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Autor(en): **Iyengar, Sundara Raja K.T.**

Objekttyp: **Article**

Zeitschrift: **IABSE publications = Mémoires AIPC = IVBH Abhandlungen**

Band (Jahr): **25 (1965)**

PDF erstellt am: **25.09.2024**

Persistenter Link: https://doi.org/10.5169/seals-20351

Contribution to the Analysis of Deep Beams

Contribution à l'étude des poutres-parois

Beitrag zur Untersuchung wandartiger Träger

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Introduction

It is well known that the ordinary straight line distribution of bending stresses derived for shallow beams in bending is not applicable for beams whose depths are comparable to their spans. Such types of beams are used in a variety of structures, e. g., bins, hoppers, hipped-plate construction, etc. Hence determination of stress distribution in such beams is an important problem for practical use. Continuous beams of this type with infinite number of spans have been treated by DISCHINGER [1, 2], CHENG and PEI [3] and THON [4]. Deep beams of two spans have been analysed by PARKUS [5] and recently by SCHLEEH [6].

Analysis of single-span deep beams, in comparison to continuous ones, presents more difficulties because of the increased number of boundary conditions to be satisfied. Approximate methods of analysis have been given by BAY [7], LI CHOW, CONWAY and WINTER [8] and UHLMANN [9] using finite-difference technique. GUZMAN and LUISONI [10] have applied Galerkin's variational method whereas ARCHER and KITCHEN [11] have presented a solution using strain energy method. In contrast to the approximate solutions stated above, solutions satisfying the equations of theory of elasticity and the boundary conditions have been given by the author [12] based on a method due to PICKETT [13]. Details of such solutions are discussed in the books by GIRKMANN [14] and TEODORESCU [15]. THEIMAR [16] and BAY [17] have collected information on deep beams continuous as well as single-span types. HENDRY and SAAD [18] using photoelastic method have reported results for a single-span deep beam with a central concentrated load. An interesting study on the distribution of gravitational stresses in a single-span deep beam has been done by Hendry and Saad [19] using photoelastic experiments.

In general, deep beams can be classified into three types based on their H/L ratios:

$$\frac{H}{L} \leqq \frac{1}{2}, \tag{I}$$

$$\frac{1}{2} < \frac{H}{L} < 2, \tag{II}$$

$$\frac{H}{L} \geqq 2. \tag{III}$$

Beams whose depths are less than half their breadths can be analysed by the elementary strength of materials theory. Beams in the other two categories are to be analysed using the elasticity theory. In beams under the second group, all the boundary conditions (i. e., on all the four edges of a rectangular beam) are to be satisfied. These are treated in references [7] to [17] mentioned above.

There is a class of beams whose heights are large when compared to their lengths, in which case, it is reasonable to assume that the effects of the top and bottom boundaries are not felt beyond the mid-depths of the beams and therefore, the top and bottom halves can be treated separately. Beams having $H/L \geqq 2$ fall into this category. In Fig. 1, beyond the hatched portion, the stresses are practically zero and hence the top boundary has practically no effect on the stress distribution.

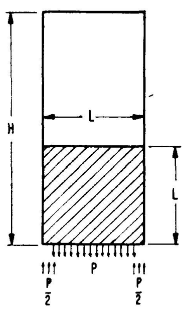

Fig. 1.

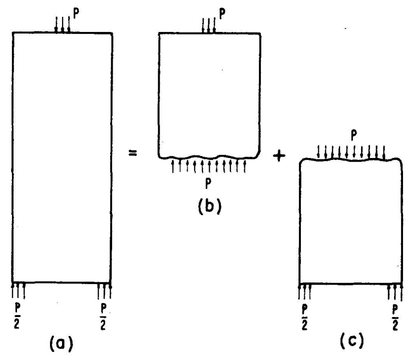

Fig. 2.

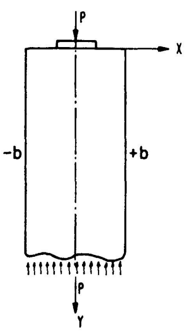

Fig. 3.

Such beams for which the height is equal to or more than twice the length are considered in detail in this paper. In such beams, the two halves can be treated separately as semi-infinite strips in which case the number of boundary conditions to be satisfied are less. The solution can be applied for all cases when $H/L \geqq 2$ whereas for beams under type (II), separate solutions have to be obtained for each H/L ratio. The two halves of the beam shown in Fig. 2a can be considered as semi-infinite strips with loadings on the narrow edge. This idealisation is shown in Figs. 2b and 2c. A semi-infinite strip with normal loading on its narrow edge has been analysed by the author [20] for the case when the load is symmetrical to the central vertical axis (of Fig. 2b) and by PICKETT and author [21] when the loading is eccentric. These solutions were applied to determine the anchorage zone stresses in post-tensioned prestressed concrete beams. Recently the author has given general solutions for arbitrary loadings on the semi-infinite strip [22, 23]. They can be easily taken over to this problem of deep beams. Since details regarding the development of basic solutions are found in the above-mentioned references, only the solutions relevant to this problem are written here. Loads on the beam, which are symmetrical to the central vertical axis are considered here.

The Basic Problem

We consider the semi-infinite strip shown in Fig. 3, occupying the region $y > 0$ and $-b < x < b$ and loaded on the top edge $y = 0$. The loading is a symmetrical function of x. Under the assumption of a two-dimensional character of the problem, it is required to determine the stress components σ_x, σ_y and τ_{xy} in the strip. Such problems are usually formulated in terms of an Airy's stress function ϕ, which satisfies the equation

$$\frac{\partial^4 \phi}{\partial x^4} + 2\frac{\partial^4 \phi}{\partial x^2 \partial y^2} + \frac{\partial^4 \phi}{\partial y^4} = 0. \tag{1}$$

The stresses are given by

$$\sigma_x = \frac{\partial^2 \phi}{\partial y^2}, \qquad \sigma_y = \frac{\partial^2 \phi}{\partial x^2}, \qquad \tau_{xy} = -\frac{\partial^2 \phi}{\partial x \, \partial y}. \tag{2}$$

The boundary conditions are

$$\begin{aligned} &\text{On } y = 0, \quad && \sigma_y = -f(x) \quad \text{and} \quad \tau_{xy} = 0. \\ &\text{On } x = \pm b, \quad && \sigma_x = \tau_{xy} = 0. \end{aligned} \tag{3}$$

At large values of y, $\qquad \sigma_y = -\dfrac{P}{2b}, \qquad \sigma_y = \tau_{xy} = 0,$

where $f(x)$ is a known function of x. The loading is assumed as compressive and P = total load applied over the edge $y = 0$ and is given by

$$P = \int_{-b}^{+b} f(x)\, dx.$$

The given load function $f(x)$ on $y = 0$ can be represented by means of Fourier cosine series as follows:

$$f(x) = \frac{P}{2b} + \sum_{m=1}^{\infty} I_m \operatorname{Cos} \frac{m \pi x}{b}, \tag{4}$$

where
$$I_m = \frac{1}{b}\int_{-b}^{+b} f(x)\,\mathrm{Cos}\frac{m\pi x}{b}\,dx. \tag{5}$$

The stress components for this case are taken from ref. [22] and are written below:

$$\begin{aligned}
\sigma_x &= \sum_{m=1}^{\infty} A_m\left[\left(-1+\frac{m\pi y}{b}\right)e^{-\frac{m\pi y}{b}}\,\mathrm{Cos}\frac{m\pi x}{b} - m(-1)^m H_m\right],\\
\sigma_y &= -\frac{P}{2b} - \sum_{m=1}^{\infty} A_m\left[\left(1+\frac{m\pi y}{b}\right)e^{-\frac{m\pi y}{b}}\,\mathrm{Cos}\frac{m\pi x}{b} - m(-1)^m F_m\right],\\
\tau_{xy} &= -\sum_{m=1}^{\infty} A_m\left[\frac{m\pi y}{b}e^{-\frac{m\pi y}{b}}\,\mathrm{Sin}\frac{m\pi x}{b} - m(-1)^m S_m\right],
\end{aligned} \tag{6}$$

where
$$\begin{aligned}
H_m &= 4b^3\int_0^{\infty}\frac{[\alpha x\,\mathrm{Sinh}\,\alpha x - (1+\alpha b\,\mathrm{Coth}\,\alpha b)\,\mathrm{Cosh}\,\alpha x]}{[(\alpha b)^2+(m\pi)^2]^2\left(\mathrm{Cosh}\,\alpha b + \frac{\alpha b}{\mathrm{Sinh}\,\alpha b}\right)}\alpha^2\,\mathrm{Cos}\,\alpha y\,d\alpha,\\
F_m &= 4b^3\int_0^{\infty}\frac{[\alpha x\,\mathrm{Sinh}\,\alpha x + (1-\alpha b\,\mathrm{Coth}\,\alpha b)\,\mathrm{Cosh}\,\alpha x]}{[(\alpha b)^2+(m\pi)^2]^2\left[\mathrm{Cosh}\,\alpha b + \frac{\alpha b}{\mathrm{Sinh}\,\alpha b}\right]}\alpha^2\,\mathrm{Cos}\,\alpha y\,d\alpha,\\
S_m &= 4b^3\int_0^{\infty}\frac{[\alpha x\,\mathrm{Cosh}\,\alpha x - \alpha b\,\mathrm{Coth}\,\alpha b\,\mathrm{Sinh}\,\alpha x]}{[(\alpha b)^2+(m\pi)^2]^2\left[\mathrm{Cosh}\,\alpha b + \frac{\alpha b}{\mathrm{Sinh}\,\alpha b}\right]}\alpha^2\,\mathrm{Sin}\,\alpha y\,d\alpha.
\end{aligned} \tag{7}$$

It can be verified that the expressions for the stresses given in Eq. (6) satisfy the equations of equilibrium and compatability conditions, since these have been derived from a stress function satisfying the differential Eq. (1). The boundary conditions of Eq. (3) are also satisfied when A_m's are given by the set of simultaneous equations

$$A_m = I_m + 16\pi^2 m^2 \sum_{r=1}^{\infty}(-1)^{r+m}\,r\,A_r\,E(r,m), \tag{8}$$

where
$$E(r,m) = \int_0^{\infty}\frac{x^3\tanh x\,dx}{\left[1+\frac{2x}{\mathrm{Sinh}\,2x}\right](x^2+r^2\pi^2)^2(x^2+m^2\pi^2)^2}. \tag{9}$$

Eq. (8) can be written as follows:

$$A_m(1-Q_{mm}) = I_m + \sum_{r=1,2\ldots} Q_{mr}A_r, \quad (m \neq r) \tag{10}$$
$$m = 1, 2\ldots$$

where
$$Q_{mr} = 16\pi^2 m^2 r(-1)^{m+r} E(r,m).$$

Solving Eq. (10), we can write

$$A_m = \sum_{r=1,2\ldots} L_{mr} I_r, \tag{11}$$

where L_{mr} is the inverse matrix of the coefficients of A_m in Eq. (10). Knowing A_m, the stresses are determined from Eq. (6).

Numerical Results

To apply the solutions derived above in practical cases, numerical calculations have been done with the help of the IBM 650 digital computer at the Technische Hochschule, Hannover. Details of calculations including the numerical method of finding the infinite integrals, etc., are given in ref. [22]. Introducing the following notations

$$\frac{P}{2b} = \text{average stress} = p, \qquad \frac{x}{b} = \xi, \qquad \frac{y}{b} = \eta, \tag{12}$$

we can write the stress components as

$$\begin{aligned}
\sigma_x &= \sum_m A_m [(-1 + m\pi\eta) e^{-m\pi\eta} \operatorname{Cos} m\pi\xi - 4m(-1)^m H'_m], \\
\sigma_y &= -p - \sum_m A_m [(1 + m\pi\eta) e^{-m\pi\eta} \operatorname{Cos} m\pi\xi - 4m(-1^m) F'_m], \\
\tau_{xy} &= -\sum_m A_m [(m\pi\eta) e^{-m\pi\eta} \operatorname{Sin} m\pi\xi - 4m(-1)^m S'_m],
\end{aligned} \tag{13}$$

where
$$\begin{aligned}
H'_m &= \int_0^\infty \frac{[\beta\xi \operatorname{Sinh}\beta\xi - (1 + \beta \operatorname{Coth}\beta) \operatorname{Cosh}\beta\xi]}{(\beta^2 + m^2\pi^2)^2 \left[\operatorname{Cosh}\beta + \frac{\beta}{\operatorname{Sinh}\beta}\right]} \beta^2 \operatorname{Cos}\beta\eta \, d\beta, \\
F'_m &= \int_0^\infty \frac{[\beta\xi \operatorname{Sinh}\beta\xi + (1 - \beta \operatorname{Coth}\beta) \operatorname{Cosh}\beta\xi]}{(\beta^2 + m^2\pi^2)^2 \left[\operatorname{Cosh}\beta + \frac{\beta}{\operatorname{Sinh}\beta}\right]} \beta^2 \operatorname{Cos}\beta\eta \, d\beta, \\
S'_m &= \int_0^\infty \frac{[\beta\xi \operatorname{Cosh}\beta\xi - \beta \operatorname{Coth}\beta \operatorname{Sinh}\beta\xi]}{(\beta^2 + m^2\pi^2)^2 \left[\operatorname{Cosh}\beta + \frac{\beta}{\operatorname{Sinh}\beta}\right]} \beta^2 \operatorname{Sin}\beta\eta \, d\beta.
\end{aligned} \tag{14}$$

From Eq. (10), $A_m = \sum_r L_{mr} I_r$. Substituting this in Eq. (13), the expression for σ_x can be written as

$$\sigma_x(\xi, \eta) = \sum_m \{\sum_r L_{mr} I_r [(-1 + m\pi\eta) e^{-m\pi\eta} \operatorname{Cos} m\pi\xi - 4m(-1)^m H'_m]\}. \tag{15}$$

This and the other stresses can be put into the following form,

$$\begin{aligned}
\sigma_x(\xi, \eta) &= \sum_m X_m I_m, \\
\sigma_y(\xi, \eta) &= -p + \sum_m Y_m I_m, \\
\tau_{xy}(\xi, \eta) &= \sum_m Z_m I_m.
\end{aligned} \tag{16}$$

In practical calculations, it would be sufficient if the stresses are calculated at points selected in some systematic manner. If the coefficients in Eq. (16), i.e., X_m, Y_m, Z_m are calculated for selected values of ξ and η for various values of m, the calculations of the stress components at these points becomes a simple affair. These coefficients calculated for twelve terms for the following values of ξ and η are given in ref. [22].

$$\xi = 0,\ \tfrac{1}{4},\ \tfrac{1}{2},\ \tfrac{3}{4} \text{ and } 1,$$

$$\eta = 0,\ \tfrac{1}{6},\ \tfrac{1}{4},\ \tfrac{1}{3},\ \tfrac{1}{2},\ \tfrac{2}{3},\ \tfrac{3}{4},\ 1,\ 1\tfrac{1}{2} \text{ and } 2.$$

For most of the usual loadings, taking 12 terms in the series, the convergence of the results has been good and in cases of slow convergence near the narrow edge, methods of improving the convergence can be successfully used. Hence with the help of these tables, one can get fairly accurate values for the stresses. Having known these values at selected points, they can be obtained at other locations by suitable interpolation. Knowing the stress components at these points, calculation of the magnitudes and directions of principal stresses is straightforward.

Five types of loading have been analysed. These loadings are the same as chosen by Li Chow, Conway and Winter [8] who have used the finite-difference method. These are indicated in Fig. 4. In this paper, all these loadings are analysed for $H/L = 2$ by superposing the solutions for a semi-infinite strip. Stresses are calculated using the tables as mentioned previously.

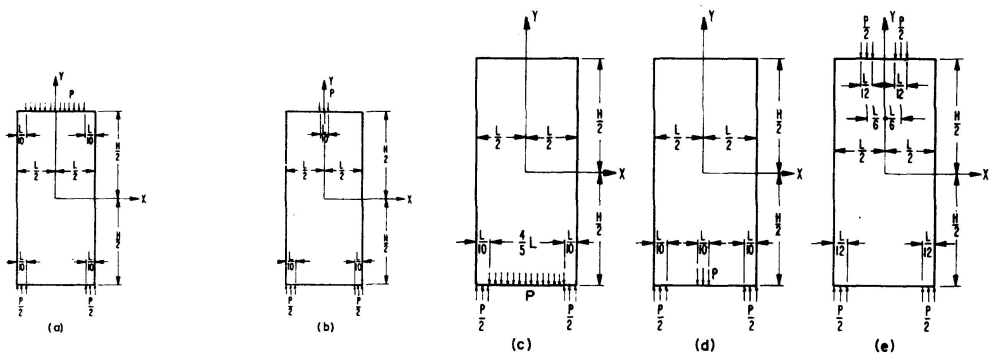

Fig. 4.

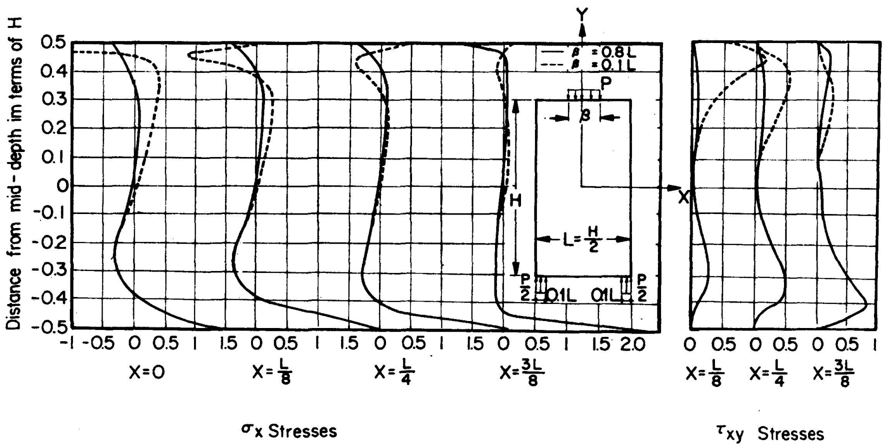

Fig. 5. Stresses in terms of $p = P/L$.

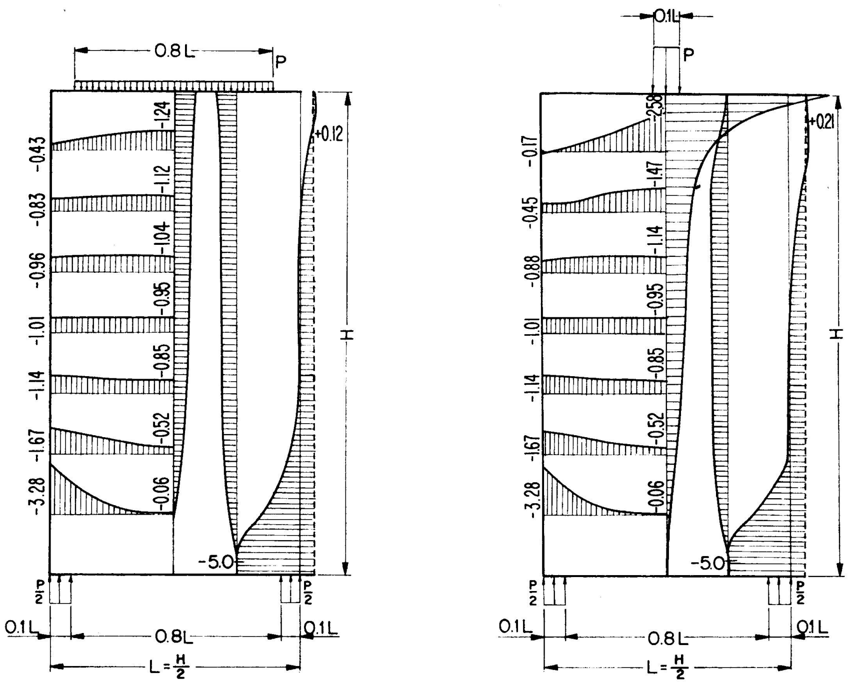

Fig. 6. σ_y stresses in terms of $p = P/L$.

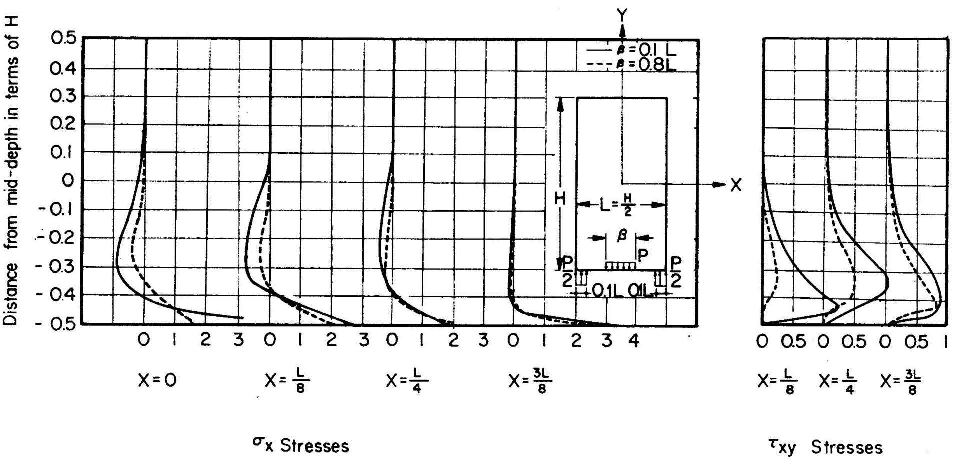

Fig. 7. Stresses in terms of $p = P/L$.

The results are shown in Figs. 5 to 10 where the normal and shear stresses have been plotted at various cross sections.

For $H/L > 2$, a similar procedure can be used.

Gravitational Stresses

The analysis of a deep beam in the foregoing was done by solving Eq. (1), on the assumption that body forces were absent. When a deep beam forms part of a building structure and particularly when it is a reinforced concrete deep beam, the self weight of the beam forms a major proportion of the loading

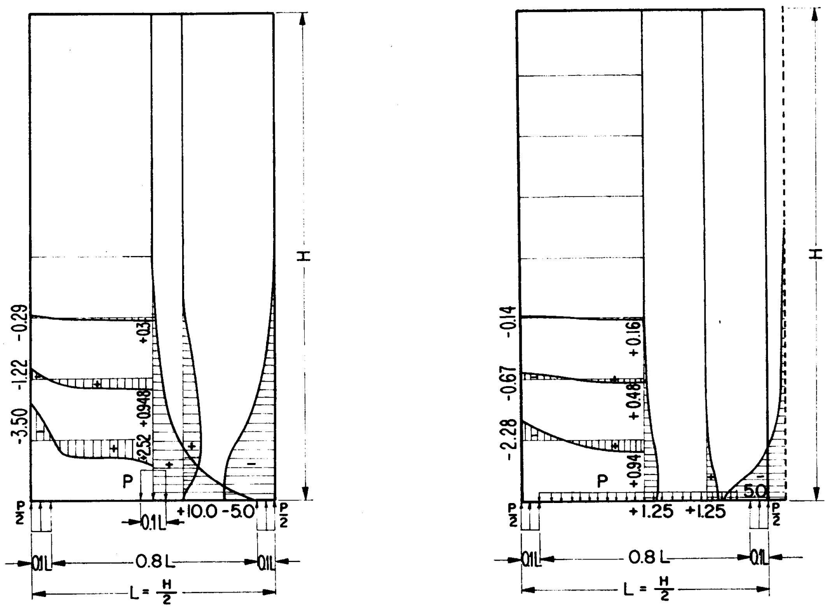

Fig. 8. σ_y stresses in terms of $p = P/L$.

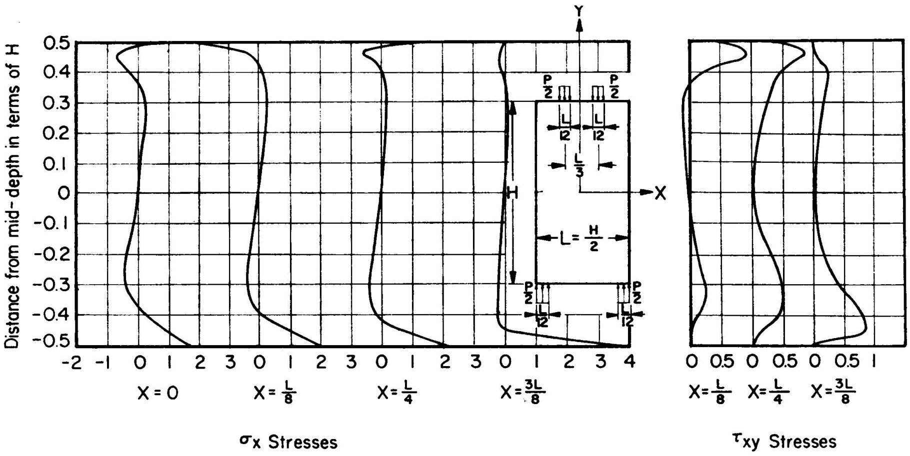

Fig. 9. Stresses in terms of $p = P/L$.

which cannot be neglected and it is necessary to be able to estimate these gravitational stresses. Only very few workers have considered this problem.

BAY [17] in his book has described a solution to this problem using finite-difference method. Recently a systematic series of photoelastic studies were done by HENDRY and SAAD [19] using a large centrifuge in conjunction with frozen stress technique. When gravity is the only body force, the problem can be easily converted into a problem with boundary loadings without body force. Such a solution is given here for a deep beam whose ratio $H/L \geqq 2$, using the analysis developed for the problem without body force.

Consider a deep beam of dimensions H and L supported as shown in Fig. 11. Each support has a width αL. The solution to this problem can be taken as the sum of two parts:

$$\begin{aligned} \sigma_x &= \sigma'_x + \sigma''_x, \\ \sigma_y &= \sigma'_y + \sigma''_y, \\ \tau_{xy} &= \tau'_{xy} + \tau''_{xy}, \end{aligned} \tag{17}$$

we take

$$\begin{aligned} \sigma''_x &= 0, \\ \sigma''_y &= -\rho g\left(\frac{H}{2} - y\right), \\ \tau''_{xy} &= 0, \end{aligned} \tag{18}$$

where ρg is the density of the material. The stress components in Eq. (18) satisfy the equations of equilibrium and compatability. This system gives rise to a uniform compressive stress $\sigma''_y = -\rho g H$ on the bottom edge i. e. $y = -\frac{1}{2} H$. To remove this and to take into account the supports, we consider the beam shown in Fig. 12a with no body forces.

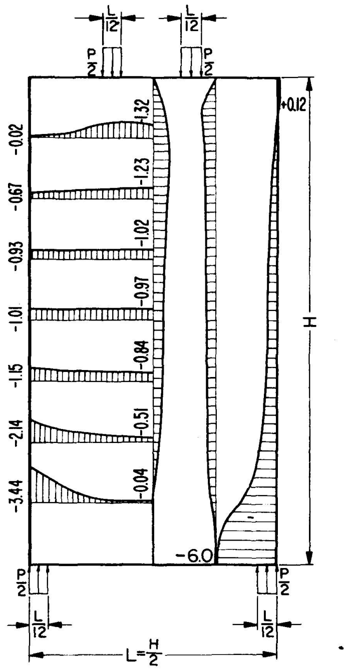

Fig. 10. σ_y stresses in terms of $p = P/L$.

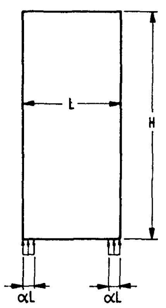

Fig. 11.

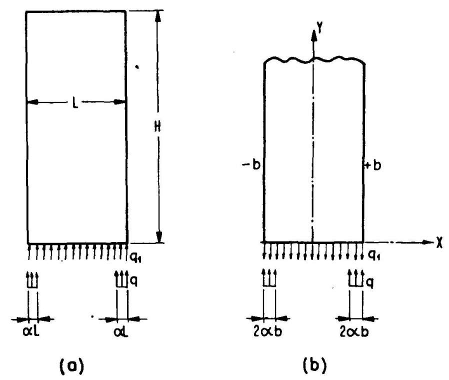

Fig. 12.

Here

$$\begin{aligned} q_1 &= \rho g H, \\ q &= -\frac{\rho g H}{2\alpha}. \end{aligned} \tag{19}$$

When $H/L \geqq 2$, this beam can be treated as a semi-infinite strip with boundary loadings as shown in Fig. 12b for which solutions are already given in Eqs. (6) and (16). Using these and the coefficients X_m, Y_m and Z_m, all the stress components are determined. The case of a deep beam with $H/L = 1.98$ and $\alpha = 0.112$ which has been analysed by Hendry and Saad [19] has been analysed here. In this case,

$$I_m = -\frac{\rho g H}{\alpha m \pi} \operatorname{Sin} m\pi(1 - 2\alpha).$$

The distribution of normal and shear stresses along various cross sections is shown in Figs. 13 and 14.

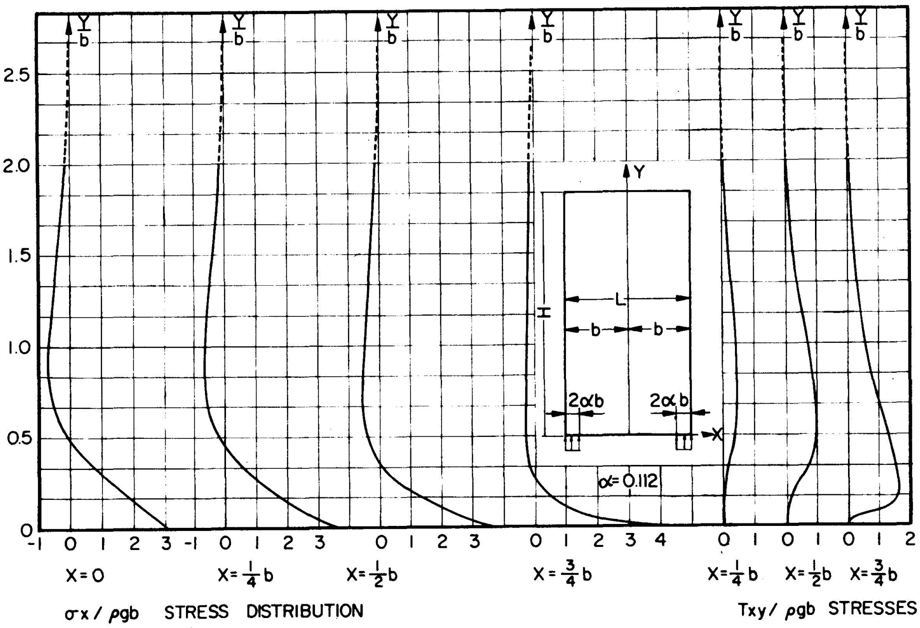

Fig. 13.

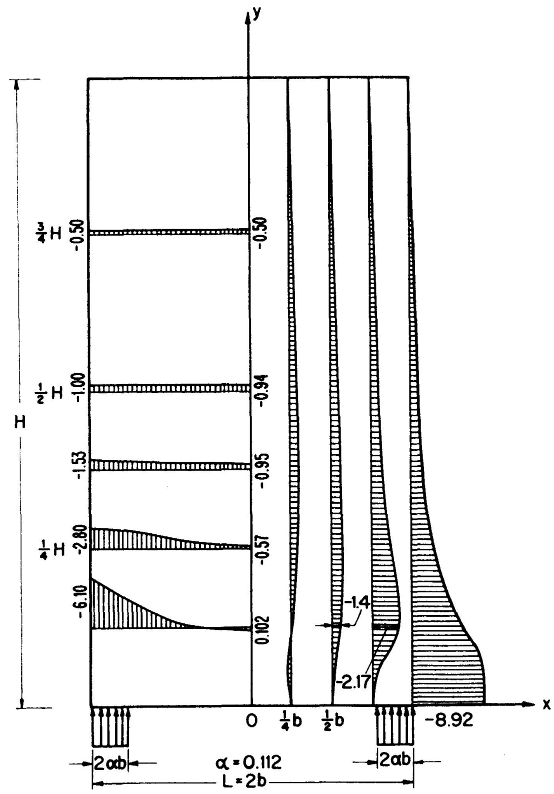

Fig. 14. $\sigma_y/\rho g_L$ stress distribution.

Further Applications of the Solution

The analysis of a class of problem in deep beams was done in this paper using the solution for a semi-infinite strip with normal symmetric loading on the narrow edge, which is the usual type of loading occuring in deep beams. But the general solution of a semi-infinite strip with loading on the narrow edge may be applied in a variety of civil engineering design problems. Transmission of heavy forces into an elastic structure by applying these forces on a small portion of the surface of the structure is a common problem in civil engineering. Stress distribution in the anchorage zones of prestressed concrete beams, stresses in footings of foundations and stresses in bridge piers under the bearing blocks are some of the examples. Most of these problems in reality are three-dimensional in nature. But a first approximation to get a practical solution is to treat these as two-dimensional in character and with this assumption, the solution given in this paper can be applied to many of these problems, when the loading is symmetrical about the central line. Solutions for other types of loadings (normal and tangential) are given in ref. [22]. To the problem of anchorage zone stresses in prestressed concrete beams, several approximate solutions exist, notably by GUYON [24], BLEICH [25] and SIEVERS [26]. In a recent paper [27] the author has applied the solutions given in this paper to this problem and has compared the results of all the existing approximate solutions.

Notations

H = Height of the beam.

L = Length of the beam $= 2b$.

$\sigma_x, \sigma_y, \tau_{xy}$ = Stress components.

$\xi = \frac{x}{b}$, $\eta = \frac{y}{b}$ = Dimensionless variables.

p = Average stress $= \frac{P}{2b}$.

P = Total load.

q, q_1 = Intensities of loading defined in Eq. (19).

ϕ = Airy's stress function.

I_m = Fourier coefficient of the applied load.

A_m = Coefficient in series of Eq. (6).

H_m, H'_m; F_m, F'_m; S_m, S'_m = Infinite integrals.

L_{mr} = Inverse matrix of coefficients of A_m.

$$\left.\begin{array}{l} X_m \\ Y_m \\ Z_m \end{array}\right\} \quad = \text{Coefficients to determine the stresses.}$$

ρg = Density of the material.

References

1. DISCHINGER, F.: «Beitrag zur Theorie der Halbscheibe und des wandartigen Trägers.» Publications, IABSE, Vol. 1, 69—93, 1932.
2. Portland Cement Association: "Design of deep girders." Pamphlet No. ST 66.
3. CHENG, D. H. and M. L. PEI: "Continuous deep beams." Proc. ASCE, Vol. 80, Paper No. 450, 1954.
4. THON, R.: «Beitrag zur Berechnung und Bemessung durchlaufender wandartiger Träger.» Beton und Stahlbetonbau, Bd. 53, 12, 1958.
5. PARKUS, H.: «Der wandartige Träger auf drei Stützen.» Österr. Ing.-Archiv, 2, 185, 1948.
6. SCHLEEH, W.: «Die statisch unbestimmt gestützte durchlaufende Scheibe.» Beton und Stahlbetonbau, Febr. 1965, 25—34.
7. BAY, H.: «Über den Spannungszustand in hohen Trägern und die Bewehrung von Eisenbetontragwänden». Stuttgart, 1931.
8. LI CHOW, H. D. CONWAY and G. WINTER: "Stresses in deep beams." Trans. ASCE., Vol. 118, 686—702, 1953.
9. UHLMANN, H. L. B.: "The theory of girder walls with special reference to reinforced concrete design." The Structural Engineer, Vol. 30, 8, 172—181, 1952.
10. GUZMAN, A. M. and C. J. LUISONI: «Solucion variacional del problema de la viga rectangular simplemente apoyada de gran altura.» Ciencia y Tecnica, Buenos Aires, Vol. 111, 1948.
11. ARCHER, F. E. and E. M. KITCHEN: "Stresses in single-span deep beams." Australian Journal of Applied Science, Vol. 7, 4, 314—326, 1956.
12. SUNDARA RAJA IYENGAR, K. T.: "Analysis of deep beams." Proc. of the 2nd Congress on Theoretical and Applied Mechanics, New Delhi, 87—94, 1956.
13. PICKETT, G.: "Application of the Fourier method to the solution of certain boundary problems in the theory of elasticity." Journal of Appl. Mech., Vol. 11, A 176, 1944.
14. GIRKMANN, K.: «Flächentragwerke.» Springer-Verlag, Vienna, 1959.
15. TEODORESCU, P. P.: «Probleme plane in Theoria Elasticitath.» Vol. I, Rumania, 1961.
16. THEIMER, O. F.: «Hilfstafeln zur Berechnung wandartiger Stahlbetonträger.» Verlag von Wilhelm Ernst und Sohn, 1958.
17. BAY, H.: «Wandartiger Träger und Bogenscheibe.» Verlag Konrad Witter, Stuttgart, 1960.
18. SAAD, S. and A. W. HENDRY: "Stresses in a deep beam with a central concentrated load." Experimental Mechanics, Vol. 1, 6, 192—198, 1961.
19. SAAD, S. and A. W. HENDRY: "Gravitational stresses in deep beams." The Structural Engineer, Vol. 39, 6, 185—194, 1961.
20. SUNDARA RAJA IYENGAR, K. T.: "On a two-dimensional problem in the end-block design of post-tensioned prestressed concrete beams." Proc. of 1st Congress on Theoretical and Applied Mechanics, Kharagpur, 107—112, 1955.
21. PICKETT, G. and K. T. SUNDARA RAJA IYENGAR: "Stress concentration in post-tensioned prestressed concrete beams." Journal of Technology, Vol. 1, 105—112, 1956.

22. SUNDARA RAJA IYENGAR, K. T.: «Der Spannungszustand in einem elastischen Halbstreifen und seine technischen Anwendungen.» Dissertation, Hannover, 1960.
23. SUNDARA RAJA IYENGAR, K. T.: «Über den Spannungszustand in einem elastischen Halbstreifen.» Österreichisches Ing.-Archiv, Bd. 16, 3, 185—199, 1962.
24. GUYON, Y.: «Contraintes dans les pièces prismatiques soumises à des forces appliquées sur leurs bases, au voisinage de ces bases.» Mem. A.I.P.C., Vol. 11, 165—226, 1951.
25. BLEICH, F.: «Der gerade Stab mit Rechteckquerschnitt als ebenes Problem.» Der Bauingenieur, H. 9 und H. 10, 1923.
26. SIEVERS, H.: «Die Berechnung von Auflagerbänken und Auflagerquadern von Brückenpfeilern.» Der Bauingenieur, 27, 209—213, 1952.
27. SUNDARA RAJA IYENGAR, K. T.: "On the two-dimensional theories of anchorage zone stresses in post-tensioned prestressed beams." Journal of American Concrete Institute, Vol. 59, 10, 1443—1466, 1962.

Summary

In a beam whose depth is comparable to its span, the distribution of bending and shear stresses differs appreciably from those given by the ordinary flexural theory. In this paper, general solution has been given for the analysis of a rectangular single-span beam under normal loadings for a beam whose height is equal to or more than twice its length. For such beams, each half is considered as a semi-infinite strip and the solution previously derived by the author has been applied. Gravitational stresses are also determined. Numerical results for several loading cases are given.

Résumé

Lorsque la hauteur d'une poutre est du même ordre de grandeur que sa portée, la répartition des contraintes normales et tangentielles diffère notablement de celle donnée par la théorie classique. Dans la présente contribution, l'auteur présente une solution générale au problème de la poutre sur appuis simples, de section rectangulaire, soumise à des charges normales, et dont la hauteur atteint ou dépasse le double de la portée. Chaque moitié de la poutre est assimilée à un prisme mince indéfini dans une direction, auquel on applique une solution établie auparavant. On étudie également les contraintes dues aux forces massiques et on donne des résultats numériques pour divers cas de charge.

Zusammenfassung

In wandartigen Trägern, deren Höhe und Spannweite von gleicher Größenordnung sind, zeigt die Verteilung der Normal- und Schubspannungen nennenswerte Unterschiede gegenüber derjenigen nach der klassischen Biegelehre. In diesem Beitrag wird eine allgemeine Lösung für die Spannungen in einem einfachen Balken mit rechteckigem Querschnitt angegeben, dessen Höhe gleich groß oder größer als die doppelte Spannweite ist. Für solche Träger wird jede Hälfte als ein Halbstreifen betrachtet und die früher abgeleiteten Beziehungen angewendet. Die Spannungen infolge Eigengewicht sowie numerische Ergebnisse für einige Belastungsfälle werden angegeben.