On the shear capacity of girder webs

On the Shear Capacity of Girder Webs

De la capacité de cisaillement des âmes de poutres

Über die Schubkapazität von Trägerstegen

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The post critical shear capacity of webs has been known for a long time [1, 2, 3]. However, a simple calculation was first started by BASLER [4, 5] when he introduced the shear capacity as the sum of the shear taken by shear stress in the web and the shear taken by a tension diagonal. There has been raised some critic against Basler's equations, for instance by FUJII [9]. Considerable work has been done on improvements of Basler's equations, notably by ROCKEY and ŠKALOUD [6, 7, 8].

In the following will be demonstrated some errors made by BASLER. Further on a more general deduction will be given, which includes the corrected Basler equations and the Rockey-Škaloud equations.

In Fig. 1 is given a girder with tension field due to shear load into the post critical range. BASLER's idea that the shear capacity is the sum:

$$V_u = V_\tau + V_d \tag{1}$$

is generally accepted. V_τ is the shear force taken by the shear stresses in the

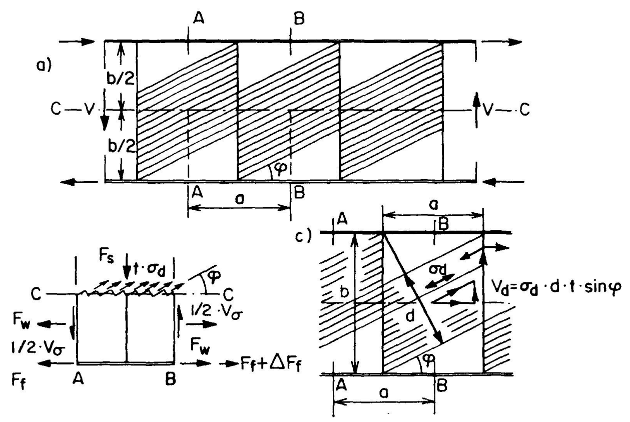

Fig. 1. Equilibrium due to shear force V.

web and V_d is the shear force taken by the tension diagonal or tension field, see Fig. 1.

For the V_τ BASLER postulates:

$$V_\tau = \tau_c b\, t, \tag{2}$$

where τ_c is the critical shear buckling stress:

$$\tau_c = \tau_{cE} = \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2 k_s \tag{2a}$$

for an elastic material. $k_s = 5{,}34 + 4\left(\frac{1}{\alpha}\right)^2.$

BASLER assume that the development of a tension field has no effect on the value of τ_c. Accordingly the ultimate capacity corresponds to the angle ϕ giving max V_u or V_d:

$$\frac{dV_u}{d\phi} = \frac{dV_d}{d\phi} = 0. \tag{3}$$

As will be seen from Fig. 1c we have

$$V_d = \sigma_d\, d\, t \sin\phi, \tag{4}$$

where

$$d = b\cos\phi - a\sin\phi \tag{4a}$$

and

$$\frac{dV_d}{d\phi} = 0 = \frac{d}{d\phi}(\sigma_d dt \sin\phi) = \sigma_d t \frac{d}{d\phi}(b\cos\phi\sin\phi - a\sin^2\phi). \tag{5}$$

BASLER neglects obviously that σ_d will be a function of ϕ, as seen from Eq. (8). However, this error will be small, as easily controlled.

The error may in rare cases mean up to 5% of V_d and is consequently of no importance.

Eq. (5) gives:

$$\operatorname{tg}\phi = \sqrt{1+\alpha^2} - \alpha; \qquad \alpha = \frac{a}{b} \tag{6}$$

and

$$V_d = \sigma_d t b \frac{1}{2(\sqrt{1+\alpha^2}-\alpha)} = \frac{1}{2}\sigma_d t b(\sqrt{1+\alpha^2}-\alpha) = \frac{1}{2}\sigma_d t b \operatorname{tg}\phi. \tag{7}$$

BASLER deduced the equation:

$$V_d = \sigma_d t b \frac{1}{2\sqrt{1+\alpha^2}}.$$

This result is due to the incomplete picture of forces in Fig. 1b used by BASLER in his deductions [9].

As readily seen, Eq. (7) give results well below the Basler equation.

Between σ_d and τ we have the following equation due to the deviation yield hypothesis:

$$\sigma_d = \sqrt{\sigma_y^2 - \tau_c^2\left[3 - \left(\frac{3}{2}\sin 2\phi\right)^2\right]} - \frac{3}{2}\tau_c \sin 2\phi. \tag{8}$$

As given by BASLER, the following equation:

$$\sigma_d \approx \sigma_y - \sqrt{3}\,\tau_c \tag{8a}$$

may be used without any important loss of accuracy. However, we shall here use the complete Eq. (8).

Introducing this we get for V_u:

$$V_u = \sigma_d t b \tfrac{1}{2} \operatorname{tg}\phi + \tau_c t b \tag{9}$$

or
$$V_u = \sigma_y t b \left\{\frac{1}{2}\operatorname{tg}\phi\left[\sqrt{1 - \left(\frac{\tau_c}{\sigma_y}\right)^2 \left(3 - \frac{3}{2}\sin 2\phi^2\right)} - \frac{3}{2}\left(\frac{\tau_c}{\sigma_y}\right)\sin 2\phi\right] + \frac{\tau_c}{\sigma_y}\right\}. \tag{9a}$$

The angle ϕ is given by Eq. (6) and τ_c is known from (2a) for an elastic material. For stresses above the proportionality limit we may introduce.

$$\tau_c = \tau_y - \left(\frac{1}{\kappa} - 1\right)\frac{\tau_p^2}{\tau_{cE}}, \tag{10}$$

where $\tau_y = \frac{1}{\sqrt{3}}\sigma_y$; $\tau_p = \kappa\,\tau_y$ is the proportionality limit.

For most structural steel $\kappa \approx 0.8$, but due to welding stresses, out of planes of the web etc. $\kappa = 0.5$ is a better value for technical use. In the diagrams, Figs. 2 and 7, the value $\kappa = 0.8$ is used.

Fig. 2 gives the values of $\frac{V_u}{V_y} \approx \frac{\tau_u}{\tau_y}$ for webs with varying $b/t = \beta$ values and $a/b = \alpha$ values 1.0 and 2.0.

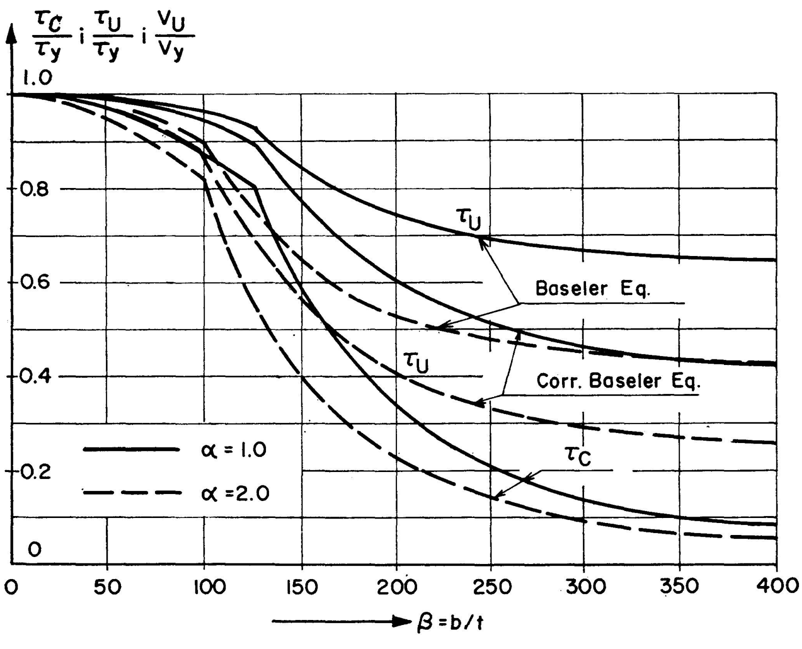

Fig. 2. Diagram of V_u/V_y for $\alpha = a/b$ values 1.0 and 2.0 and web slenderness $\beta = b/t$, giving original and corrected Basler results.

As will be seen from Fig. 2 the corrected Basler equation (9) gives considerably less shear capacity than the original Basler equation. Especially will it be so for thin webs $\beta < 200$. Eq. (9) is also given by Fujii [9].

The original Basler equations give results which are in good agreement with experiments, especially for α values between 1 and 1.5. With the error in V_d corrected a V_u corresponding to the experiments will only be possible with an increased shear force V_τ, and it is little reason to believe that τ_c will not be affected by a tension field or tension diagonal across the web, which in turn gives a raise to V_τ.

In Fig. 3 is given a diagram for the shear buckling τ_s' as effected by a uniform tension in one direction [10, 11]. As will be seen from Fig. 3 the "critical" shear stress may be considerably greater than given by (2a).

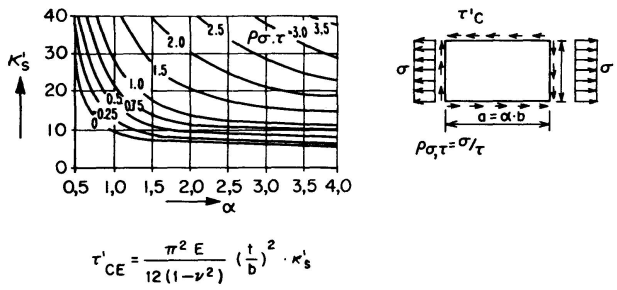

Fig. 3. Effect of uniform tension on shear buckling stress τ_{cE}.

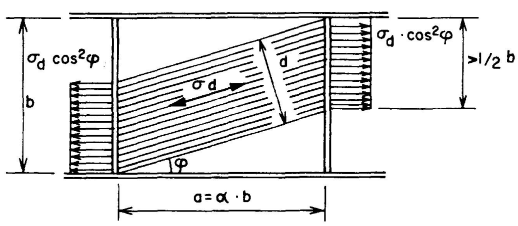

Fig. 4. Tension field or diagonal in shear loaded web.

In Fig. 4 is drawn for a comparison a sketch of the tension field or diagonal. It is readily seen that the effect must be very much the same as in Fig. 3.

We may for instance assume the following expression for the shear buckling stress τ_c.

$$\tau_c = \tau_{c4} + f(\sigma_d; \beta; \alpha),$$

where τ_{c4} is the critical shear stress in a rectangular plate without any tension field. σ_d is given by Eq. (8).

However, just to demonstrate the arbitrariness of the Basler assumption concerning τ_c, we shall handle it in a different manner.

The tension field, see Fig. 4, will act as an elastic support on the web, like a hammock. As a simplification we consider this effect as equal to a support which follows the middle of the tension field, see Fig. 5. We have then two trapezoidal plates AEE'D and BEE'C instead of the original plate ABCD. The effect of this support might increase τ_{cE} up to 4 times the previous value, as will be seen by substituting the two trapezoids by two rectangles.

However, it will emphasize the arbitrariness better to calculate the critical shear load of triangular plates AEF and CE'F' [12]. The stiffeners, BE and DE' will have some effect, but a calculation of critical shear stress τ_c as for a triangle is a simplification on the conservative side. With d and ϕ known, Eqs. (4a) and (6), we get:

$$l = \tfrac{1}{2}(b \operatorname{cotg}\phi + a); \qquad e = \tfrac{1}{2}(b \operatorname{cotg}\phi - a) \tag{11}$$

and

$$\tau_{cE} = \frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{l}\right)^2 k_s', \tag{12}$$

where k_s' is given by:

$$k_s' \approx 5.34(1.0+\xi^2) + \frac{1.8\,\xi^2}{1.0+\xi}; \qquad \xi = \operatorname{cotg}\phi. \tag{13}$$

Eq. (13) is an approximation of the results given in [12]. Within the elastic range $\tau_c = \tau_{cE}$, in the post proportionality range τ_c is given by Eq. (10).

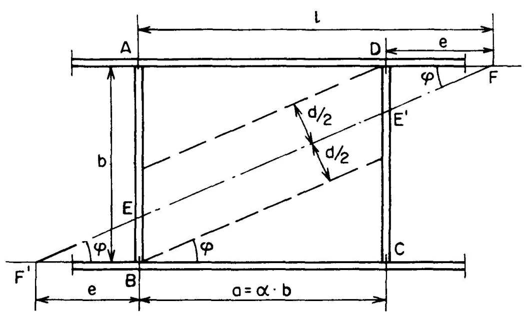

Fig. 5. Effect of tension field substituted by a support CC'.

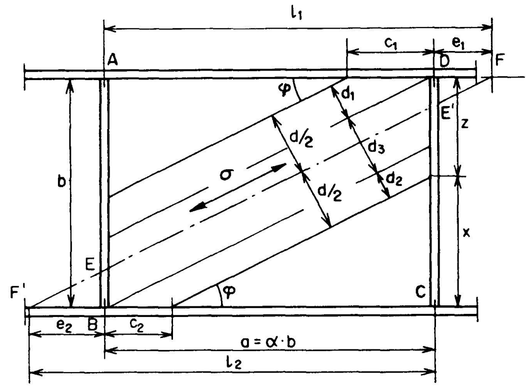

Fig. 6. Tension field with different stiffness of flanges.

With τ_c given, σ_d is found from Eq. (8) and V_u from Eq. (9). In Fig. 7 is given the results for a calculation with α values 1.0 and 2.0. The curves to be compared with the results in Fig. 2 are marked with $c/a = 0$.

In a series of publications [6, 7, 8] Rockey and Škaloud have made investigations on the shear capacity of webs. They introduced a tension field which is symmetrical about the diagonal, and the width of which is given by the stiffness of the upper and lower flanges. The angle ϕ is given by $\operatorname{tg}\phi = b/a$.

This model is an unnecessary limitation on the tension diagonal. We shall here investigate the conditions shown in Fig. 6. c_1 and c_2 are defined by the stiffness of the flanges, see Eq. (23).

The tension diagonal d is devided in 3 strings. We have:

$$d = d_1 + d_2 + d_3,$$
$$d_1 = c_1 \sin\phi; \quad d_2 = b\cos\phi - a\sin\phi; \quad d_3 = c_2 \sin\phi \tag{14}$$

and for the shear capacity V_d.

$$V_d = \sigma_d t \left\{ c_1\left(1 - \frac{c_1}{2a}\right)\sin^2\phi + b\cos\phi\sin\phi - a\sin^2\phi + c_2\left(1 - \frac{c_1}{2a}\right)\sin^2\phi \right\}$$

or

$$V_d = \sigma_d t b\{\cos\phi\sin\phi - \alpha_c \sin^2\phi\}; \qquad \alpha_c = \frac{a}{b}\left(1 - \frac{c_1+c_2}{a} + \frac{c_1^2+c_2^2}{2a^2}\right) \tag{15}$$

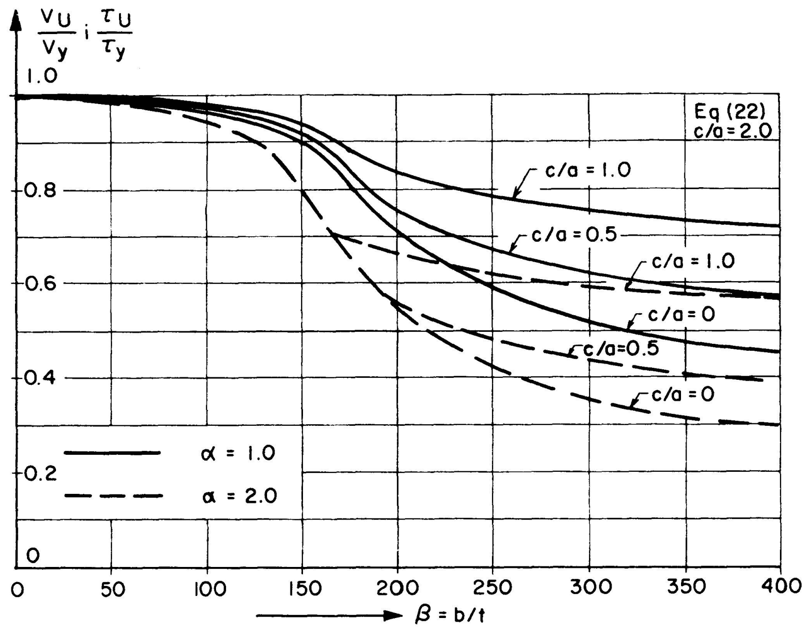

Fig. 7. V_u/V_y diagram $\alpha = 1$ and 2; $\beta = b/t$; $c/a = 0$; 0.5 and 1.0, due to Eq. (19).

with further modification:

$$V_u = \sigma_d t b \frac{(1 - \alpha_c \operatorname{tg}\phi)\operatorname{tg}\phi}{1 + \operatorname{tg}^2\phi} + \tau_c t b. \tag{16}$$

The deduction of Eqs. (14)–(16) imply $\operatorname{tg}\phi \leqq b/a$. However, it can be proved that the equations are valid for $\operatorname{tg}\phi \lesseqgtr b/a$.

The special situation investigated by Rockey and Škaloud [6, 7, 8] and others is given by $c_1 = c_2$ and $\operatorname{tg}\phi = b/a$. If we introduce this in Eqs. (15) or (16), we get the Rockey solution:

$$V_u = 2\,\sigma_d t c \sin^2\phi \left(1 - \frac{c}{2\,a}\right) + \tau_c t b. \tag{17}$$

The relation between σ_d and τ_c is given by Eq. (8).

If we, like Basler, neglect the influence of ϕ on the stresses σ_d and τ_c we get from Eq. (16):

$$\frac{dV_u}{d\phi} = \frac{dV_d}{d\phi} = 0; \quad \text{and} \quad \operatorname{tg}\phi = \sqrt{1 + \alpha_c^2} - \alpha_c; \quad \alpha_c = \frac{a}{b}\left(1 - \frac{c_1 + c_2}{a} + \frac{c_1^2 + c_2^2}{2\,a^2}\right) \tag{18}$$

and the following simple formula is deduced:

$$V_u = \sigma_d t b \tfrac{1}{2} \operatorname{tg}\phi + \tau_c t b, \tag{19}$$

which is identical to formula (9); the value ϕ beeing different. τ_c is given by Eqs. (12) and (10); σ_d by Eq. (8).

In Fig. 8 is shown the results of calculations with Eqs. (16) and (19). The values given by Eq. (19) are given in the centre of the diagrams, and then Eq. (16) is solved for different ϕ values, $\phi + \Delta\phi$. As will be seen Eq. (19) is

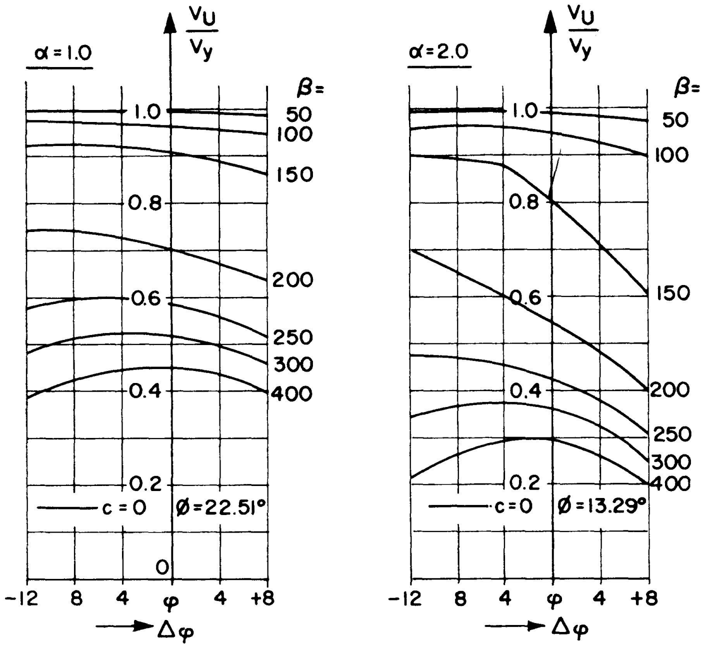

Fig. 8. Comparison of results from Eqs. (16) and (19). $\alpha = 1.0$ and 2.0; $c/a = 0$.

fairly good in most cases, the maximum value of V_u are always greater than given by Eq. (19).

In Fig. 7 is given the results V_u/V_y for $\alpha = 1.0$ and 2.0 and $c/a = 0$; 0.5; 1.0; with $c_1 = c_2$ we have $c = 2\,c_1$; with $c_1 \lesseqgtr c_2$ we have:

$$c = 2\,a - \sqrt{2}\,\sqrt{(a-c_1)^2 + (a-c_2)^2} \tag{20}$$

with absolute stiff flanges we get $c_1 = c_2 = a$, $c = 2\,a$, $\operatorname{tg}\phi = 1$ and

$$V_d = \tfrac{1}{2}\sigma_d\,t\,b\,, \tag{21}$$

which result was obtained by WAGNER [2], and the ultimate capacity for a very thin web, $\beta > 400$, with stiff flanges is:

$$V_u = V_d = \frac{1}{2}\sigma_y\,t\,b \quad \text{or} \quad \frac{V_u}{V_y} = \frac{\frac{1}{2}\sigma_y}{\tau_y} = \frac{\sqrt{3}}{2} = 0.865. \tag{22}$$

The bending moment in the flanges may be controlled by the following equations, see Fig. 9.

$$c_1^2\left(1 - \frac{c_1}{2\,a}\right)^2 = \frac{4\,M_{Fy}}{t\,\sigma_d\,\sin^2\phi} \tag{23}$$

and the yield hinge in the flange is placed at

$$x = c_1 - \frac{c_1^2}{2\,a}. \tag{23a}$$

Normally the flanges and in consequence the yield moment in the flange M_{Fy} is known, and c_1 is found by a trial and error method as σ_d will be a function of c_1.

In general the c_1 and c_2 values will be small compared to a. However, in some structures as for instance composite beams one of them may be of considerable importance, see Fig. 10.

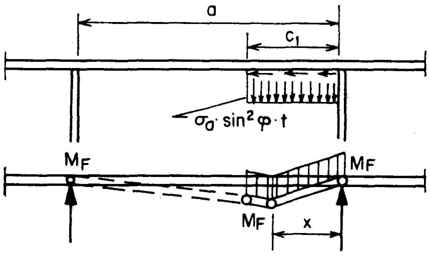

Fig. 9. Bending of flange.

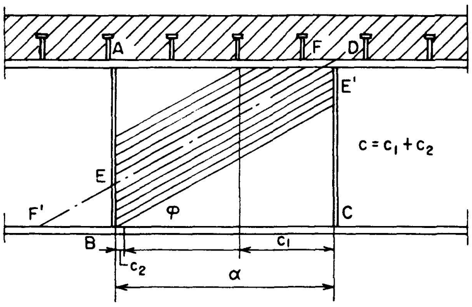

Fig. 10. Bending of flanges in a composite beam.

In this situation the triangles AEF and CE'F are different in size, and a mean value:

$$\tau_{cE} = \sqrt{\tau_{cE_1}\,\tau_{cE_2}} \tag{24}$$

is better used for calculating V_τ.

The triangles are defined by the following expressions, see Fig. 6:

$$\begin{aligned} e_1 &= \frac{d}{2}\,\frac{1}{\sin\phi} - c_1; \qquad l_1 = a + e_1, \\ e_2 &= \frac{d}{2}\,\frac{1}{\sin\phi} - c_2; \qquad l_2 = a + e_2. \end{aligned} \tag{25}$$

d is given by Eq. (14).

The method demonstrated above is compared to a number of tests with astonishing good results, which in itself is really interesting compared to the arbitrariness in the assumptions leading to the method.

In the following shall only be referred the results from two tests H_1 and H_2 in [13].

The relation V_{exp}/V are for this beams:

	Basler	Corr. Basler	Fujii	[13]	This method	Rockey
$H_1\, V_{exp}/V$ =	1.05	1.24	1.09	1.00	1.09	2.85
$H_2\, V_{exp}/V$ =	1.04	1.26	1.02	0.95	1.03	2.16

V_{exp} is the shear capacity observed by the experiment. V is the calculated capacity when the moment effect is considered as well. In the control is the effect of the moment except for the Rockey method handled as suggested by Basler [15]. The beams are characterized by the following data:

$$H_1: \ \alpha = 2.0;\ \beta = 249; \qquad H_2: \ \alpha = 3.0;\ \beta = 242.$$

In Fig. 11 is given the load deflection diagram for this beams. As will readily be seen the ultimate loads are connected with great deformations. In reality a structure will be destroyed before deformations of this magnitude can take place.

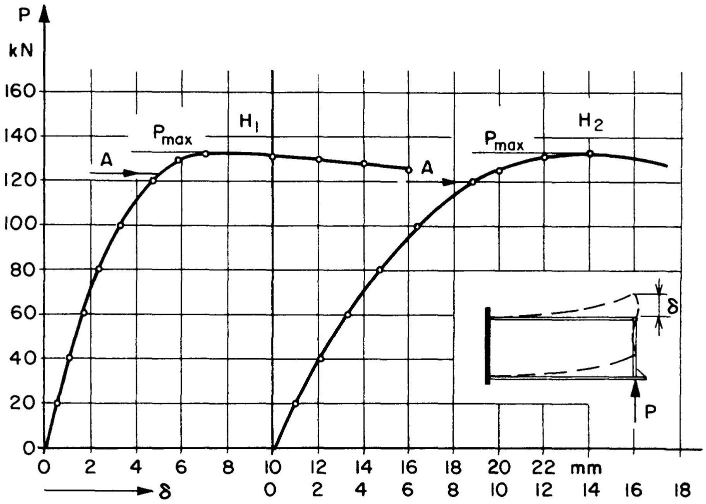

Fig. 11. Load deflection of test beams H_1 and H_2 |13|.

For design work the load capacities at the beginning of the really great deformations, f. inst. points A in Fig. 11, are the important ones. A design procedure founded on the theory of plates with large deformations as f. inst. given by [14] or [16] seems to fulfil the purpose. A load limit where the first yielding take place in the middle plane of the plate will satisfy any design purpose, and the large deflection elastic plate theory will still be sufficiently valid. It may be well to keep in mind that the web failure always is a result of yield in tension, the web "stability" is not the main problem, stability failures are due to failure in f. inst. the flanges which is quite a different problem.

Notation

a = Length of web panel, distance between vertical stiffeners.
b = Distance between flanges.
$c_{1,2}$ = Length of tension field action on upper resp. lower flange.
c $\approx$ c_1+c_2. Eq. (20).
d = Width of tension field.
$e_{1,2}$ = Where center line of tension field cross the flanges.
t = Web thickness.
α = a/b.
β = b/t.
κ = τ_p/τ_y.
σ_d = Stress in tension field.

τ_c = Critical shear stress.
τ_{cE} = Critical shear stress, elastic material.
ϕ = Angle between tension field and flanges.
V = Shear force. Index u, y, d, τ refer to ultimate, yield, tension diagonal and shear tension.

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Summary

Some errors are discussed and corrected in the Basler method, especially the effect of the tension field. The corrected value gives a considerable reduction in shear capacity compared to the Basler equations. The main point is however the arbitrariness of the assumption, made by Basler, Rockey and others, that the critical shear stresses of the web are not affected by the tension field.

To demonstrate this point, a method is presented where the effect of the tension field is substituted by a simple lateral support. The method gives

results which for slender webs ($\beta > 200$) are in very good agreement with experiments. All methods give reasonable results for stout webs ($\beta < 100$).

However, the ultimate loads are connected with deformations of such a magnitude that actual structures, f. inst. bridges, will be destroyed before this load limit. A design method based on large deflection theory of elastic plates and taking the first yielding in the middle plane of the plate as the ultimate load limit will serve the purpose and be more realistic to conditions in actual structures.

Résumé

On corrige des fautes dans la méthode de Basler, en particulier l'effet du champ de tension. Les valeurs corrigées donnent une réduction considérable de la capacité de cisaillement. Le point capital est pourtant l'objection contre la supposition faite par Basler, Rockey et d'autres comme quoi la tension critique de cisaillement dans l'âme ne serait influencée par le champ de tension.

Pour illustrer ce point on présente une méthode où l'effet du champ de tension est remplacé par un simple support latéral. Pour des âmes élancées ($\beta > 200$) les résultats correspondent très bien aux valeurs d'essai. Tous les procédés donnent des résultats raisonnables pour des âmes épaisses ($\beta < 100$).

Pourtant la charge ultime est associée à des déformations de sorte que des structures existantes, p. ex. des ponts seront détruits avant la charge limite. Une méthode de calcul fondée sur la théorie des plaques élastiques, en tenant compte des grands déplacements est donc plus convenable et plus réaliste pour des structures existantes. La charge critique est alors atteinte à l'écoulement de la fibre moyenne de la section de la plaque.

Zusammenfassung

Es werden Fehler der Methode von Basler korrigiert, speziell der Einfluss des Zugfeldes. Die Korrekturwerte ergeben eine beachtliche Reduktion der Schubkapazität. Wesentlich ist aber der Einwand gegen die willkürliche Annahme von Basler, Rockey und anderen, wonach die kritische Schubspannung im Steg nicht vom Zugfeld beeinflusst wird.

Zur Illustration dient ein Verfahren, bei dem der Einfluss des Zugfeldes durch eine einfache Festhaltung ersetzt wird. Die Ergebnisse stimmen für schlanke Stege ($\beta > 200$) sehr gut mit den Versuchswerten überein. Sämtliche Verfahren ergeben vernünftige Resultate für dicke Stege ($\beta < 100$).

Zur kritischen Last gehören so grosse Verformungen, dass bestehende Konstruktionen (z. B. Brücken) vor Erreichen derselben zerstört werden. Ein auf der elastischen Plattentheorie beruhendes Berechnungsverfahren unter Berücksichtigung der grossen Verschiebungen ist deshalb zweckmässig und für bestehende Tragwerke realistischer. Die kritische Last wird dann beim Fliessen der mittleren Faser des Plattenquerschnitts erreicht.

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