$ Fig. 8 show the deflections and bending moments calculated from measured strains of stiffening trusses for the four loading conditions mentioned above. The $\circ$-marks in the figures are calculated values of bending moments obtained from measured strains of upper chords of stiffening trusses and the $\times$-marks those of lower chords, the solid lines and dashed lines are theoretical values obtained from the deflection theory in the preceding chapter.

A comparison of the experimental and theoretical results is summarized in the following:

1. In regard to deflections of stiffening trusses, the experimental results are in comparatively good agreement with theoretical ones for all types and for all loading conditions.

2. In regard to bending moments of stiffening trusses, the experimental results are somewhat scattered, but as a trend, there is agreement with theory.

3. Comparison of the bending moments in Fig. 4 and Fig. 5 shows that although bending moments obtained from measured strains are generally lower than the values obtained from theory, the results coincide approximately with each other. The large displacement method, which considers the equilibrium conditions in the state subsequent to deflection, was also applied to evaluate the test results. The results of this theory are shown by the dash-dot lines in Fig. 4 and Fig. 5 and are in good agreement with those derived from the beforementioned deflection theory, which is a kind of the force method. The large displacement method is so advantageous because the influences of elongation and inclination of hangers can be included in the calculations, but in case numerous comparison studies of designs are to be made, the method is inconvenient since much time is consumed in computations and preparation of input data. As stated above, the deflection theory suits the experimental results comparatively well. For the auxiliary cable continuous suspension bridges having hangers at the branching points of main and auxiliary cables as well as ordinary continuous suspension bridges, it is appropriate for analyses of deflections and stresses to be made by the deflection theory developed in the preceding chapter.

4. The model suspension bridges shown in Fig. 6, Fig. 7 and Fig. 8 are of forms in which hangers are not joined at branching points of main and auxiliary cables, and in these cases the peculiar vertical force cannot be directly sustained. At first the experimental results were compared with the theoretical ones derived from the deflection theory on the assumption that there were ordinary hangers at the branching points. The use of this assumption results in considerable safety as to bending moment obtained from measured strains. As an another assumption, virtual hangers having the extremely small sectional area were placed at the branching points and the finite difference method was applied to the analysis. The dashed lines in the figures indicate analytical

results obtained by assuming that the sectional area of the virtual hangers is 10^{-4} of that of ordinary hangers. The experimental results on deflection are situated between the solid and dashed lines, but those on bending moment show closer to the dashed lines. Computations based on the large displacement method were also made and the results were in relatively good agreement with the dashed lines. From these results, it is reasonable to deduce that analyses of deflections and stresses can be performed using the finite difference method of the deflection theory by placing virtual slender hangers at branching points in cases when there are no hangers at branching points.

5. The branching points at main and auxiliary cables were deflected and moved laterally with loadings in order to decrease bending moments in the stiffening trusses. This indicates an advantageous structural characteristic of the auxiliary cable continuous suspension bridge.

6. The bending moments of stiffening trusses in the model Type 3 is the lowest in maximum value in a series of the experiments. This fact suggests that there should be a way of determining the optimum auxiliary cable geometry in an auxiliary cable continuous suspension bridge.

d) Results of Vibration Tests

Fig. 9 shows the vibration modes and natural frequencies of the model suspension bridges. The solid lines and dashed lines are the results of analytical computation of free vibration, in which each model is assumed to be a multi-mass system. The ○-marks and ●-marks in the figure are experimental values of vibration modes obtained from the analysis of the dynamic data. The solid lines and ○-marks are the values on the conditions of no loading and the dashed lines and ●-marks those of the loading of Case 3. The first values in the sides indicate the natural frequencies obtained from the analytical computation of free vibration and the second values in round brackets those obtained from experimental results. The computational results agree rather well with the experimental ones. Fig. 9 also shows that there are no significant differencies in vibration modes among these suspension bridge models. It may be considered from this fact the vibration properties of an auxiliary cable continuous suspension bridge having a comparatively short length of auxiliary cable are almost the same as those of an ordinary continuous suspension bridge.

Table 1

Type of Bridge	Load Condition: No Load	Load Condition: Case 3
Type 1	0.0176	0.0265
Type 2	0.0225	0.0288
Type 3	0.0265	0.0427

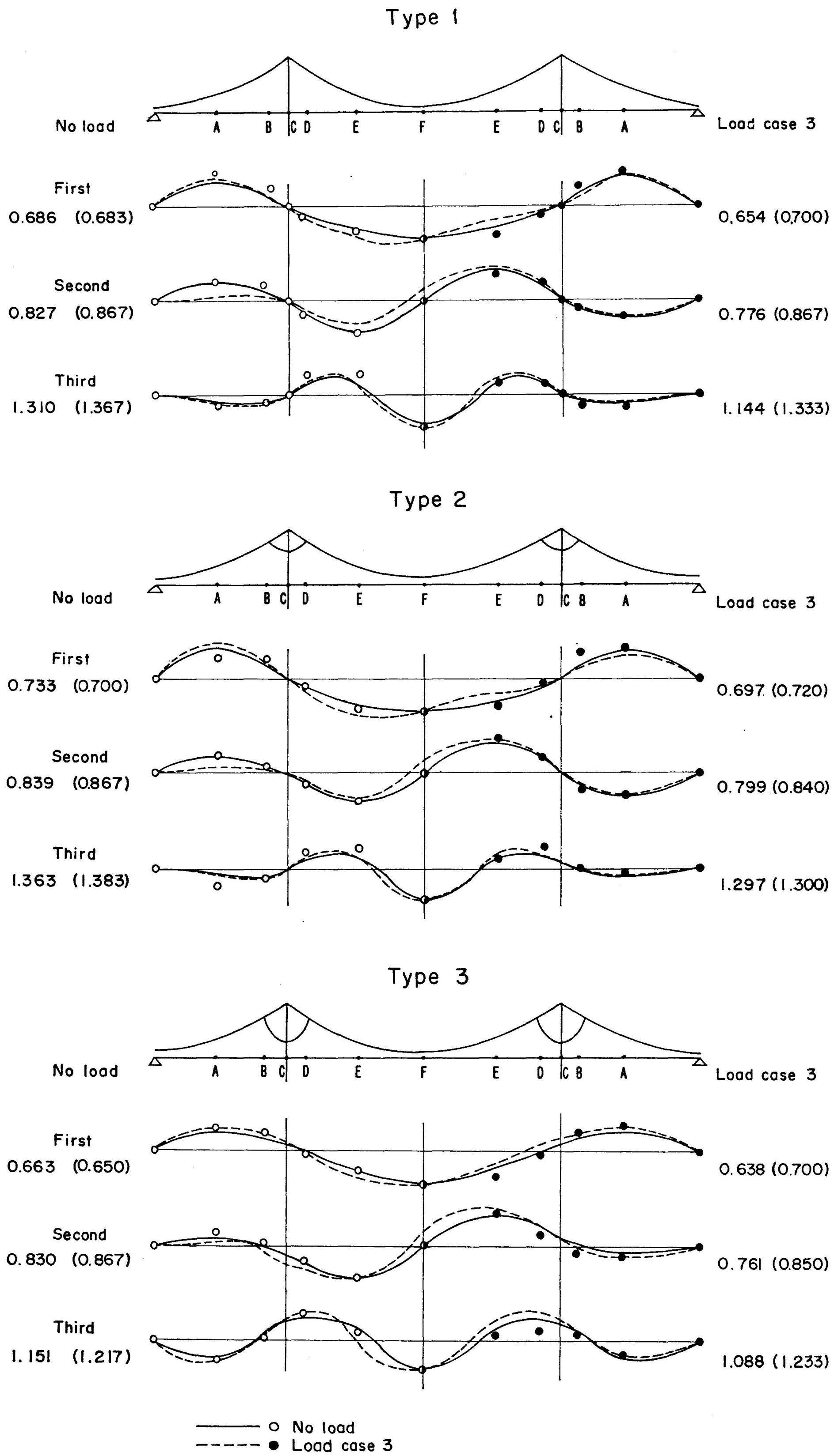

Fig. 9. Vibration test results.

Logarithmic decrements were also obtained from the vibration records of deflections. Table 1 shows the results. Auxiliary cables seemed to behave as small dampers and the damping effect augmented slightly by increasing length of auxiliary cable. The vibration was observed to be damped with increasing weight of structures.

6. Comparison of Suspension Bridge of 300 + 1100 + 300 m Span

The theory developed in this paper was applied to the analysis of an actual three-span continuous suspension bridge in order to test the practicability and, at the same time, to investigate the essential nature of auxiliary cable construction. The structure selected for the numerical application of the theory had a 1100 m main span and two 300 m side spans, and had been previously designed as a continuous suspension bridge for combined highway and railway passage along the new routes connecting Honshu and Shikoku in Japan as shown in Fig. 1a. The stiffening trusses had a constant depth of 10.37 m throughout and were spaced 33 m, center to center. The sectional area and moment of inertia were 0.68 m^2 and 2.687 m^4 respectively.

Various modifications of auxiliary cable continuous suspension bridges were studied. The geometry of the auxiliary cable can be determined from Eq. (12) as a function of l_{21}, l_{22} and f_2. Assuming that $l_{21} = l_{22}$ and varying both span length l_2 and sag f_2 of auxiliary cable, the maximum bending moments of stiffening trusses were computed for three varieties of fixed loads as shown in Fig. 11, where the highway and railway load per meter per cable was taken to be 2.16 t/m and 3.71 t/m, respectively, and maximum length of train, 400 m.

The results obtained from these analysis are presented in Fig. 10. The notations M in the figure are as follows:

M_{10}, M_{20}: maximum negative bending moments at branching points of main and auxiliary cables.

M_{11}: maximum positive bending moment at center of side spans.

M_{21}: ditto at center of auxiliary cable spans.

M_{31}: ditto at one-quarter point of main span.

M_{32}: ditto at center of main span.

According to these results, there are hardly any effects on bending moments of the side spans and center span when the span length and geometry of auxiliary cables are varied. However, the negative bending moments at branching points of main and auxiliary cables and the positive bending moments near main towers are increased with increasing span length l_2 and sag-span ratio f_2/l_2 of auxiliary cable. If f_2 is decreased, bending moments near

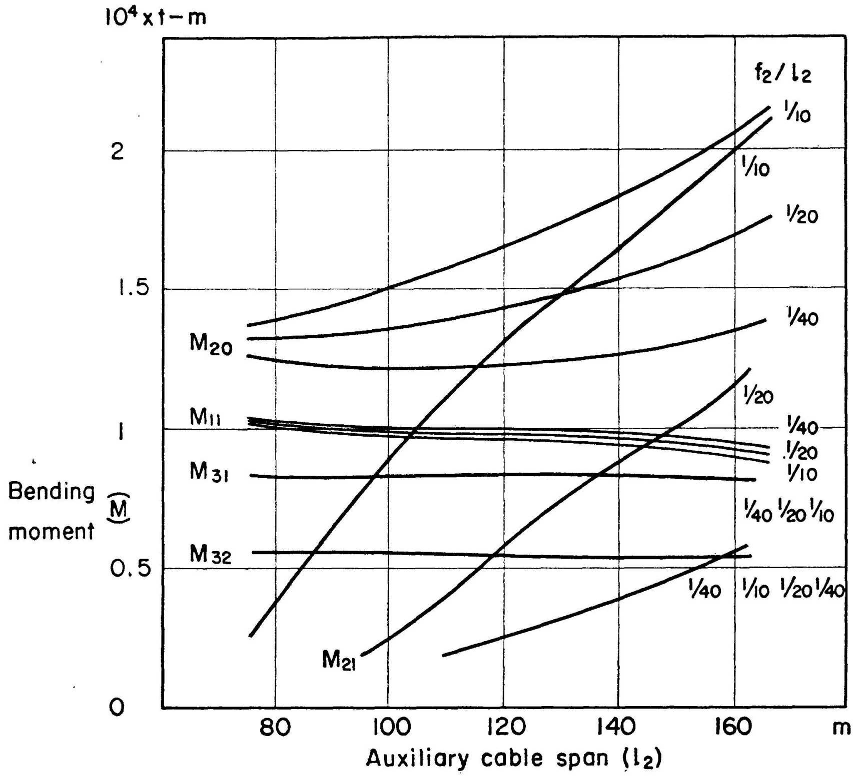

Fig. 10. Relationship between span of auxiliary cable and bending moment.

main towers are distributed advantageously and the deflections of stiffening trusses are alleviated, it may be difficult to make the sag too small owing to limitations imposed by requirements in cable construction. If l_2 is decreased within the range indicated in Fig. 10, bending moments in the vicinities of main towers also decrease advantageously, but tensile force of auxiliary cables varies in a wide range with live load, which may bring greater error in calculation. Considering these requirements, an auxiliary cable continuous suspension bridge with auxiliary cable span of 100 m and sag-span ratio $1/20\,(f=5\,\text{m})$ as shown in Fig. 1b was selected for comparison with the ordinary continuous suspension bridge.

Fig. 11 shows a comparison of the results of calculations by the deflection theory on the two types of continuous suspension bridges shown in Fig. 1. Upper part of the figure indicates influence lines of horizontal components of cable forces and of bending moments of stiffening trusses. Regarding influence lines of the former, the solid lines show the values of main cable (H_1) and auxiliary cables (H_2, H_4) in the modified bridge and the dashed lines, those in the original bridge. Influence lines of the latter were obtained for such locations as being used in discussing the length of auxiliary cable. Lower part of the figure indicates bending moments and deflections produced in stiffening trusses applying three varieties of fixed loads shown in the figure, which were selected considering the influence lines of bending moments. The dashed lines

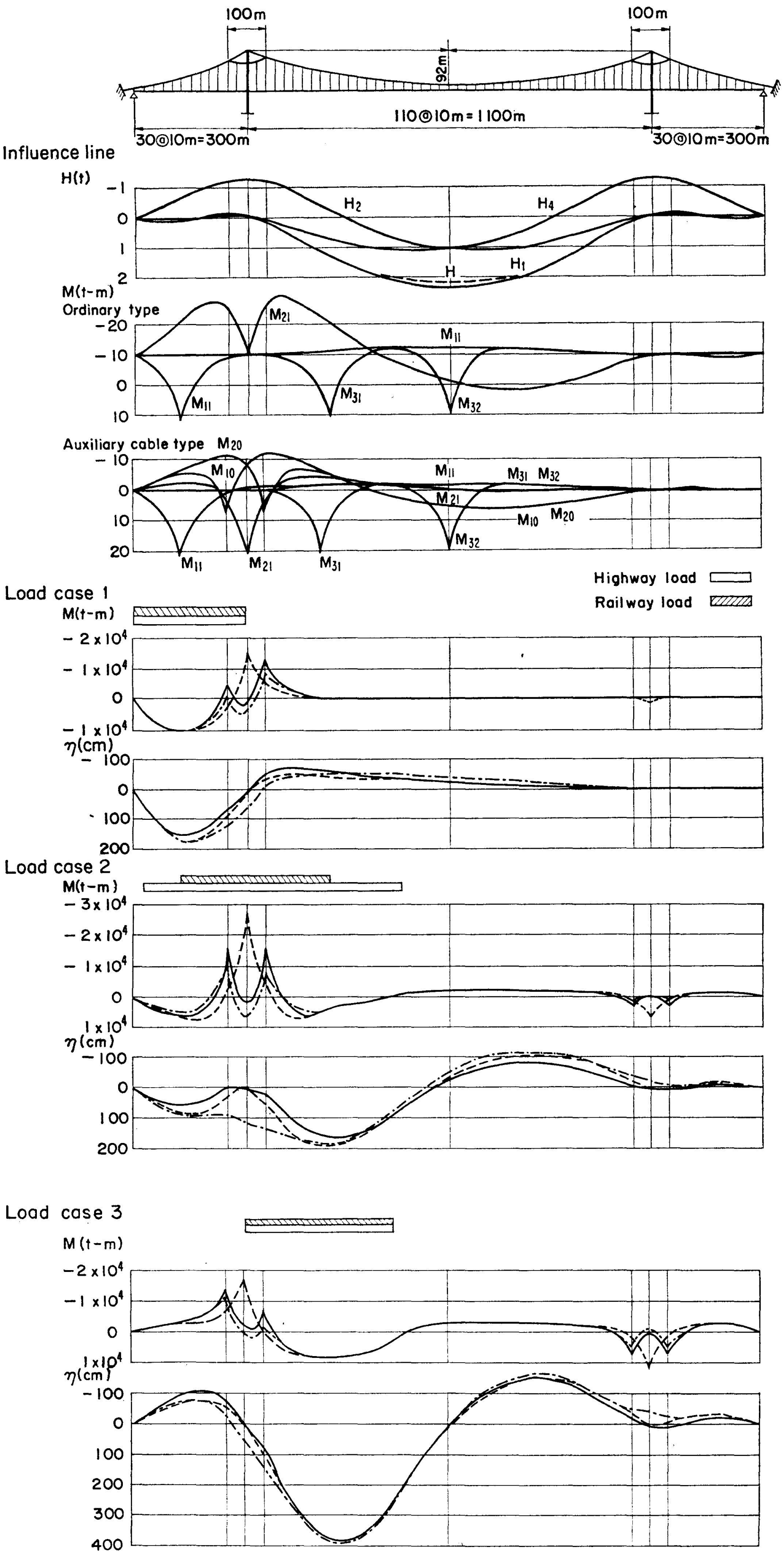

Fig. 11. Influence lines, bending moments and deflections of continuous suspension bridge.

are for the original continuous suspension bridge and the solid lines for the modified auxiliary cable continuous suspension bridge. The figure indicates that the negative bending moments in the vicinities of main towers are exceedingly lesser in the modified bridge than in the original bridge. The ratio of both maximums amounts to approximately one-half. The deflection changes in the vicinities of main towers are also evened out. Nevertheless, there are almost no differences in maximum bending moments and deflections at mid point of side spans, mid point and one-quarter points of center span. Since the parts with auxiliary cables comprise only an extremely small portion of the total length of the bridge, influences do not appear prominently in parts other than near the main towers.

According to the results of the experiments mentioned in the preceding chapter, bending moments and deflections of stiffening trusses would become of more advantageous natures if a structual type having no hangers at branching points of main and auxiliary cables is used, so that a structure from which hangers were eliminated at the branching points of main and auxiliary cables of the modified bridge shown in Fig. 1 b was conceived and calculations were performed for this structure. For calculations in this case, the cross-sectional areas of hangers at branching points were assumed to be significantly smaller than the cross-sectional areas of other hangers, and the finite difference method of the deflection theory was employed. The dash-dot lines in lower part of Fig. 11 illustrate the results.

According to the results, the positive and negative bending moments of stiffening trusses are more evened out and the maximum negative bending moments near main towers are approximately one-third of those for the original continuous suspension bridge. Deflection changes are also more gradual. There are cases when compressive forces are produced under live loads at hangers near the branching points of an auxiliary cable continuous suspension bridge, but in the type having no hangers at branching points, the compressive forces are reduced and can be adequately offset by tensile forces due to dead load. Calculations were also made based on the large displacement method for the two cases above and the results were in good agreement with those obtained from the deflection theory.

The deflections of stiffening trusses of suspension bridges have direct relations with the running stabilities of high-speed trains. In order to investigate the serviceability of bridge, a single track train load of total length of 400 m was moved across to determine deflections of stiffening trusses. Fig. 12 shows the results. The top part of the figure shows calculation results for the original continuous suspension bridge, the middle part shows those for the auxiliary cable continuous suspension bridge of the type having hangers at branching points of main and auxiliary cables, and the bottom part shows those for the auxiliary cable continuous suspension bridge of the type having no hangers at branching points.

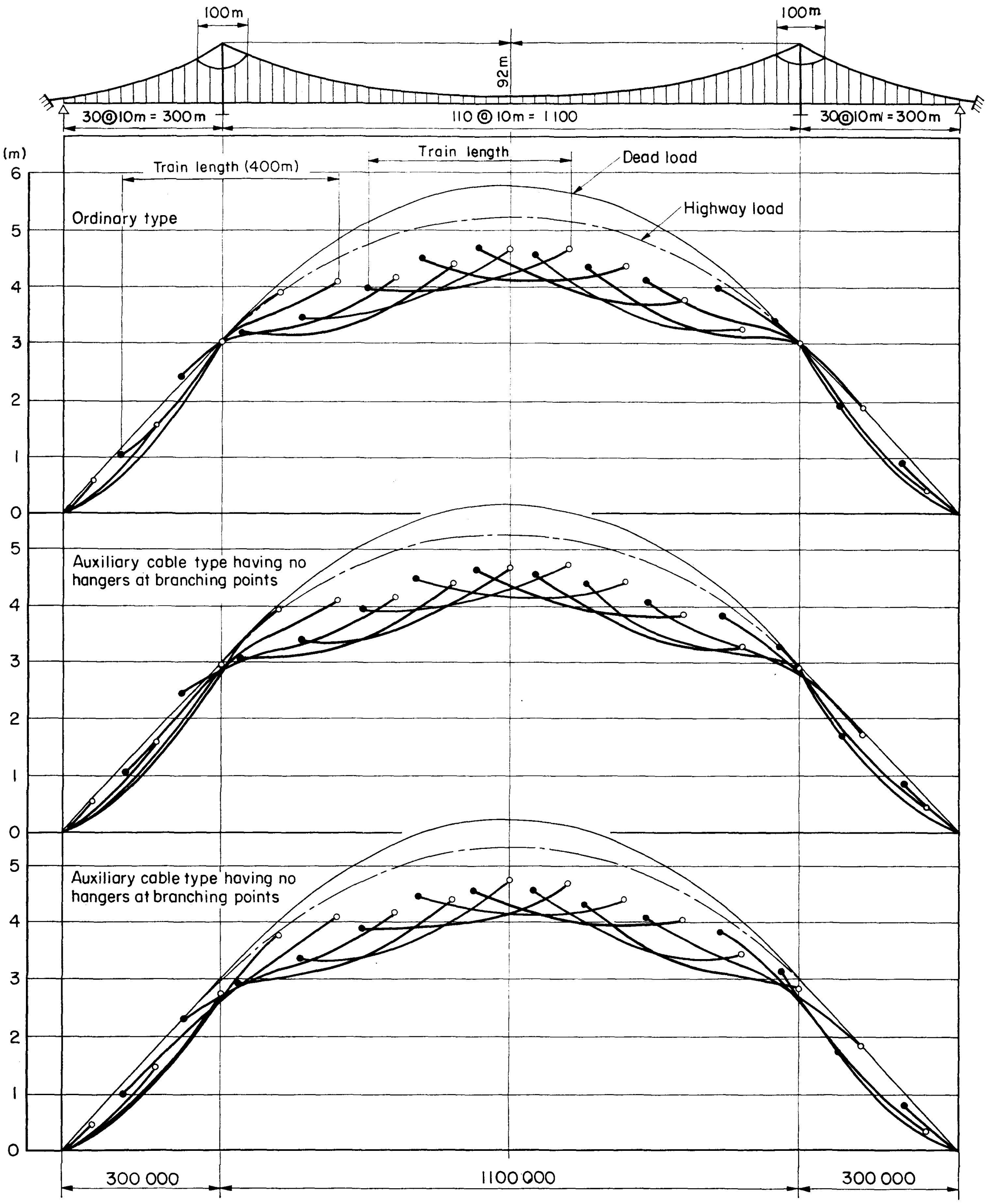

Fig. 12. Deflection change due to train passage.

The curves shown in Fig. 12 indicate the configurations of road surfaces due to dead load. The dash-dot lines indicate the deflections in case highway load is applied uniformly on the above. The solid lines indicate the deflections in the section loaded by the train when a train load is still further applied. If the train is considered to make access to the bridge from the left, the ○-marks indicate the locations of the front end of the train and the ●-marks are the locations of the rear end.

As is clear from Fig. 12, the deflection changes at the vicinities of main towers of an auxiliary cable continuous suspension bridge are more gradual than those of ordinary continuous suspension bridges and the vertical curvature produced in the bridge surfaces are alleviated. Particularly, when an auxiliary cable continuous suspension bridge of the type having no hangers at branching points of main and auxiliary cables is provided, the serviceability of running trains and automobiles are greatly improved.

7. Conclusions

The following conclusions may be reached from the present studies:

1. Auxiliary cable continuous suspension bridges can be analyzed for deflections and stresses by the deflection theory taking the horizontal components of forces acting on main and auxiliary cables as unknowns. Attention must be paid in such analyses to the fact that vertical forces different from those in ordinary continuous suspension bridges are produced in hangers at branching points of main and auxiliary cables when live loads are applied.
2. In case hangers are not joined at branching points of main and auxiliary cables, analysis can be made by the finite difference method of the deflection theory considering virtual slender hangers at these points.
3. Comparison of experimental results with theory indicates that the method presented above gives a reasonable basis for designing auxiliary cable continuous suspension bridges. Experimental results also suggested that there exist an optimum auxiliary cable geometry.
4. An auxiliary cable continuous suspension bridges, as compared to an ordinary continuous suspension bridge, has milder changes in deflections of stiffening trusses in the vicinities of main towers and vertical curvature in railway track and highway pavement surface are alleviated. Especially, the bridge having no hangers at branching points of main and auxiliary cables offers good serviceability for running trains and automobiles. This latter type of bridge also has smaller bending moments acting on stiffening trusses in the vicinities of main towers. Comparative studies on a continuous suspension bridge with the spans of $300+1100+300$ m indicate that the maximum bending moment is lessened to approximately one-half for the bridge having hangers at branching points and approximately one-third for the bridge having no hangers at these points. For the abovementioned bridges, large differences are not produced in values of bending moments and deflections at portions other than at the main towers.

5. Experimental results of vibration tests on models indicate that the dynamic properties remain almost unchanged in case auxiliary cables are installed within a limited range. It seems that this fact is valid for actual bridges.

As a general conclusion, it may be stated that the auxiliary cable type of continuous suspension bridge offers advantages over the ordinal type when designed for railway loading.

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Summary

A theory for analyzing symmetrical three span continuous suspension bridges having auxiliary cables branched out from the main cable is developed and compared with experimental results. The theory is applied to the analysis of an actual suspension bridge in order to establish data for comparisons between the new and ordinary types of continuous suspension bridge. The results indicate that the auxiliary cable continuous suspension bridge may offer advantages over the ordinary continuous suspension bridge when designed for railway loading.

Résumé

On a developpé et comparé avec les résultats expérimentaux une théorie qui étudie les ponts suspendus continus à trois portées symétriques qui possèdent des câbles auxiliaires reliés au câble principal. On applique cette théorie à l'étude d'un pont suspendu récent afin d'obtenir des données permettant de comparer ce nouveau type de pont suspendu continu avec l'ancien. Les résultats montrent que les ponts suspendus continus à câbles auxiliaires peuvent présenter des avantages sur les ponts suspendus continus ordinaires, dans le cas du dimensionnement d'un pont-rail.

Zusammenfassung

Es wird eine Theorie zur Analyse symmetrischer durchlaufender Hängebrücken mit drei Öffnungen entwickelt, bei welchen vom Hauptkabel Hilfskabel abgezweigt werden, und mit den experimentellen Resultaten verglichen. Die Theorie wird auf die Analyse einer bestehenden Hängebrücke angewandt, um Unterlagen für den Vergleich zwischen dem neuen und den üblichen Typen von Hängebrücken zu gewinnen. Die Ergebnisse zeigen, dass die mit Hilfskabeln ausgerüstete Hängebrücke gegenüber der gewöhnlichen durchlaufenden Ausführung Vorteile bietet, wenn es sich um Eisenbahnbrücken handelt.