Analysis of frames subjected to translation

Autor(en): **Wiesinger, F.P. / Lee, S.L.**

Objekttyp: **Article**

Zeitschrift: **IABSE publications = Mémoires AIPC = IVBH Abhandlungen**

Band (Jahr): **23 (1963)**

PDF erstellt am: **23.09.2024**

Persistenter Link: https://doi.org/10.5169/seals-19412

Analysis of Frames Subjected to Translation

Etude des portiques à nœuds mobiles

Die Berechnung verschieblicher Rahmen

F. P. WIESINGER
Ph. D., Associate Professor of Civil Engineering, University of Illinois, Chicago Division, Chicago, Illinois, U.S.A.

S. L. LEE
Ph. D., Professor of Civil Engineering, Northwestern University, Evanston, Illinois, U.S.A.

Introduction

The proposed method applies to two-dimensional rigid frames consisting of straight, vertical columns supported at the base, and of straight beams at each floor level uninterrupted throughout the interior of the frame. There must be a column under each joint. The usual assumptions of frame analysis are applicable.

The joints of such frames will, in general, undergo horizontal translation; the analysis will be based on allowing translation to take place during both the "fixed-end stage" and the "distribution stage". In the first stage, the frame is subjected to the applied loads while rotation of the joints is prevented by the fixed end moments. In the second stage, the unbalanced moments at the joints are relaxed, one at a time, by a distribution procedure in which the joint being balanced is rotated while all the other joints are restrained against rotation.

Historical Note

The idea of allowing frames to undergo translation during the two stages of the Moment Distribution Method is almost as old as the method itself [1]. Some of the more important contributions are listed in the References [2—10]. A similar method dealing with single-bay structures consisting of non-parallel columns and horizontal beams was presented by the authors elsewhere [9].

Fundamental Properties of Members

The Slope-Deflection Equation for a member with variable moment of inertia may be written as

$$M_i - F_i = [\theta_i + c_i \theta_j + (1 + c_i)(d/L)] k_i K, \tag{1}$$

in which i denotes the end under consideration, j the far end, M the end moment, F the fixed-end moment due to intermediate loads, θ the end rotation, d the relative translation between the two ends, L the length of the member and $K = EI/L$ where I is the moment of inertia of the section for which k is determined and E is the modulus of elasticity. The constants c and k are commonly called the carry-over factor and the stiffness factor in the Cross Method; for a prismatic member $c = 0.5$ and $k = 4$. M and θ are positive if clockwise, while d is positive if causing clockwise moment.

The fundamental quantities R_i^i, R_j^i, T_i, and Q defined by Fig. 1 can be readily verified from Eq. (1); it should be noted that $c_i k_i = c_j k_j$. R_i^i is the rotational stiffness in the Cross Method. These quantities are independent of the loading and can be considered properties of the member.

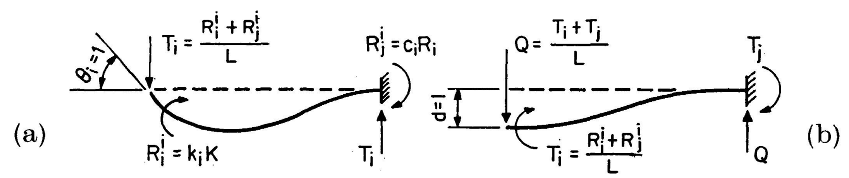

Fig. 1.

Fixed-End Stage

Fig. 2 represents a frame in the fixed-end stage, where U is the translational fixed-end moment associated with unit force applied to the structure and V is the shear in the columns. U can be computed conveniently by means of a systematic procedure presented in the following.

Comparing the columns with Fig. 1b indicates that

$$V/Q = U/T$$

for any particular end. Furthermore,

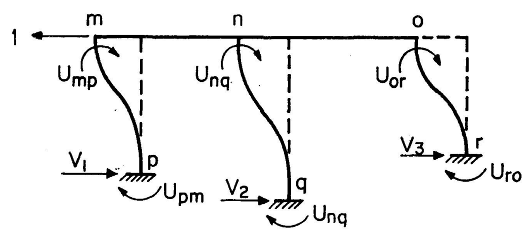

Fig. 2.

$$V/Q = \sum V/\sum Q = 1/\sum Q,$$

where the summation includes all the columns of the story under consideration, since equilibrium requires that $\sum V = 1$. Hence,

$$U_i = T_i/\sum Q. \tag{2}$$

As before, U is independent of the actual loading and can be considered a property of the frame.

Distribution Stage

During the distribution stage, joints will be rotated one-by-one while the structure is allowed to translate but rotation of the other joints is prevented, as shown in Fig. 3a, where joint n is subjected to unit rotation. This figure will be considered as the superposition of Fig. 3b and c. The system shown in Fig. 3b represents the Cross Method, where translation is prevented by a pair of horizontal forces. Comparing the center column with Fig. 1a indicates that these forces are T_{nq}. They are removed together in Fig. 3c. The moments shown can be readily verified by comparing the figure with Fig. 2. Note that this system does not cause moments in the beams.

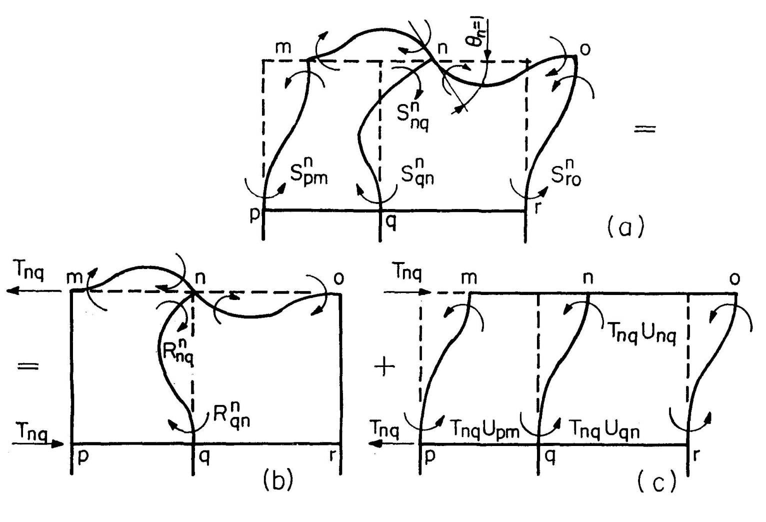

Fig. 3.

The moments shown in Fig. 3a will be designated as the stiffness S. It is not necessary to write explicit expressions giving their value, since the computations will be carried out in tabular form corresponding to the two superimposed cases.

Numerical Example

The frame shown in Fig. 4 will be analyzed as an illustration of the proposed method. All members are prismatic. K is given on row 4 of Table 1. On row 3 column ends of the first and second story are identified with the symbols I and II respectively, while beam ends are designated with B. For prismatic members $T = (R_i^i + R_j^i)/L = (k_i K + c_i k_i K)/L = (4K + 2K)/L = 6K/L$, computed on row 6. $Q = (T_i + T_j)/L$, hence for the first story $\sum Q = 24/20 + 36/15 + 24/10 + 24/20 + 36/15 + 24/10 = 12$ and for the second story $\sum Q = (30 + 30 + 45 + 45)/8 = 18.75$. On row 7, U is computed according to Eq. 2 by dividing T with the appropriate value of $\sum Q$.

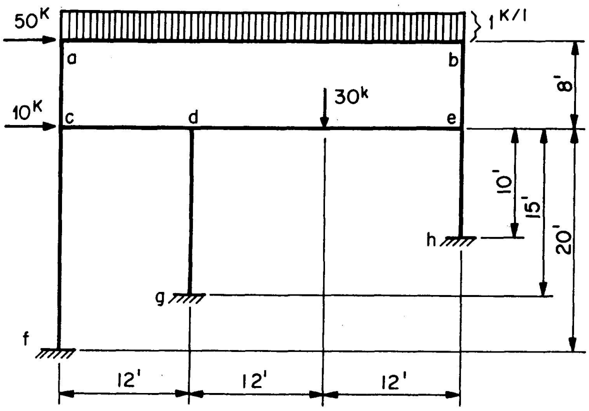

Fig. 4.

The several joints are considered next one-by-one, starting with joint a. On row 8 under ac $R_{ac}^a = k_{ac} K = 4 \times 40 = 160$ is entered, while at the far end of the same member, i.e. at ca $R_j^i = R_{ca}^a = c_{ac} R_{ac} = 0.5 \times 160 = 80$ is shown. R_{ab}^a and R_{ba}^a are similarly computed. This row represents the moments associated with unit rotation of joint a with translation prevented, corresponding to a system similar to that of Fig. 3b. The values shown are the rotational stiffnesses and their products with the carry-over factors of the Cross Method.

On row 9 the moments associated with a system similar to that of Fig. 3c are entered. These are computed as $T_{ac} = 30$ multiplied by the appropriate values of U of the second story. The negative sign follows from Fig. 3c.

The stiffnesses S associated with unit rotation of joint a are obtained on row 10 by summing the two previous rows; these represent a system similar to that of Fig. 3a. Rotating joint a does not involve the columns of the first story as can be readily verified by inspecting Fig. 3. Equilibrium requires that the shear in the second story should be zero. This can be used for checking by summing S/L for all column ends; i.e. $(112 - 72 + 32 - 72)/8 = 0$.

The rest of the table is completed in a similar fashion. Rotating joints c and e will cause translation of both stories. The appropriate computations are shown respectively on rows 15 and 21. As a final check, a vertical summation on S/L is carried out on row 23 for the column ends, for instance $(272 - 72 - 48)/20 = 7.6$ under $c3$. The summation of the values on row 23 should, of course, be zero.

The S values represent a stiffness matrix, the diagonal symmetry of which can also be used for checking.

It should be mentioned that in case of non-prismatic members it is more

Table 1. Properties of Frame Used in Numerical Example (rows 24 through 30 are discussed at the end of the paper)

1		Joint	*a*		*b*		*c*			*d*			*e*			*f*	*g*	*h*
2		End	*ac*	*ab*	*be*	*ba*	*ca*	*cf*	*cd*	*dg*	*dc*	*de*	*eb*	*eh*	*ed*	*fc*	*gd*	*he*
3		Type	*II*	*B*	*II*	*B*	*II*	*I*	*B*	*I*	*B*	*B*	*II*	*I*	*B*	*I*	*I*	*I*
4		K	40	60	60	60	40	80	200	90	200	100	60	40	100	80	90	40
5		L	8	—	8	—	8	20	—	15	—	—	8	10	—	20	15	10
6		$T=6K/L$	30	—	45	—	30	24	—	36	—	—	45	24	—	24	36	24
7		$U=T/\sum Q$	1.6	—	2.4	—	1.6	2.0	—	3.0	—	—	2.4	2.0	—	2.0	3.0	2.0
8		R	160	240		120	80											
9	*a*	$T_{ac}U_{II}$	-48		-72		-48						-72					
10		S	112	240	-72	120	32						-72					
11		R		120	240	240							120					
12	*b*	$T_{be}U_{II}$	-72		-108		-72						-108					
13		S	-72	120	132	240	-72						12					
14		R	80				160	320	800		400					160		
15	*c*	$T_{ca}U_{II};T_{cf}U_{I}$	-48		-72		-48	-48		-72			-72	-48		-48	-72	-48
16		S	32		-72		112	272	800	-72	400		-72	-48		112	-72	-48
17		R							400	360	800	400			200		180	
18	*d*	$T_{dg}U_{I}$						-72		-108				-72		-72	-108	-72
19		S						-72	400	252	800	400		-72	200	-72	72	-72
20		R			120							200	240	160	400			80
21	*e*	$T_{eb}U_{II};T_{eh}U_{I}$	-72		-108		-72	-48		-72			-108	-48		-48	-72	-48
22		S	-72		12		-72	-48		-72		200	132	112	400	-48	-72	32
23		Check	0		0		0	7.6		7.2			0	-0.8		-0.4	-4.8	-8.8
24		*FEM*	-80	-108	-120	+108	-80	-120		-180		-90	-120	-120	+90	-120	-180	-120
25		θ_a (*a*)	+65	+141	-42	+70	+19						-42					
26		θ_b (*b*)	+0	-3	-3	-6	+2						0					
27		θ_c (*c*)	+5		-11		+16	+40	+119	-11	+60		-11	-7		+17	-11	-7
28		θ_d (*d*)						-9	+50	+32	+100	+50		-9	+25	-9	+9	-9
29		θ_e (*e*)	-22		+4		-22	-15		-22		+61	+40	+34	+120	-14	-22	+10
30		M	-30	+30	-172	+172	-65	-104	+169	-181	+160	+21	-133	-102	+235	-126	-204	-126

Table 2. Distribution Table for Numerical Example

1	Joint	*a*		*b*		*c*			*d*			*e*			*f*	*g*	*h*
2	End	*ac*	*ab*	*be*	*ba*	*ca*	*cf*	*cd*	*dg*	*dc*	*de*	*eb*	*eh*	*ed*	*fc*	*gd*	*he*
3	Type	*II*	*B*	*II*	*B*	*II*	*I*	*B*	*I*	*B*	*B*	*II*	*I*	*B*	*I*	*I*	*I*
4	(*a*)	0.318	0.682	-0.204	0.341	0.091						-0.204					
5	(*b*)	-0.193	0.322	0.354	0.646	-0.193						0.032					
6	(*c*)	0.027		-0.061		0.095	0.230	0.675	-0.061	0.338		-0.061	-0.041		0.095	-0.061	-0.041
7	(*d*)						-0.050	0.275	0.173	0.552	0.275		-0.050	0.138	-0.050	0.050	-0.050
8	(*e*)	-0.112		0.019		-0.112	-0.075		-0.112		0.311	0.205	0.174	0.621	-0.075	-0.112	0.050
9	*F*		-108		+108						-90			+90			
10	$50\,U_{II}$, $60\,U_I$	-80		-120		-80	-120		-180			-120	-120		-120	-180	-120
11	+270 (*d*)						-14	+74	+47	+149	+74		-12	+37	-14	+14	-14
12	+188 (*a*)	+60	+128	-38	+64	+17						-38					
13	+164 (*e*)	-18		+ 3		-18	-12		-18		+51	+33	+29	+102	-12	-18	+ 8
14	+153 (*c*)	+ 4		- 9		+15	+35	+103	- 9	+51		- 9	- 6		+14	- 9	- 6
15	- 75 (*d*)						+ 4	-21	-13	-41	-21		+ 4	-10	+ 4	- 4	+ 4
16	+ 21 (*e*)	- 2		+ 0		- 2	- 2		- 2		+ 7	+ 4	+ 4	+13	- 2	- 2	+ 1
17	+ 21 (*c*)	+ 1		+ 1		+ 2	+ 5	+14	- 1	+ 7		- 1	- 1		+ 2	- 1	- 1
18	+ 15 (*a*)	+ 5	+10	- 3	+ 5	+ 1						- 3					
19	- 11 (*d*)						+ 1	- 3	- 2	- 6	- 3		+ 1	- 2	+ 1	- 1	+ 1
20	- 9 (*b*)	+ 2	- 3	- 3	- 6	+ 2						- 0					
21	+ 5 (*e*)	- 1		+ 0		- 1	- 0		- 1		+ 1	+ 1	+ 1	+ 3	- 0	- 1	+ 0
22	+ 2 (*a*)	+ 1	+ 1	- 0	+ 1	+ 0						- 0					
23	- 1 (*b*)	+ 0	- 0	- 0	- 1	+ 0						- 0					
24	*M*	-28	+28	-171	+171	-64	-103	+167	-179	+160	+19	-133	-101	+233	-127	-202	-127

convenient to compute first R_i^i and R_j^i (rows 8, 11, etc.) and then compute T (row 6) from these; for instance, $T_{ac} = (R_{ac}^a + R_{ca}^a)/L = (160+80)/8 = 30$.

The distribution table (Table 2) is assembled next. The distribution factors on rows 4 through 8 are obtained by dividing S by the summation of S at the joint under consideration. Thus, for instance, row 4 is obtained by dividing row 10 of Table 1 by $112 + 240 = 352$. A check (not shown) analogous to that shown on row 23 of Table 1 is carried out and yields $-.001 = 0$.

The fixed-end moments on row 9 are computed in the usual way. On row 10 the translational fixed-end moments are entered. These are computed by multiplying the total shear in the story by the appropriate value of U. The value under dg, for instance, is obtained as $60 \times 3.0 = 180$ ft. kips. The negative sign is attached by inspection.

The distribution starts on row 11. Joint d, where the unbalanced moment $-180-90 = -270$ is the largest, is balanced first by applying a distributed moment of $+270$ to joint d. This causes moments at the several ends which are proportionate to the moments associated with the rotation of joint d, hence 270 is multiplied by the values on row 7. This procedure is continued until the desired accuracy is achieved. The final moments on row 24 are obtained by summing rows 9 through 23.

Discussion

An alternate form of the Slope-Deflection Equation, obtained by subtracting from Eq. 1 an equation which is c_j times Eq. 1 with i and j reversed, is

$$(M_i - F_i) - c_j (M_j - F_j) = [\theta_i + (d/L)](1 - c_i c_j) k_i K. \tag{3}$$

Hence, for a member with the far end hinged R_i^i and R_j^i can be defined as shown in Fig. 5, while the original definition of T and Q are still applicable. By using these values, the necessity of distributing moments back and forth to a hinged column base can be eliminated. It should be noted that T for a prismatic member with the far end hinged is $(R_i^i + R_j^i)/L = (3K+0)/L = 3K/L$, furthermore, T is zero at a hinged end.

Once the analysis is completed, deformations can be readily obtained by means of Eq. 3 by first computing θ for the beam ends (i.e., joints) and then d for the columns. This provides a complete check, since d must be the same for all columns in the story. In the example, d is -6.3 and -4.5 for the first and second stories respectively.

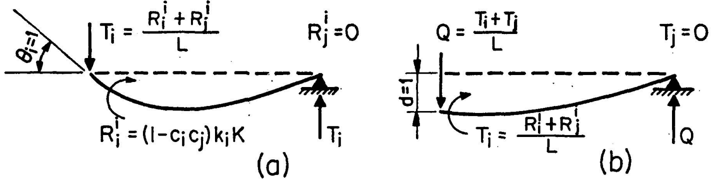

Fig. 5.

The numerical work can be simplified by computing T, Q, and U on the basis of a relative length $\bar{L}$ instead of the real length L. For instance, in the numerical example $\bar{L}$ for both columns in the second story could have been taken as unity, while for the first story 1, 1.5, and 2 could have been used. Designating quantities computed by means of $\bar{L}$ with a bar, $T = \bar{T}\,\bar{L}/L$ and $Q = \bar{Q}\,(\bar{L}/L)^2$. When computing the translational fixed-end moments (row 10 of Table 2) $U = \bar{U}\,L/\bar{L}$ should be used, but Table 1 can be assembled as before, since $T\,U = \bar{T}\,\bar{U}$.

Lateral loads acting on the columns between joints can be treated by means of the simple substitution shown in Fig. 6. First the ordinary fixed-end moments F are computed, with translation prevented by the reactions of the

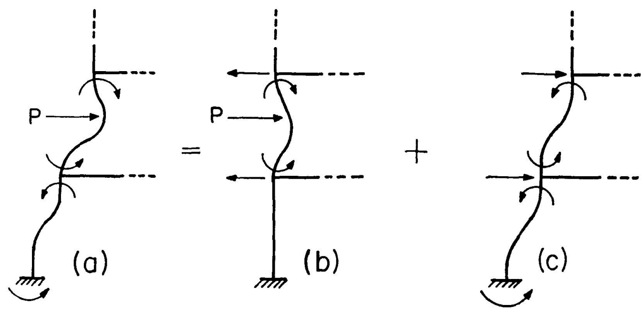

Fig. 6.

fixed-end member (Fig. 6b); in addition to these, the translational fixed-end moments caused by the removal of the reactions must also be included (Fig. 6c).

It can be readily seen by reviewing the foregoing derivations that the proposed method is applicable without modification to frames with sloping beams, if such frames meet the other restrictions presented in the introduction.

If a digital computer program is available, the proposed method may be applied in a different form. At joint a in the example, for instance, the fixed-end moment is $-80-108 = -188$, as indicated on rows 9 and 10 of Table 2. Rotating joint a through an angle θ_a will cause a moment at joint a equal to $(112+240)\,\theta_a = 352\,\theta_a$. This follows from the definition of S, the values of which are obtained from row 10, column a of Table 1. Rotating joint b will cause a moment at joint a equal to $(-72+120)\,\theta_b = 48\,\theta_b$; the coefficients being obtained from row 13 of Table 1. The influence of rotating joints c, d, and e is evaluated in similar fashion. The equilibrium of joint a requires that

$$352\,\theta_a + 48\,\theta_b + 32\,\theta_c + 0 - 72\,\theta_e - 188 = 0.$$

Similar considerations lead to

$$\begin{bmatrix} 352 & 48 & 32 & 0 & -72 \\ 48 & 372 & -72 & 0 & 12 \\ 32 & -72 & 1184 & 328 & -120 \\ 0 & 0 & 328 & 1452 & 128 \\ -72 & 12 & -120 & 128 & 644 \end{bmatrix} \begin{bmatrix} \theta_a \\ \theta_b \\ \theta_c \\ \theta_d \\ \theta_e \end{bmatrix} = \begin{bmatrix} 188 \\ 12 \\ 200 \\ 270 \\ 150 \end{bmatrix},$$

which yields

$$\theta_a = 0.586; \quad \theta_b = -0.024; \quad \theta_c = 0.147; \quad \theta_d = 0.125; \quad \theta_e = 0.302.$$

The final moments are computed on rows 24 through 30 of Table 1, where the values on row 25, for instance, are obtained by multiplying the values on row 10 by 0.586.

Conclusion

The proposed method yields final moments in a single distribution operation, taking joint translation into account automatically. Considerable reduction in labor is possible, since the required number of steps increases approximately linearly with increasing number of unknowns, while in any other method this increase is according to some power of the unknowns. If it is necessary to analyze the frame for several loading conditions, as is usually the case in practice, the same set of distribution factors can be used and only the distribution operation has to be repeated.

Acknowledgement

The material of this paper represents an extension of a portion of a dissertation [8] written by F. P. Wiesinger while on a National Science Foundation Science Faculty Fellowship under the direction of S. L. Lee.

References

1. H. Cross: "Analysis of Continuous Frames by Distributing Fixed-End Moments". Trans. ASCE, Vol. 96, 1932.
2. L. C. Maugh: "The Analysis of Vierendeel Trusses by Successive Approximation". Publ. Int. Assoc. Bridge Struct. Engrg., Vol. 3, 1935.
3. J. G. Perri: "Modified Method of Moment Distribution for Analyzing Compression Members as Parts of Rigid Frames". Ph. D. Thesis, New York University, June 1948.
4. T. C. Kavanaugh: "Analysis and Design of Columns in Frames Subject to Translation". Column Research Council, 1950.
5. P. Csonka: "Berechnung verschieblicher Rahmentragwerke". Bautechnik, Vol. 31, No. 12, Dec. 1954.
6. A. Bolton: "A New Approach to the Elastic Analysis of Two-Dimensional Rigid Frames". Struct., Engrg., Vol. XXX, No. 1, Jan. 1952.
7. C. H. Tsao: "The Analysis of the Vierendeel Truss and of the Wind Stresses in Tall Buildings". Ph. D., Thesis, Illinois Institute of Technology, 1952.
8. F. P. Wiesinger: "A Moment Distribution Method for Analyzing Rigid Frames with Joint Translation". Ph. D. Thesis, Northwestern University, June 1961.
9. S. L. Lee and F. P. Wiesinger: "Moment Distribution Analysis of Vierendeel Bents with Inclined Chords of Unequal Stiffnesses". Proc. Instn. Civil Engineers, Vol. 19, June 1961.
10. F. P. Wiesinger, S. L. Lee and D. L. Guell: "Analysis of Arch Frames". Journal of the Structural Division, Proc. ASCE, Vol. 89, No. ST 5, October 1963.

Summary

A generalization of the Moment Distribution Method which accounts automatically for translation is presented. The frame is first subjected to the applied loads while rotation of the joints is prevented by the fixed-end moments. The unbalanced moments are relaxed next, one at a time, by a distribution procedure. Translation is allowed to take place during both of these stages, hence the final moments are obtained directly, requiring neither intermittent shear corrections nor separate sidesway analyses.

As an alternate procedure, a set of simultaneous equations may be formulated with the joint rotations as the only unknowns. This approach may be desirable if a digital computer is used.

Résumé

Les auteurs présentent une généralisation de la méthode de la répartition des moments, en tenant compte automatiquement des translations des nœuds. On soumet d'abord le portique aux charges appliquées, tout en empêchant la rotation des nœuds par les moments d'encastrement parfait. Par un procédé de répartition successive, on rétablit ensuite l'équilibre des moments. Les nœuds peuvent se déplacer pendant ces deux étapes et les moments finals sont donc obtenus directement.

On peut également établir un système d'équations dont les seules inconnues sont les rotations des nœuds. Ce procédé est peut-être préférable lorsque l'on utilise une calculatrice électronique.

Zusammenfassung

Es wird eine verallgemeinerte Methode der Momentenverteilung beschrieben, bei welcher die Knotenverschiebungen automatisch berücksichtigt werden. Die Belastung wird zuerst auf den Rahmen aufgebracht, während die Knotendrehungen durch die Volleinspannmomente verhindert werden. Die Gleichgewichtsbedingungen in den einzelnen Knoten werden dann sukzessiv durch ein Verteilungsverfahren erfüllt. Verschiebungen können während der beiden Phasen stattfinden, weshalb die Endmomente direkt bestimmt werden.

Als zweites Verfahren kann ein Gleichungssystem aufgestellt werden, bei welchem die Knotendrehungen die einzigen Unbekannten sind. Diese Methode dürfte vorteilhaft sein, wenn eine elektronische Rechenmaschine benützt wird.