The design diagram of composite beams

The Design Diagram of Composite Beams

Diagrammes pour le calcul des poutres composées

Bemessungsdiagramme für Verbundträger

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1. Introduction

In determining the section of a composite beam the following elements such as slab thickness, depth and web thickness of steel girders, extreme fiber stresses of steel and concrete should be considered.

It is not always easy to find the most suitable section within the allowable limits of effective width of slab, flange area, web thickness and extreme fiber stresses.

These design diagrams have the following features. For fixed dimensions of a concrete slab, all possible sections and their resisting moments are shown by the diagram.

2. Determination of Section of the Composite Beam

a) With fiber stresses σ_C, σ_S and depth d given, to find flange sections

Referring to the composite beam section shown by fig. 1, let

$$
\begin{aligned}
F_C &= \text{concrete area},\\
A_S &= \text{steel area},\\
A_C &= \text{top flange area},\\
A_t &= \text{bottom flange area},\\
A_W &= td = \text{web area},\\
A_S &= A_C + A_W + A_t,
\end{aligned}
$$

S_C = distance from neutral axis to $c \cdot g$ of concrete,
S_S = distance from neutral axis to $c \cdot g$ of steel,
n = E_S/E_C ratio of Young's modulus of steel to concrete,
I_N = moment of inertia of composite beam,
I_C = moment of inertia of concrete referred to its $c \cdot g$ axis,
I_S = moment of inertia of steel referred to its $c \cdot g$ axis.

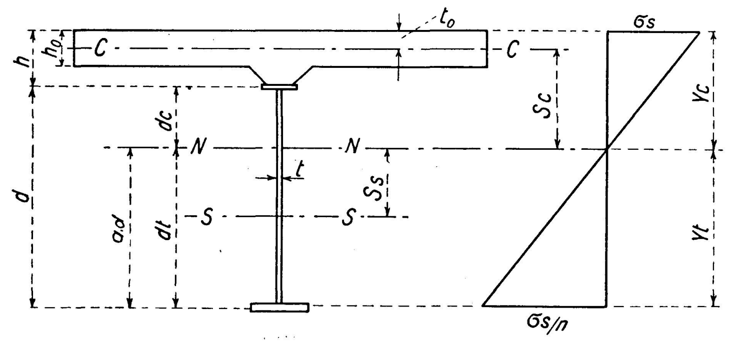

Fig. 1.

Here the axis $C-C$ is c. g. of the section of concrete, the axis $S-S$ is that of steel and the axis $N-N$ is the neutral axis of the composite beam.

h_0 = thickness of slab,
t_0 = distance from $C-C$ axis to top of concrete,
σ_C = extreme fiber stress of concrete,
σ_S = extreme fiber stress of steel.

The extreme fiber stresses σ_C and σ_S are

$$\sigma_C = \frac{M y_C}{I_N}, \qquad \sigma_S = \frac{n M y_t}{I_N} \tag{1}$$

and

$$I_N = I_C + F_C S_C^2 + n \left\{ A_C d_C^2 + A_t d_t^2 + \frac{t}{3} (d_C^3 + d_t^3) \right\}. \tag{2}$$

With regard to the neutral axis we have

$$F_C S_C = n A_S S_S,$$

or

$$n G_{SN} = G_{CN}. \tag{3}$$

G_{SN} and G_{CN} are geometrical moments of steel or concrete with respect to the neutral axis $N-N$.

To find the steel section we have by (1),

$$I_N = I_{CN} + n I_{SN} = \frac{n M y_t}{\sigma_S}. \tag{4}$$

I_{CN} and I_{SN} are moments of inertia of concrete or steel with regard to $N-N$ axis.

Further
$$\left.\begin{aligned} I_{CN} &= I_C + F_C S_C^2, \\ I_{SN} &= A_C d_C^2 + A_t d_t^2 + \frac{t}{3}(d_C^3 + d_t^3), \\ G_{SN} &= -A_C d_C + A_t d_t + \frac{t}{2}(d_C^2 - d_t^2), \\ G_{CN} &= F_C S_C. \end{aligned}\right\} \tag{5}$$

Putting
$$k = \frac{n\,\sigma_C}{\sigma_S + n\,\sigma_C},$$

we have
$$\left.\begin{aligned} y_t = d_t &= (d+h)(1-k) = d\left(1+\frac{h}{d}\right)(1-k) = a\,d, \\ \text{and} \quad d_C &= (1-a)\,d. \end{aligned}\right\} \tag{6}$$

Here
$$a = \left(1+\frac{h}{d}\right)(1-k).$$

If we put these values in eq. (3) and eq. (5) we get

$$A_C(1-a)^2 + A_t a^2 = \frac{M(1+h/d)}{(\sigma_S + n\,\sigma_C)\,d} - \frac{I_{CN}}{n\,d^2} - (\tfrac{1}{3} - a + a^2)\,A_w, \tag{7}$$

$$-A_C(1-a) + A_t a = \frac{F_C S_C}{n\,d} - (a - \tfrac{1}{2})\,A_w. \tag{8}$$

Here
$$A_w = t\,d.$$

From above we have

$$\left.\begin{aligned} A_t &= \frac{M}{\sigma_S d} - \frac{1}{a}\frac{I_{CN}}{n\,d^2} + \left(\frac{1}{a} - 1\right)\frac{F_C S_C}{n\,d} - \left(\frac{1}{2} - \frac{1}{6\,a}\right) A_w, \\ A_C &= \frac{a}{1-a}\frac{M}{\sigma_S d} - \frac{1}{1-a}\frac{I_{CN}}{n\,d^2} - \frac{a}{1-a}\frac{F_C S_C}{n\,d} - \frac{2-3a}{6(1-a)} A_w. \end{aligned}\right\} \tag{9}$$

If we assume F_C, σ_C, σ_S and d, all the terms in the right side of eq. (9) are known. Hence we get flange sections as in the case of plain plate girders. If we neglect the haunched part in the concrete section,

$$S_C = k(d+h) - h_0/2.$$

The web section A_w is so assumed that the areas A_t and A_C do not become negative.

When $a = 1$, the neutral axis lies at the boundary of concrete and steel.

When $a > 1$, tension is expected in concrete.

If $a = 1$, A_C becomes arbitrary, that is, A_C is independent of the strength of the composite beam. This is approximately true in the ordinary composite beam because the top flange is considerably understressed. In other words the top flange section can be chosen according to the size of shear connectors without giving much influence to the strength of the composite beam.

b) With fiber stress σ_S, depth d and top flange area A_C given, to find fiber stress σ_C and bottom flange section A_t

In the case of the determination of the most economical section according to the design conditions, it will be observed that the use of following equations is very efficient for finding the suitable depth.

From eq. (6) we find

$$S_C = h - t_0 + (1-a)\,d. \tag{10}$$

Here a is a variable depending on moment M and the properties of the composite section.

From eq. (3) we have

$$F_C\,S_C = n\,A_S\,S_S.$$

Hence $F_C\{(1-a)\,d + h - t_0\} = \frac{n\,t}{2}\,d^2\,(2\,a-1) - n\,A_C\,(1-a)\,d + n\,A_t\,a\,d.$

Further $A_S = A_C + t\,d + A_t.$

Finally

$$A_S = \frac{1}{a}\left\{\frac{F_C}{n}\left(1+\frac{h-t_0}{d}\right) + \frac{t\,d}{2} + A_C\right\} - \frac{F_C}{n} \tag{11}$$

and

$$A_t = \frac{1}{a}\left\{\frac{F_C}{n}\left(1+\frac{h-t_0}{d}\right) + \frac{t\,d}{2} + A_C\right\} - \left(t\,d + A_C + \frac{F_C}{n}\right). \tag{12}$$

Here A_C is assumed constant, being the least possible amount for the attachment of shear connectors.

As A_C is very lowly stressed, it does not much influence the resisting moment. It can even be taken zero without serious error.

To find the most suitable depth, compare A_S for several depths and draw A_S curve, if necessary, as shown by fig. 2.

The most suitable depth corresponds to minimum A_S.

The stress σ_C is given by eq. (15).

It should be noted that σ_C is not necessary high to obtain minimum A_S.

With regard to σ_S and from eq. (4) we have

$$I_N - \frac{n\,M\,a\,d}{\sigma_S} = 0. \tag{13}$$

Remembering $d_t = a\,d$ and $d_c = (1-a)\,d$ and substituting A_t from eq. (12) for A_t of eq. (2) and using eq. (13) to find a variable a,

$$I_C + F_C\{(h-t_0)+d\}^2 + n\,A_C\,d^2 + \frac{n\,t}{3}\,d^3$$
$$-a\left\{F_C\,(h-t_0)\,d + \frac{n\,M}{\sigma_S}\,d + (F_C + n\,A_C)\,d^2 + \frac{n\,t}{2}\,d^3\right\} = 0\,.$$

Finally we have

$$\left.\begin{aligned} a &= \frac{I_C + F_C\,(h-t_0)^2 + 2\,F_C\,(h-t_0)\,d + (F_C + n\,A_C)\,d^2 + \frac{n\,t}{3}\,d^3}{\left\{F_C\,(h-t_0) + \frac{n\,M}{\sigma_S}\right\} d + (F_C + n\,A_C)\,d^2 + \frac{n\,t}{2}\,d^3}, \\ &\text{or} \\ \frac{1}{a} &= \frac{\left\{F_C\,(h-t_0) + \frac{n\,M}{\sigma_S}\right\} d + (F_C + n\,A_C)\,d^2 + \frac{n\,t}{2}\,d^3}{I_C + F_C\,(h-t_0) + 2\,F_C\,(h-t_0)\,d + (F_C + n\,A_C)\,d^2 + \frac{n\,t}{3}\,d^3}. \end{aligned}\right\} \quad (14)$$

And then we can easily obtain the following relation in fig. 1, i. e.,

$$\sigma_C = \frac{\sigma_S}{n}\left\{\frac{1}{a}\left(1 + \frac{h}{d}\right) - 1\right\}. \quad (15)$$

c) Economical section

Let us now illustrate by a brief example. The conditions, M, h_0, F_C, A_C and σ_S are given as follows,

$$\begin{aligned} M &= 30{,}000{,}000 \text{ kg/cm}, \\ h_0 &= 20 \text{ cm}, \\ F_C &= 12\,h_0{}^2 = 20 \times 240 = 4{,}800 \text{ cm}^2, \end{aligned}$$

(here the width of slab is taken 12 times its thickness),

$$\begin{aligned} \text{haunch} &= 0, \\ A_C &= 0, \\ \sigma_S &= 1{,}300 \text{ kg/cm}^2. \end{aligned}$$

Taking the depth variable ($d = 110$, 120, 130 cm etc.) and using the above eqs. (11), 12), (14) and (15), we can easily obtain the following result.

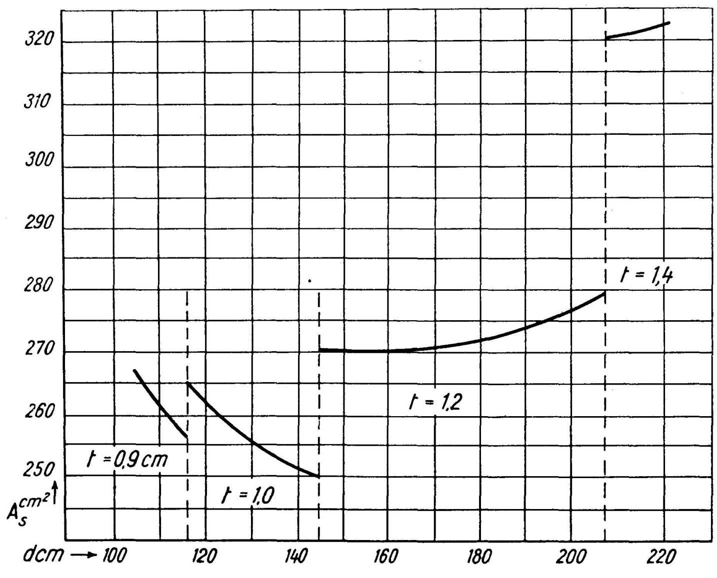

Fig. 2.

Table 1

d cm (t cm)	110 (0.9)	116 (0.9)	116 (1.0)	120 (1.0)	130 (1.0)	140 (1.0)	144 (1.0)	144 (1.2)	150 (1.2)	160 (1.2)	170 (1.2)	207 (1.2)	207 (1.4)
a	0.7735	0.7797	0.7789	0.7826	0.7913	0.7989	0.8017	0.7992	0.8029	0.8084	0.8133	0.8265	0.8086
$A_t^{\mathrm{cm}^2}$	161.95	151.24	147.83	141.10	125.39	116.36	106.08	97.58	89.80	77.63	66.31	30.61	31.38
$A_S^{\mathrm{cm}^2}$	260.95	255.64	263.83	261.10	255.39	251.36	250.08	270.38	269.80	269.96	270.31	278.61	321.38
$\sigma_C \frac{\mathrm{kg}}{\mathrm{cm}^2}$	68	65	66	64	60	56	55	55	53	51	49	41	41

As is shown by table 1 and fig. 2 the economy of section depends considerably on the selection of depth for the given steel stress σ_S and the allowable range of concrete stress σ_C.

The most economical section shows 24% decrease compared to the most unfavorable section.

The design diagram consisting of the sets of curves as represented in fig. 2 gives all possible sections and their resisting moments for the fixed dimensions of a concrete slab. Therefore we can easily select any section corresponding to the design requirements.

N.B. The curve of fig. 2 becomes discontinuous due to the market sizes of plate thickness $t=8$, 9, 10, 12 mm etc., and the restriction $t>\sqrt{d}/12$ according to the Japanese Highway Bridge Specification.

3. Method of Drawing the Diagram

Determining the slab thickness, the depth of haunch, the concrete area and the top flange area of steel, we can easily obtain the curves in the design diagram by use of the above equations.

a) σ_C curve

Assuming σ_C and depth d, the variable a is determined by

$$a = \frac{\sigma_S}{n\,\sigma_C + \sigma_S}\left(1 + \frac{h}{d}\right), \tag{16}$$

and sets of A_S curves can be drawn for assumed σ_C by substituting eq. (16) into eq. (11).

Here σ_S = allowable tensile stress of steel.

b) Moment curve

We can draw groups of A_S curves depending on M by means of eq. (14) and eq. (11), because $$\frac{1}{a} = \frac{\left\{F_C(h-t_0)+\frac{nM}{\sigma_S}\right\} d + (F_C+n A_C) d^2 + \frac{nt}{2} d^3}{I_C+F_C(h-t_0)^2+2F_C(h-t_0)d+(F_C+nA_C)d^2+\frac{nt}{3}d^3},$$

$$\Delta\left(\frac{1}{a}\right) = C \Delta M, \tag{17}$$

in which C is a constant independent of moment.

If we take ΔM = constant, $\Delta(1/a)$ = constant. Hence ΔA_S = constant. So if we draw only one series of curves, the other curves can be set equidistant with regard to the increment of M.

Fig. 3 is shown as the combination of two sets of curves. Here

$$A_t = A_S - t d - A_C, \quad A_C = 13 \text{ cm}^2.$$

c) *Boundary curves*

The boundary curves are given by the restriction of allowable fiber stresses, web thickness and the condition $A_t \geqq 0$. For instance, if the allowable fiber stress of concrete is assumed 70 kg/cm², the curve ($\sigma_C = 70$) denotes, the limit of the section that can be obtained.

The curve shown by $a = 1$ also denotes the limit at which the neutral axis lies in the steel section.

Putting $a = 1$ in eq. (11), we get the curve ($a = 1$). When $a > 1$, tension is induced in concrete.

4. Example

The diagram such as shown by fig. 3 can be used in the case when the following conditions are known, i. e.,

slab thickness 15 cm,
haunch depth 5 cm,
top flange area 13 cm²,
$\sigma_C \leqq 70$ kg/cm²,
$\sigma_S = 1{,}300$ kg/cm².

Let us now illustrate the application to a particular example.

For $M = 11{,}000{,}000$ kg/cm,
$d = 66 \sim 144$ cm
and $A_S = 143 \sim 165$ cm²,

the most economical section is given as follows

$d = 92$ cm,
$t = 8$ mm,
$A_S = 143$ cm²,
$A_C = 13$ cm²,
$A_t = 56.4$ cm²,
$\sigma_C = 55$ kg/cm²,
$\sigma_S = 1{,}300$ kg/cm².

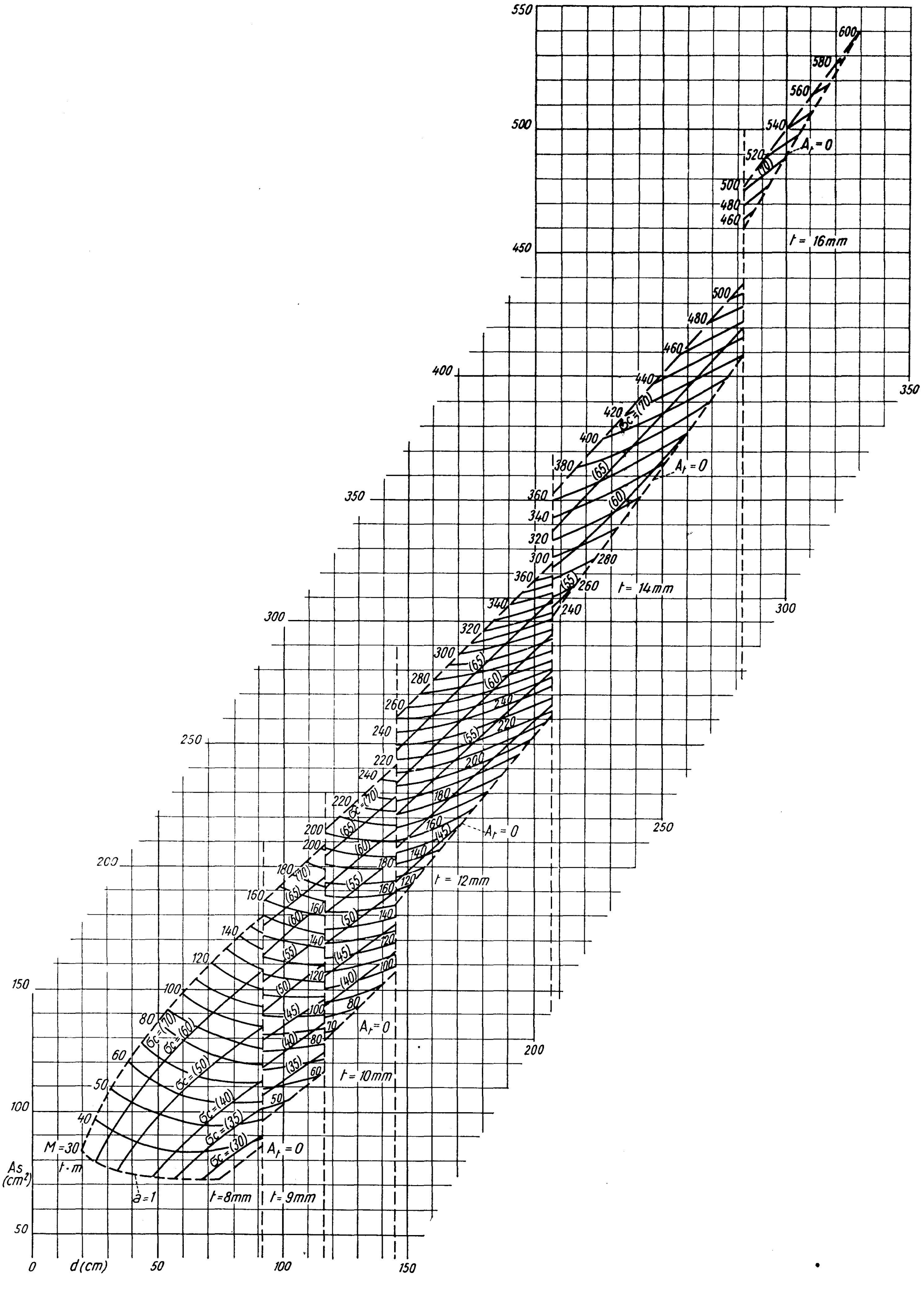

Fig. 3. The design diagram for composite beams.

slab thickness = 15 cm haunch depth = 5 cm

$A_C = 13\ \mathrm{cm}^2$ $\sigma_C \leqq 70\ \mathrm{kg/cm}^2$ $\sigma_S = 1300\ \mathrm{kg/cm}^2$

Figures M 30, 40, 50 denote B. M. in tm. Figures (70), (60) denote concrete stress $\mathrm{kg/cm}^2$.

Summary

Following the necessity of easy and rapid determination of the most suitable and economical section of composite beams according to the design conditions, the authors have composed convenient design diagrams. These diagrams can be used for determining the section of a composite beam in the case of composite action for both dead and live load.

Résumé

En vue de la détermination rapide et aisée des sections les plus favorables et les plus économiques pour les poutres composées, les auteurs ont établi des diagrammes tenant compte des conditions effectives du problème. Ces diagrammes permettent de déterminer la section d'une poutre composée soumise simultanément à la charge du poids mort et à la charge utile.

Zusammenfassung

Zur leichten und raschen Bestimmung des geeignetsten und wirtschaftlichsten Querschnitts von Verbundträgern haben die Verfasser unter Berücksichtigung der Bemessungsbedingungen entsprechende Diagramme zusammengestellt. Mit diesen kann der Querschnitt eines Verbundträgers ermittelt werden, der unter der gemeinsamen Belastung von totem Gewicht und Nutzlast steht.

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