The analysis of multispan suspension bridge systems

The Analysis of Multispan Suspension Bridge Systems

Calcul des systèmes constitués par des ponts suspendus à travées multiples

Untersuchungen von Hängebrückensystemen mehrfacher Spannweite

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Introduction

Of the many methods which have been devised for solving the classical equations of a suspension bridge, perhaps the best known and most easily applied is that given by TIMOSHENKO [1]. In this procedure the mildly nonlinear nature of the governing equations of a bridge is recognised, and their solution is obtained using a first order iteration method known as the method of successive approximations. This is essentially a numerical process, so that when several cycles of iteration are required it is necessary to replace algebraic symbols by numbers, otherwise unduly complicated algebraic expressions result. Thus an explicit solution is not in general possible by this method.

It is the object of this paper to show how by use of the perturbation method a closed form of solution can be generated for the governing equations of a suspension bridge. This alternative method has the advantage that it yields explicit algebraic formula for the solution in terms of the basic linear solution, together with a convergent sequence of correction functions which must be added to take account of the nonlinear influence. The method is applied to the general case of several suspension spans coupled together in series.

It is by recognising at the outset that the governing equations for a suspension bridge are only slightly nonlinear, and that the solution being sought is in the region of the linearized solution, that the use of the perturbation method can be justified. The perturbation equations are all second order linear differential equations, and are solved by expanding the solution in terms of Fourier Series. This technique is particularly advantageous if point loads are applied to the deck, since the discontinuities they introduce are "smoothed" by introducing harmonic components for the applied loads. This permits a single equation to be used for describing the deck deformation.

Although the perturbation method strictly only applies for slightly nonlinear systems, it has been found to give reliable answers for cases with considerable nonlinearity. This is attributable to the well behaved nature of the governing equations of a bridge. However, for such cases care must be exercised to ensure that the method has converged properly.

General Theory

Fig. 1 shows the general arrangement of a set of R suspended bridge spans connected together in series. The decks are all simply supported at their ends

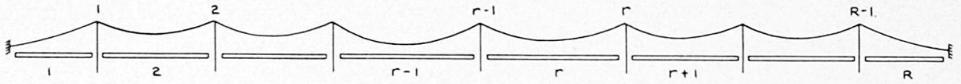

Fig. 1.

so that they do not carry end moments. The intermediate support towers are all considered to be flexible and of known stiffness. The cables are rigidly clamped to the towers, and in the two terminal spans the ends of the cable are rigidly anchored in the ground.

Consider any span r of the system shown in Fig. 1. This is given in detail in Fig. 2. Then the equation of equilibrium for the deck at any section x is given by

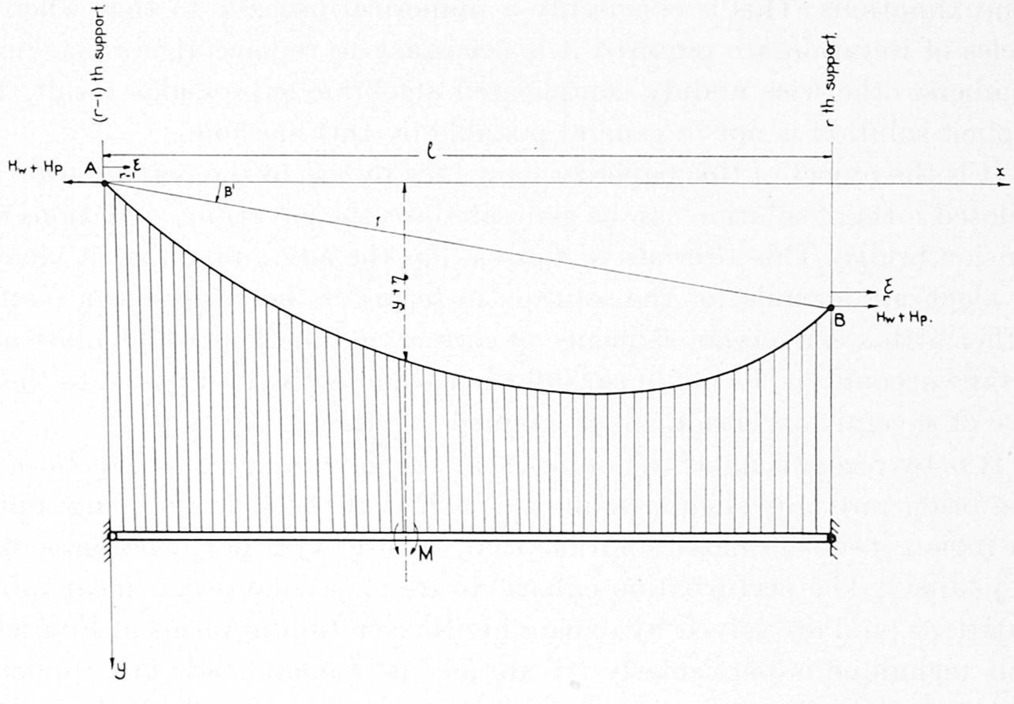

Fig. 2.

Eq. (21) of TIMOSHENKO

$$ {}_rE\,{}_rI\,\frac{d^2\,{}_r\eta}{dx^2}-({}_rH_w+{}_rH_p)\,{}_r\eta = {}_rH_p\,\frac{{}_rw\,x}{2\,{}_rH_w}({}_rl-x)+{}_rM_p\,, $$

where ${}_rE$ = Young's Modulus for the deck,
${}_rI$ = second moment of area of the deck,
${}_r\eta$ = cable deflection,
${}_rH_w$ = horizontal component of cable tension under dead load,
${}_rH_p$ = additional horizontal tension due to live load and temperature,
${}_rw$ = dead weight of bridge per unit length,
${}_rl$ = cable span,
${}_rM_p$ = live load bending moment.

This equation differs from Eq. (21) on the right hand side where y has been replaced by its equivalent expression Eq. (2), and hogging moments have been taken positive. This latter modification merely changes the sign of ${}_rM_p$. Suffix r indicates the rth span.

The equation for horizontal compatibility of the cable is given by Eq. (15) of TIMOSHENKO. When applying this equatio TIMOSHENKO assumes the cable fixed ended. If this is not the case, and the cable deflects small amounts ${}_{r-1}\zeta$ and ${}_r\zeta$ at A and B respectively (see Fig. 2), then on integrating Eq. (15) between 0 and ${}_rl$ and making use of TIMOSHENKO's Eq. (o) and Eq. (p) gives

$$ {}_r\zeta-{}_{r-1}\zeta = \frac{{}_rH_p\,{}_rL}{{}_rE_c\,{}_rA_c}+{}_r\epsilon\,{}_rt\,{}_rL_1-\frac{{}_rw}{{}_rH_w}\int\limits_0^{{}_rl}{}_r\eta\,dx-\tfrac{1}{2}\int\limits_0^{{}_rl}({}_r\eta')^2\,dx\,, $$

where ${}_rL$ = is defined by Eq. (o) TIMOSHENKO; it is approximately $l\sec^3\beta'$,
${}_rE_c$ = Young's Modulus for the cable,
${}_rA_c$ = cross-sectional area of the cable,
${}_r\epsilon$ = thermal expansion coefficient of the cable,
${}_rt$ = rise in temperature of the cable,
${}_rL_1$ = is defined on p. 232 by TIMOSHENKO; it is appoximately $l\sec^2\beta'$.

The term ${}_r\epsilon\,{}_rt\,{}_rL_1$ due to thermal expansion has been included in accordance with Eq. (25), and H_p is now the change in tension due to live loading and temperature effects. Note that TIMOSHENKO uses H_s for this. The above two equations are the well known governing equations for a single suspended span.

When using the above two equations it is convenient to adopt nondimensional forms by writing

$$ {}_rH_p = {}_rh\,{}_rH_w; \qquad {}_r\eta = {}_rv\,{}_rl; \qquad {}_rM_p = {}_r\mu\,{}_rH_w\,{}_rl; \qquad {}_r\alpha = \frac{{}_rE\,{}_rI}{{}_rH_w\,{}_rl^2}; $$

$$ {}_r\beta = \frac{{}_rw\,{}_rl}{{}_rH_w}; \qquad {}_r\gamma = \frac{{}_rH_w\,{}_rL}{{}_rl\,{}_rE_c\,{}_rA_c}; \qquad {}_r\delta = \frac{{}_r\epsilon\,{}_rt\,{}_rL_1}{{}_rl} \quad \text{and} \quad x = \frac{x}{{}_rl}. $$

Substituting these expressions into the governing equations gives

$$ {}_r\alpha\, {}_r v'' - (1 + {}_r h)\, {}_r v = {}_r h\, {}_r\beta \frac{x\,(1-x)}{2} + {}_r\mu\,, \tag{1} $$

$$ \frac{{}_r\zeta - {}_{r-1}\zeta}{{}_r l} = {}_r h\, {}_r\gamma + {}_r\delta - {}_r\beta \int_0^1 {}_r v\, dx - \tfrac{1}{2}\int_0^1 {}_r v'^{\,2}\, dx\,. \tag{2} $$

Now if all spans are erected with the same initial horizontal component of tension H_w, then the deflection of the $(r-1)$ and rth towers is

$$ {}_{r-1}\zeta = ({}_r h - {}_{r-1}h)\, H_w\, {}_{r-1}s' \quad \text{and} \quad {}_r\zeta = ({}_{r+1}h - {}_r h)\, H_w\, {}_r s' $$

respectively. Where ${}_{r-1}s'$ and ${}_r s'$ are the flexibilities of the $(r-1)$ and rth towers respectively. Substituting these into Eq. (2) gives

$$ \left[\frac{H_w\, {}_{r-1}s'}{{}_r l} \quad -\left\{({}_{r-1}s' + {}_r s')\frac{H_w}{{}_r l} + {}_r\gamma\right\} \quad \frac{H_w\, {}_r s'}{{}_r l}\right] \begin{bmatrix} {}_{r-1}h \\ {}_r h \\ {}_{r+1}h \end{bmatrix} = {}_r\delta - {}_r\beta \int_0^1 {}_r v\, dx - \tfrac{1}{2}\int_0^1 {}_r v'^{\,2}\, dx\,. \tag{3} $$

Equations similar to (1) and (3) are available for all R spans. It is convenient to group together the R equations of type (3) into the matrix form

$$ \underline{\overline{D}}\, \underline{h} = [-{}_r\delta + {}_r\beta \int_0^1 {}_r v\, dx + \tfrac{1}{2}\int_0^1 {}_r v'^{\,2}\, dx]\,, \tag{4} $$

where $\underline{\overline{D}} = \begin{bmatrix} & & \vdots & & \\ \cdots & -\dfrac{H_w\, {}_{r-1}s'}{{}_r l} & ({}_{r-1}s + {}_r s')\dfrac{H_w}{{}_r l} + {}_r\gamma & \dfrac{-H_w\, {}_r s'}{{}_r l} & \cdots \\ & & \vdots & & \end{bmatrix}$ and $\underline{h} = \begin{bmatrix} {}_1h \\ \vdots \\ {}_R h \end{bmatrix}$.

$\underline{\overline{D}}$ is a tridiagonal matrix whose elements on the rth row are as indicated. The square brackets on the right hand side of Eq. (4) indicate a column matrix whose rth row is of the general form shown.

Examination of Eqs. (1) and (4) show that the dependent variables are ${}_r h$ and ${}_r v$. Since by hypothesis these are considered to be small relative to unity and ${}_r y/{}_r l$ then it is reasonable to assume that squares or products of ${}_r h$ and ${}_r v$ are of second order significance compared with other terms occuring in the equations. Since the nonlinear terms in Eqs. (1) and (4) arise from such products, it follows that these are of second order significance. This is signified by attaching the symbolic perturbation parameter Δ to them as shown below

$$ {}_r\alpha\, {}_r v'' - {}_r v - \frac{{}_r\beta\, x\,(1-x)\, {}_r h}{2} = {}_r\mu + \Delta\, {}_r h\, {}_r v\,, \tag{5} $$

$$ \underline{\overline{D}}\, \underline{h} - [{}_r\beta \int_0^1 {}_r v\, dx] = \left[-{}_r\delta + \frac{\Delta}{2}\int_0^1 {}_r v'^{\,2}\, dx\right]. \tag{6} $$

The perturbation method then proceeds to solve Eqs. (5) and (6) by expanding the solutions for ${}_r h$ and ${}_r v$ in a power series [2] of the type

$$ {}_r h = {}_r h_0 + \Delta\, {}_r h_1 + \Delta^2\, {}_r h_2 + \Delta^3\, {}_r h_3 + \Delta^4\, {}_r h_4 + \cdots \tag{7} $$

$$ {}_r v = {}_r v_0 + \Delta\, {}_r v_1 + \Delta^2\, {}_r v_2 + \Delta^3\, {}_r v_3 + \Delta^4\, {}_r v_4 + \cdots \tag{8} $$

Substituting Eqs. (7) and (8) into Eqs. (5) and (6) and equating powers of Δ then the following sets of linear equations are obtained

Δ^0
$$ {}_r\alpha\, {}_r v_0'' - {}_r v_0 - \tfrac{1}{2} x (1-x)\, {}_r\beta\, {}_r h_0 = {}_r\mu\,, \tag{9a} $$

$$ \underline{\overline{D}}\, \underline{h}_0 - [{}_r\beta \int_0^1 {}_r v_0\, dx] = [-{}_r\delta]. \tag{9b} $$

Δ^1
$$ {}_r\alpha\, {}_r v_1'' - {}_r v_1 - \tfrac{1}{2} x (1-x)\, {}_r\beta\, {}_r h_1 = {}_r h_0\, {}_r v_0\,, \tag{10a} $$

$$ \underline{\overline{D}}\, \underline{h}_1 - [{}_r\beta \int_0^1 {}_r v_1\, dx] = [\tfrac{1}{2} \int_0^1 {}_r v_0'^{\,2}\, dx]. \tag{10b} $$

Δ^2
$$ {}_r\alpha\, {}_r v_2'' - {}_r v_2 - \tfrac{1}{2} x (1-x)\, {}_r\beta\, {}_r h_2 = {}_r h_1\, {}_r v_0 + {}_r h_0\, {}_r v_1\,, \tag{11a} $$

$$ \underline{\overline{D}}\, \underline{h}_2 - [{}_r\beta \int_0^1 {}_r v_2\, dx] = [\int_0^1 {}_r v_0'\, {}_r v_1'\, dx]. \tag{11b} $$

Δ^3
$$ {}_r\alpha\, {}_r v_3'' - {}_r v_3 - \tfrac{1}{2} x (1-x)\, {}_r\beta\, {}_r h_3 = {}_r h_2\, {}_r v_0 + {}_r h_1\, {}_r v_1 + {}_r h_0\, {}_r v_2\,, \tag{12a} $$

$$ \underline{\overline{D}}\, \underline{h}_3 - [{}_r\beta \int_0^1 {}_r v_3\, dx] = [\tfrac{1}{2} \int_0^1 ({}_r v_1'^{\,2} + 2\, {}_r v_0'\, {}_r v_2')\, dx]. \tag{12b} $$

Δ^4
$$ {}_r\alpha\, {}_r v_4'' - {}_r v_4 - \tfrac{1}{2} x (1-x)\, {}_r\beta\, {}_r h_4 = {}_r h_3\, {}_r v_0 + {}_r h_2\, {}_r v_1 + {}_r h_1\, {}_r v_2 + {}_r h_0\, {}_r v_3\,, \tag{13a} $$

$$ \underline{\overline{D}}\, \underline{h}_4 - [{}_r\beta \int_0^1 {}_r v_4\, dx] = [\int_0^1 ({}_r v_0'\, {}_r v_3' + {}_r v_1'\, {}_r v_2')\, dx]. \tag{13b} $$

Higher order functions corresponding to Δ^5, Δ^6, etc. could similarly be obtained if required.

It will be noticed that the above R pairs of linear second order differential equations are all uncoupled, and hence in principle are easily solved

In practice the live load in any span may consist of a number of point loads each of which introduces a discontinuity into the live load bending moment diagram ${}_r\mu$. Strictly speaking it is necessary to use a separate set of equations (1) and (2) between each successive pair of discontinuities [3], and to determine the constants of integration for these from the compatibility conditions at each discontinuity. With a large number of point loads such a procedure would be unduly laborious, and it is desirable to expand the true moment diagram in a Fourier Series.

$$ {}_r\mu = \sum_{m=1}^{M} {}_r M_m \sin (m \pi x). \tag{14} $$

This "smooths" out the discontinuities from the actual bending moment diagram and makes μ a continuous function across the span. Eqs. (1) and (2) are then sufficient to describe the behaviour of the whole span.

The technique of solving Eqs. (9) to (13) is to start with (9a) and (9b) and obtain ${}_r v_0$ and ${}_r h_0$. These are of course the well known solutions to the "Linearized Deflection Theory". Next Eqs. (10a) and (10b) can then be solved for ${}_r v_1$

and ${}_rh_1$. In this way the functions $({}_rv_2, {}_rh_2)$, $({}_rv_3, {}_rh_3)$, etc. may be obtained in sequence.

Solution of the Perturbation Equations Using Fourier Series

Having expressed ${}_r\mu$ in harmonic components it seems reasonable to extend this idea and expand ${}_rv_0$, ${}_rv_1$, etc. in Fourier Series, thus

$$ {}_rv_0 = \sum_{m=1}^{M} {}_ra_m \sin(m\pi x)\,, \tag{15a} $$

$$ {}_rv_1 = \sum_{m=1}^{M} {}_rb_m \sin(m\pi x)\,, \tag{15b} $$

$$ {}_rv_2 = \sum_{m=1}^{M} {}_rc_m \sin(m\pi x)\,, \tag{15c} $$

$$ {}_rv_3 = \sum_{m=1}^{M} {}_rd_m \sin(m\pi x)\,, \tag{15d} $$

$$ {}_rv_4 = \sum_{m=1}^{M} {}_re_m \sin(m\pi x)\,. \tag{15e} $$

These functions all satisfy the essential boundary conditions

$$ v(0) = v(1) = E\,I\,v''(0) = E\,I\,v''(1) = 0\,. $$

Further we can write

$$ x(1-x) = \frac{4}{\pi^3} \sum_{m=1}^{M} \frac{\Phi_m}{m^2} \sin(m\pi x)\,, \tag{16} $$

where
$$ \Phi_m = \frac{1-\cos(m\pi)}{m}. \tag{17} $$

Introducing Eqs. (15) and (16) into the set of Eqs. (9) to (13) and equating coefficients of $\sin(m\pi x)$ gives

$$ -{}_ra_m / {}_rS_m - {}_rh_0\, {}_rR_m = {}_rM_m\,, \tag{18a} $$

$$ \underline{\bar{D}}\,\underline{h}_0 - \left[\frac{{}_r\beta}{\pi} \sum_{m=1}^{M} {}_ra_m\, \Phi_m\right] = [-{}_r\delta]\,, \tag{18b} $$

$$ -{}_rb_m / {}_rS_m - {}_rh_1\, {}_rR_m = {}_rh_0\, {}_ra_m\,, \tag{19a} $$

$$ \underline{\bar{D}}\,\underline{h}_1 - \left[\frac{{}_r\beta}{\pi} \sum_{m=1}^{M} {}_rb_m\, \Phi_m\right] = \left[\frac{\pi^2}{4} \sum_{m=1}^{M} {}_ra_m^2\, m^2\right], \tag{19b} $$

$$ -{}_rc_m / {}_rS_m - {}_rh_2\, {}_rR_m = {}_rh_1\, {}_ra_m + {}_rh_0\, {}_rb_m\,, \tag{20a} $$

$$ \underline{\bar{D}}\,\underline{h}_2 - \left[\frac{{}_r\beta}{\pi} \sum_{m=1}^{M} {}_rc_m\, \Phi_m\right] = \left[\frac{\pi^2}{2} \sum_{m=1}^{M} m^2\, {}_ra_m\, {}_rb_m\right], \tag{20b} $$

$$-{}_r d_m/{}_r S_m - {}_r h_3\,{}_r R_m = {}_r h_2\,{}_r a_m + {}_r h_1\,{}_r b_m + {}_r h_0\,{}_r c_m, \tag{21a}$$

$$\underline{D}\,\underline{h}_3 - \left[\frac{{}_r\beta}{\pi}\sum_{m=1}^{M} {}_r d_m\,\Phi_m\right] = \left[\frac{\pi^2}{4}\sum_{m=1}^{M} m^2\left({}_r b_m^2 + 2\,{}_r a_m\,{}_r c_m\right)\right], \tag{21b}$$

$$-{}_r e_m/{}_r S_m - {}_r h_4\,{}_r R_m = {}_r h_3\,{}_r a_m + {}_r h_2\,{}_r b_m + {}_r h_1\,{}_r c_m + {}_r h_0\,{}_r e_m, \tag{22a}$$

$$\underline{D}\,\underline{h}_4 - \left[\frac{{}_r\beta}{\pi}\sum_{m=1}^{M} {}_r e_m\,\Phi_m\right] = \left[\frac{\pi^2}{2}\sum_{m=1}^{M} m^2\left({}_r a_m\,{}_r d_m + {}_r b_m\,{}_r c_m\right)\right], \tag{22b}$$

where

$$ {}_r S_m = \frac{1}{{}_r\alpha\, m^2\pi^2 + 1} \quad \text{and} \quad {}_r R_m = 2\,\Phi_m \frac{{}_r\beta}{\pi^3 m^2}. \tag{23}$$

The technique of solving the algebraic equations given in (18a, b), (19a, b), etc., is to the start off with the first pair (18a, b) and from these determine ${}_r a_m$ and ${}_r h_0$. Next Eqs. (19a, b) are used to obtain ${}_r b_m$ and ${}_r h_1$. In this way (20a, b), (21a, b), etc. are worked through in sequence to obtain $({}_r c_m, {}_r h_2)$, $({}_r d_m, {}_r h_3)$, etc.

Substituting ${}_r a_m$ from (18a) into (18b) gives

$$\underline{h}_0 = -\underline{D}^{-1}\left[{}_r\delta + \frac{{}_r\beta}{\pi}\sum_{m=1}^{M}\Phi_m\,{}_r M_m\,{}_r S_m\right], \tag{24a}$$

$${}_r a_m = -{}_r S_m\left({}_r R_m\,{}_r h_0 + {}_r M_m\right). \tag{24b}$$

Similarly

$$\underline{h}_1 = \underline{D}^{-1}\left[\sum_{m=1}^{M}\left(\frac{\pi^2}{4} m^2\,{}_r a_m^2 - \frac{{}_r\beta}{\pi}\Phi_m\,{}_r S_m\,{}_r h_0\,{}_r a_m\right)\right], \tag{25a}$$

$${}_r b_m = -{}_r S_m\left({}_r R_m\,{}_r h_1 + {}_r h_0\,{}_r a_m\right), \tag{25b}$$

$$\underline{h}_2 = \underline{D}^{-1}\left[\sum_{m=1}^{M}\left\{\frac{\pi^2 m^2}{2}\,{}_r a_m\,{}_r b_m - \frac{{}_r\beta\,\Phi_m\,{}_r S_m}{\pi}\left({}_r h_1\,{}_r a_m + {}_r h_0\,{}_r b_m\right)\right\}\right], \tag{26a}$$

$${}_r c_m = -{}_r S_m\left({}_r R_m\,{}_r h_2 + {}_r h_1\,{}_r a_m + {}_r h_0\,{}_r b_m\right), \tag{26b}$$

$$\underline{h}_3 = \underline{D}^{-1}\left[\sum_{m=1}^{M}\left\{\frac{\pi^2 m^2}{4}\left({}_r b_m^2 + 2\,{}_r a_m\,{}_r c_m\right) - \frac{{}_r\beta\,\Phi_m\,{}_r S_m}{\pi}\left({}_r h_2\,{}_r a_m + {}_r h_1\,{}_r b_m + {}_r h_0\,{}_r c_m\right)\right\}\right], \tag{27a}$$

$${}_r d_m = -{}_r S_m\left({}_r R_m\,{}_r h_3 + {}_r h_2\,{}_r a_m + {}_r h_1\,{}_r b_m + {}_r h_0\,{}_r c_m\right), \tag{27b}$$

$$\underline{h}_4 = \underline{D}^{-1}\left[\sum_{m=1}^{M}\left\{\frac{\pi^2 m^2}{2}\left({}_r a_m\,{}_r d_m + {}_r b_m\,{}_r c_m\right) - \frac{{}_r\beta\,\Phi_m\,{}_r S_m}{\pi}\left({}_r h_3\,{}_r a_m + {}_r h_2\,{}_r b_m + {}_r h_1\,{}_r c_m + {}_r h_0\,{}_r d_m\right)\right\}\right]. \tag{28a}$$

$${}_r e_m = -{}_r S_m\left({}_r R_m\,{}_r h_4 + {}_r h_3\,{}_r a_m + {}_r h_2\,{}_r b_m + {}_r h_1\,{}_r c_m + {}_r h_0\,{}_r d_m\right), \tag{28b}$$

where

$$\underline{D} = \begin{bmatrix} & & \vdots & & \\ \cdots & -\dfrac{H_w\,{}_{r-1}s'}{{}_r l} & \left\{{}_r\gamma + \dfrac{H_w\left({}_{r-1}s' + {}_r s'\right)}{{}_r l} + \dfrac{{}_r\beta}{\pi}\displaystyle\sum_{m=1}^{M}\Phi_m\,{}_r S_m\,{}_r R_m\right\} & -\dfrac{H_w\,{}_r s'}{{}_r l} & \cdots \\ & & \vdots & & \end{bmatrix}$$

is a tridiagonal matrix whose rth row is of the form shown. By virtue of the tridiagonal form of $\underline{D}$ the solution of the set of R simultaneous equations given in (24 a), (25 a), (26 a), (27 a) and (28 a) can easily be effected [4].

Referring to Eqs. (24 a) to (28 b) it will be seen that they provide simple explicit formula for the calculation of ${}_ra_m$, ${}_rb_m$, ${}_rh_0$, ${}_rb_1$, etc. Once these are known then the equations

$$
\begin{aligned}
{}_rh &= {}_rh_0 + {}_rh_1 + {}_rh_2 + {}_rh_3 + {}_rh_4\,,\\
{}_rv &= \sum_{m=1}^{M} ({}_ra_m + {}_rb_m + {}_rc_m + {}_rd_m + {}_re_m)\sin(m\,\pi\,x)\,,\\
\frac{{}_rM}{H_{w\,r}l} &= -\pi^2\,{}_r\alpha \sum_{m=1}^{M} m^2\,({}_ra_m + {}_rb_m + {}_rc_m + {}_rd_m + {}_re_m)\sin(m\,\pi\,x)\,.
\end{aligned}
$$

can be used to calculate the horizontal tension increment, girder deflection, and girder bending moment respectively.

Althougt the higher order corrections $({}_rh_3, {}_rd_m)$ and $({}_rh_4, {}_re_m)$ have been given, it is worth noting that only in cases with considerable non-linearity are they required.

Analytical investigation of the convergence of the perturbation equation is difficult. In any particular numerical application however, the speed of convergence can be studied by observing how fast the successive correcting functions diminish.

Application to a Three Span Bridge

In a previous paper [5] the Author has described a fundamental method of analysing a bridge consisting of a mainspan and two sidespans by treating it as a special type of open panel truss. Having an exact solution of this type available, it is instructive to compare solutions obtained by that method with those of the present paper. For this purpose the symmetrical bridge shown in Fig. 3 is considered.

Bridge data: Left and right side span; these are identical.
Hangers-26, spaced equidistant apart at 60 ft.
$\tan\beta' = -0.196$ for left side span.
$\tan\beta' = 0.196$ for right side span.
$EI = 3\times 10^8$ tons-ft^2.

Main span: Hangers-54, spaced at 60 ft. apart.
The towers are of the same height, hence $\tan\beta' = 0$.
$EI = 3\times 10^8$ tons-ft^2.
All hangers are considered extensible.

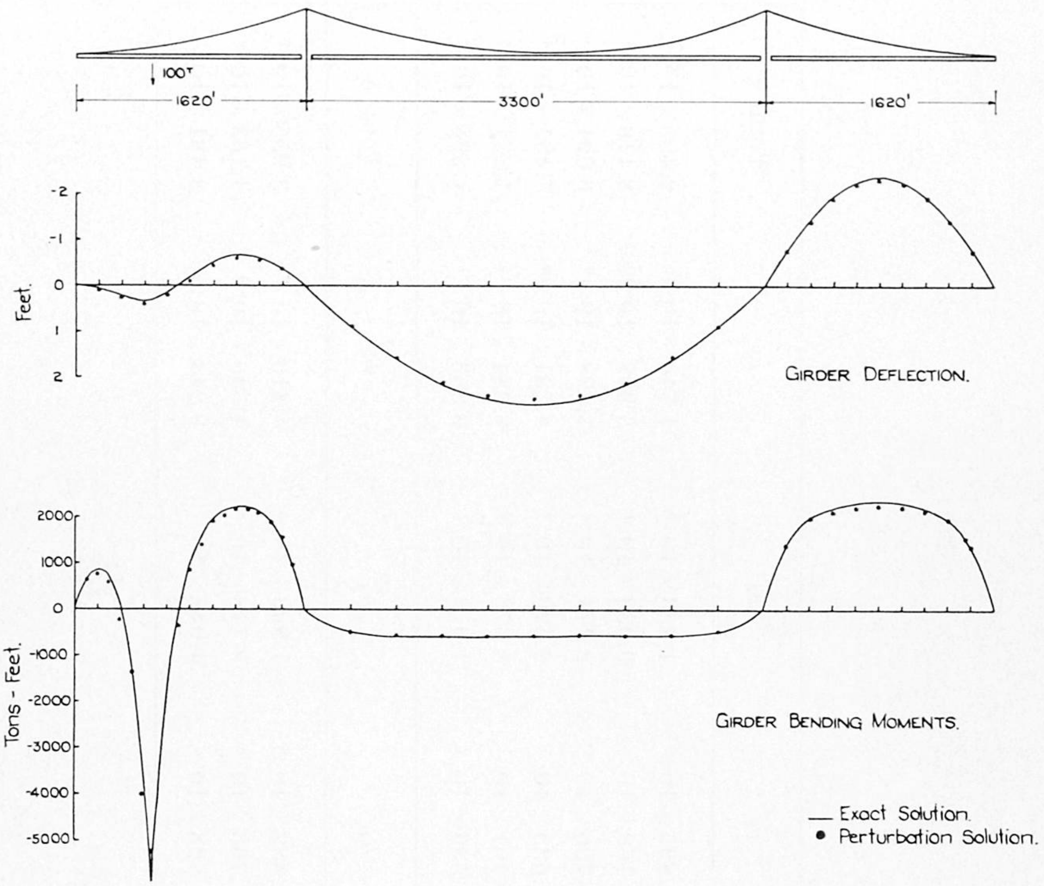

Fig. 3.

Cable data: $H_w = 10000$ ton.
$E_c A_c = 4 \times 10^6$ tons.
The dead weight acting at each hanger is 140 tons.
s' (tower flexibility) $= 0.01$ ft/ton.

It is required to analyse the bridge with a 100 ton load applied to the deck at hanger number 9 of the left side span (i. e. at the one-third point), and with live loads of 7 tons acting at each hanger of the main span. No live loading is applied to the right side span, and no change in temperature occurs.

The results of analysing this structure by both methods are given in Fig. 3. From these it will be seen that the two methods show good agreement. The perturbation method used 20 modes. The increments in the horizontal components of cable tension were:

Left span, $h = 396$ tons (perturbation solution); 394 tons (exact solution).
Centre span, $h = 416$ tons (perturbation solution); 414 tons (exact solution).
Right span, $h = 348$ tons (perturbation solution); 348 tons (exact solution).

An indication of the convergence of each of the five perturbation equations is apparent from Table 1.

Table 1

Span		v_0	v_1	v_2	v_3	v_4	Total v
Left	$\frac{1}{3}$ point	2.429×10^{-4}	-2.441×10^{-6}	1.895×10^{-8}	1.953×10^{-9}	-1.740×10^{-10}	2.405×10^{-4}
Left	$\frac{1}{2}$ point	-8.675×10^{-5}	5.800×10^{-6}	-2.169×10^{-7}	9.039×10^{-9}	-3.998×10^{-10}	-8.116×10^{-5}
Centre	$\frac{1}{3}$ point	6.767×10^{-4}	-1.828×10^{-5}	6.706×10^{-7}	-2.274×10^{-8}	7.198×10^{-10}	6.591×10^{-4}
Centre	$\frac{1}{2}$ point	7.634×10^{-4}	-2.071×10^{-5}	7.625×10^{-7}	-2.599×10^{-8}	8.291×10^{-10}	7.435×10^{-4}
Right	$\frac{1}{3}$ point	-1.265×10^{-3}	3.508×10^{-5}	-7.401×10^{-7}	8.676×10^{-9}	8.284×10^{-11}	-1.231×10^{-3}
Right	$\frac{1}{2}$ point	-1.437×10^{-3}	4.014×10^{-5}	-8.536×10^{-7}	1.018×10^{-8}	9.063×10^{-11}	-1.398×10^{-3}
Span		h_0	h_1	h_2	h_3	h_4	Total h
Left		3.962×10^{-2}	-6.839×10^{-5}	7.888×10^{-9}	-1.893×10^{-8}	1.410×10^{-9}	3.955×10^{-2}
Centre		4.174×10^{-2}	-1.130×10^{-4}	1.796×10^{-6}	-9.418×10^{-8}	4.540×10^{-9}	4.163×10^{-2}
Right		3.470×10^{-2}	1.211×10^{-4}	-5.928×10^{-6}	1.048×10^{-7}	9.244×10^{-10}	3.481×10^{-2}

Due to space limitations values for the $\frac{1}{3}$ and $\frac{1}{2}$ points only have been given. Other points in the girder do of course show a similar convergence pattern.

Conclusions

This paper has shown how the perturbation method can be used to solve the classical nonlinear equations of a suspension bridge. It provides an alternative to the more usual method of successive approximations which is used to compute the horizontal component of cable tension. As might be expected, when it is used to solve problems with considerably nonlinearity, four or five perturbation equations may be required to obtain a reasonably accurate solution. This makes it more laborious than the conventional methods, however this disadvantage must be set against the advantage of having an explicit analytical solution. Finally it can be said that in solving multispan bridge systems where the towers are of sufficient stiffness to prevent the complete equalization of the horizontal component of cable tension, then the methods of this paper have an advantage over previous methods.

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Summary

This paper describes an analytical method of solving the classical equations of a suspension bridge. It is based on the perturbation method, and generates a closed form solution in terms of a convergent power series. It provides an alternative method of solution to the more usual method of successive approximations. An application to the solution of multispan interconnected suspension bridge systems is given.

Résumé

La présente communication décrit une méthode analytique de résolution des équations classiques des ponts suspendus. Elle est basée sur la méthode des perturbations et fournit une solution sous forme d'intégrale en fonction d'une série de puissances convergente. Elle peut remplacer le procédé plus courant des approximations successives. On présente son application au calcul d'un pont suspendu composé de travées multiples.

Zusammenfassung

In dieser Studie ist ein analytisches Verfahren zur Lösung der klassischen Gleichungen einer Hängebrücke beschrieben. Dieses Verfahren ergibt eine geschlossene Lösung in Form einer konvergenten Potenzreihe. Damit wird eine andere Lösung für die üblicheren fortgesetzten Näherungsverfahren angegeben. Ein Anwendungsbeispiel für die Lösung von Hängebrückensystemen mehrfacher Spannweite wird angegeben.