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Sollicitations dynamiques de poutres sous charges mobiles

Ueber die dynamischen Beanspruchungen von Trägern infolge beweglicher Lasten

A study of dynamic influences of moving loads on girders

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An investigation into the dynamic influences of moving loads on bridges is being made at the Institution of Structural Engineering and Bridge Building at the Royal Institute of Technology, Stockhom, Sweden. To begin with, the simplest cases are thoroughly studied, and then the various effects are added one by one in order to determine their separate and cumulative action. The investigation comprises both theoretical and experimental studies.

In the years 1943 to 1946 the main part of the work was done by Rolf Lerfors, C. E., and from the end of 1946 the investigation has been continued by the Author.

In the first place, I studied the case of a concentrated load moving at a constant speed along a girder of uniform section. Methods of solution for this case have been given by several authors, but these methods are either very laborious or else so roughly approximative that the solution is too inaccurate. Therefore, I have tried to simplify and to rationalize the arithmetical computations in a method due to Prof. Inglis (1), which I consider to be one of the most reliable methods available for this purpose. This method was simplified so that it was possible to calculate a great number of cases and thus to form an estimate of the dynamic increment in all practical cases.

It is very important that, if damping is left out of account, the dynamic

(1) Inglis, *A Mathematical Treatise on Vibrations in Railway Bridges*, Cambridge, 1934.

increment can be shown to be completely determined by two dimensionless quantities defined as follows :

$$\alpha = \frac{\text{velocity of load}}{2 \times \text{natural frequency of girder} \times \text{length of girder}}$$

$$\nu = \frac{\text{mass of load}}{\text{mass of girder}}$$

For practical purposes, the approximate limits of these quantities are

$$0 < \alpha < 0{,}15$$

$$0 < \nu < 5$$

For $\nu = 0$, Timoshenko (2) has shown that the dynamic increment in centre deflection is approximately given by

$$\frac{\alpha}{1 - \alpha}.$$

In the more difficult case where $\nu \neq 0$, a great many methods were studied, and, as has been mentioned above, that due to Inglis was found to be most suitable. He expresses the concentrated load per unit length of span by the Fourier series

$$p = \frac{2\,P}{l} \sum_{i=1}^{\infty} \sin i\varphi \cdot \sin \frac{i\pi x}{l}.$$

The notations are given in fig. 1.

In the computations, Inglis uses only the first term of this series.

The deflection is assumed to be

$$y = q(\varphi) \cdot \sin \frac{\pi x}{l}$$

and we obtain, for the determination of $q(\varphi)$, the differential equation

$$\frac{d^2q}{d\varphi^2}\left(\frac{1}{2\nu} + \sin^2\varphi\right) + \frac{dq}{d\varphi}\sin 2\varphi + q\left(\frac{1}{2\nu\alpha^2} - \sin^2\varphi\right) = \frac{y_{00}}{2\nu\alpha^2}\cdot\sin\varphi$$

where y_{00} denotes the static deflection for $\varphi = \frac{\pi}{2}$.

Inglis solves the above equation by means of two series, viz., a series for a forced oscillation and a series for a free oscillation. The coefficients of these series are determined as usual.

The second series is by far the most difficult. Its determination involves much work.

It can be shown that the free oscillation is in very close agreement with the expression

$$q_{\text{free}} = \frac{A}{\sqrt{1 + 2\nu\sin^2\varphi}} \sin\left[\sqrt{\frac{1}{\alpha^2} - \nu} \int_0^{\varphi} \frac{d\varphi}{\sqrt{1 + 2\nu\sin^2\varphi}}\right]$$

which is comparatively simple to calculate, especially after tabulating the expressions

$$\frac{1}{\sqrt{1 + 2\nu\sin^2\varphi}} \quad \text{and} \quad \int_0^{\varphi} \frac{d\varphi}{\sqrt{1 + 2\nu\sin^2\varphi}}.$$

(2) See, for example, TIMOSHENKO, *Vibration Problems in Engineering*, New-York, 1937.

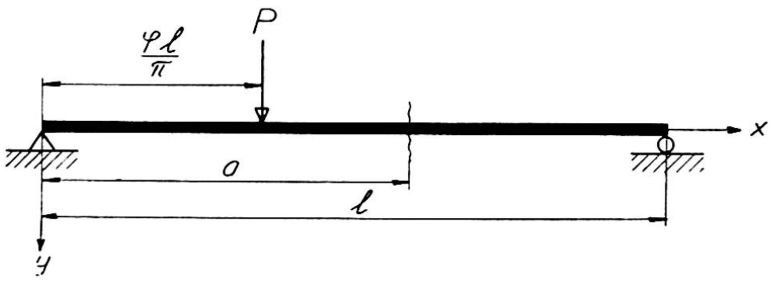

Fig. 1.

The integral can be determined by means of tables of elliptic integrals.

By using this formula, the time required for computing one case is reduced from about fifteen hours to a few hours.

By means of this simplified method, I have calculated and plotted a diagram showing the dynamic increment in deflection as a function of α and ν. It has been convenient to define this dynamic increment by

$$\varepsilon_d = \frac{(q_{\mathrm{dyn}} - q_{\mathrm{stat}})_{\max}}{q_{\mathrm{stat\,max}}}$$

where q_{dyn} has the same significance as $q(\varphi)$ above, and q_{stat} corresponds to $\alpha = 0$ and has the value

$$q_{\mathrm{stat}} = y_{00} \cdot \sin \varphi \, .$$

For small values of α and ν $(\alpha^2\nu < 0.01)$, I got a direct expression for ε, i.e.

$$\varepsilon_d = \alpha^2 \left[\frac{1+2\nu}{1-\alpha^2} + \frac{2\nu}{1-9\alpha^2}\right] + \left[1 + \alpha^2\left(\frac{1+2\nu}{1-\alpha^2} - \frac{6\nu}{1-9\alpha^2}\right)\right] \frac{\alpha}{\sqrt{1-\alpha^2\nu}\,\sqrt{1+2\nu}}$$

For higher values, I have first calculated $q(\varphi)$ and then ε.

These results were used for plotting the diagram shown in fig. 2.

When q is known, the force exerted by the load on the girder and the acceleration forces acting on the girder itself can be calculated on the same assumptions as before. Consequently, the moments and the shearing forces acting on the girder can also be computed. With the notations given in fig. 1, these values are

$$M_a = M_{0a} \cdot \frac{\frac{\varphi l}{\pi}}{a y_{00} \sin \varphi} \left[q + \alpha^2 \frac{d^2 q}{d\varphi^2} \left(1 - \frac{2\, l \sin \frac{\pi a}{l}}{\pi\,(l-a)} \, \frac{\sin \varphi}{\varphi} \right) \right]$$

$$R_a = R_{0a} \frac{\frac{\varphi l}{\pi}}{a y_{00} \sin \varphi} \left[q + \alpha^2 \frac{d^2 q}{d\varphi^2} \left(1 + \frac{2\, l}{a \pi} \cos \frac{\pi a}{l} \sin \varphi \right) \right] .$$

M_a denotes the moment at the distance a from the left end of the girder, and M_{0a} designates the static moment produced when the load is applied at this section.

R_a and R_{0a} denote the corresponding shearing forces.

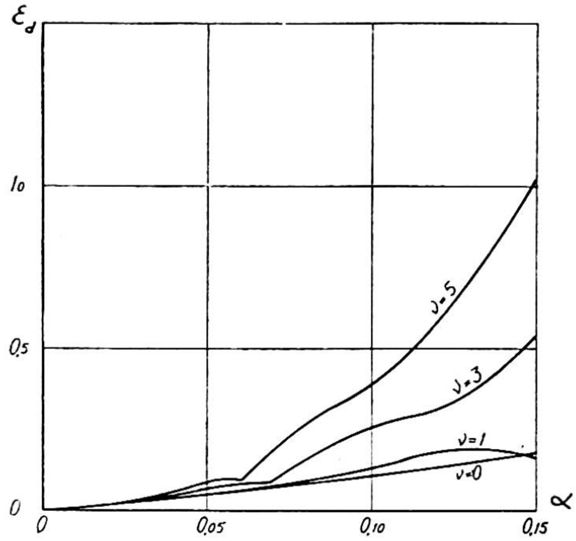

Fig. 2. Dynamic increment in centre deflection as a function of α and ν.

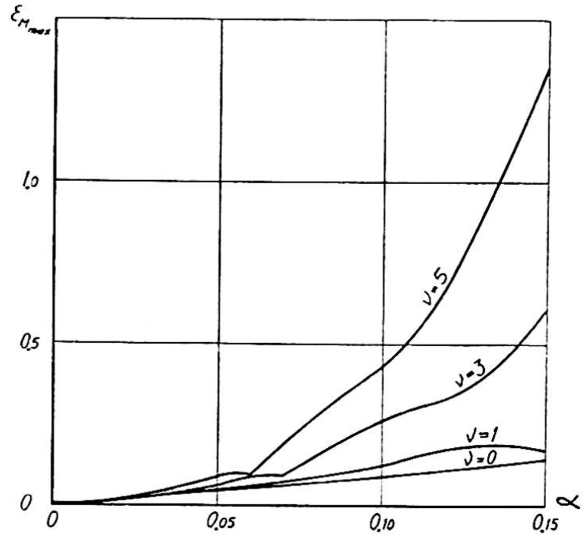

Fig. 3. Maximum dynamic increment in bending moment as a function of α and ν.

In the same way as before, we define

$$\varepsilon_{M\,a} = \frac{\left(M_{a_{dyn}} - M_{a_{stat}}\right)_{max}}{M_{a_{stat_{max}}}}$$

$$\varepsilon_{R\,a} = \frac{\left(R_{a_{dyn}} - R_{a_{stat}}\right)_{max}}{R_{a_{stat_{max}}}} .$$

The maximum value of ε for a given ν and α is of great interest. This maximum value is obtained when $\frac{\varphi l}{\pi} = a$ and is

$$\varepsilon_{M\max} = \left\{ \frac{1}{y_{00} \sin \varphi} \left[q + \alpha^2 \frac{d^2 q}{d\varphi^2} \left(1 - \frac{2 \sin^2 \varphi}{\varphi(\pi - \varphi)}\right)\right]\right\}_{max} - 1$$

$$\varepsilon_{R\max} = \left\{ \frac{1}{y_{00} \sin \varphi} \left[q + \alpha^2 \frac{d^2 q}{d\varphi^2} \left(1 + \frac{\sin 2\varphi}{\varphi}\right)\right]\right\}_{max} - 1 .$$

The values of $\varepsilon_{M\max}$ are shown in fig. 3. $\varepsilon_{R\max}$ differs very little from $\varepsilon_{M\max}$ and is always less than the latter value.

To verify the theoretical results, model tests are being made. The test set-up is shown in fig. 4.

The girder is made of steel and has the approximate dimensions $5 \times 50 \times 1\,100$ mm. The load is a ball of the type used in ball-bearings, which rolls along a track on the girder. The bending stresses are measured by resistance strain gauges at several sections of the girder, and are recorded by an oscillograph which also indicates the time and the instants at which the load passes through definite points.

So far, tests have been made for $\nu = 3.5$ only. Fig. 5 shows some of the oscillograph records obtained in the tests, compared with the corresponding dynamic influence lines computed theoretically. In fig. 6 the test

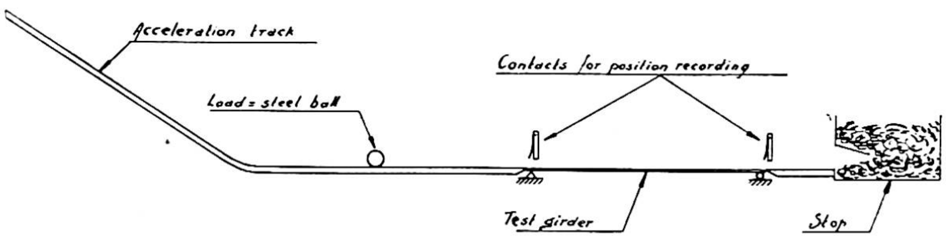

Fig. 4. Set-up used for laboratory tests.

values of ε expressed as function of α for several sections are compared with the computed values.

The agreement between the theoretical and experimental results shows that the theory can be regarded as fairly accurate. Nevertheless, further tests must be made before the first chapter of the investigation can be completed. After that, the questions relating to damping, spring-borne masses, etc., will be studied.

I hope that we shall be able to publish a more detailed account of the results later on. In the meantime, suggestions or questions are welcome.

The Author expresses his gratitude to Professor Georg Wästlund having stimulated the research described in this article.

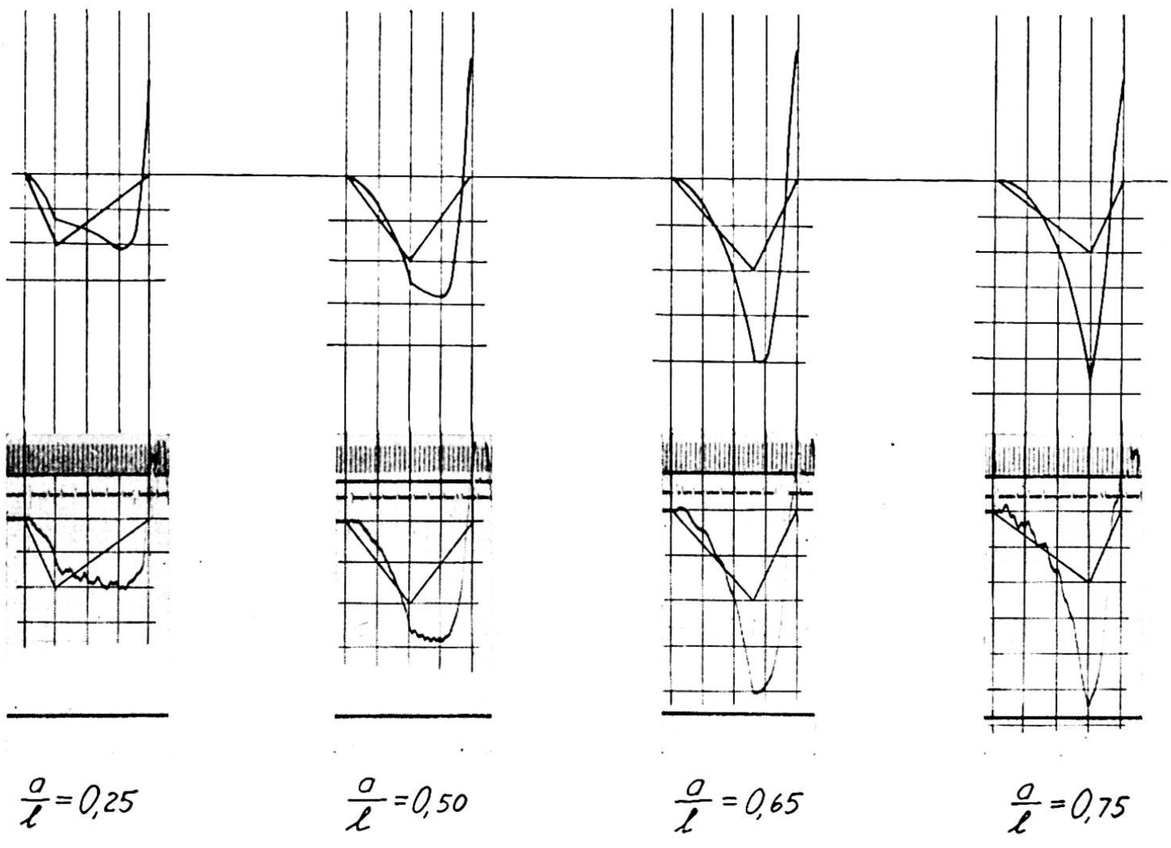

Fig. 5. Theoretically computed dynamic influence lines for bending stresses compared with test values ν = 3.5, α = 0.2.

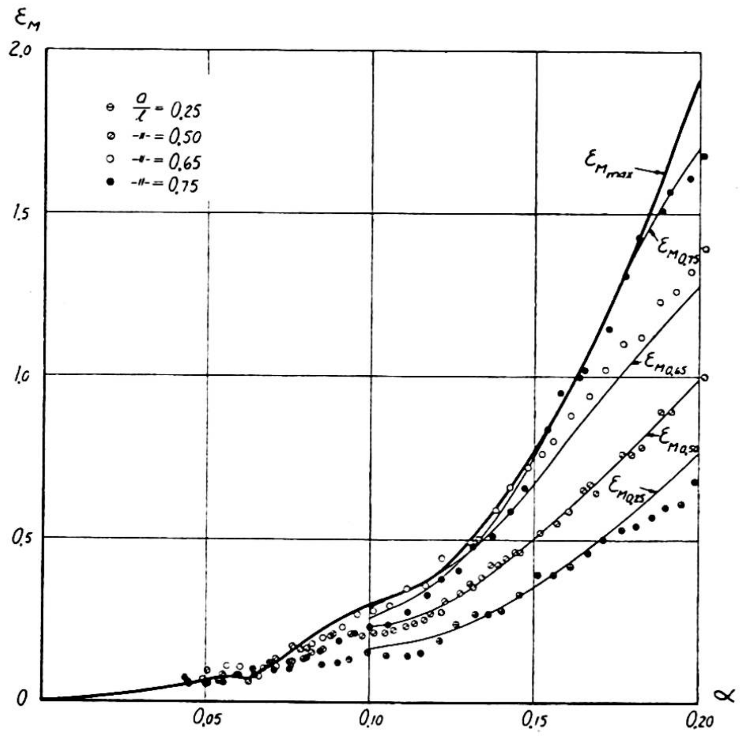

Fig. 6. Theoretical curves showing $\varepsilon_{M\max}$ and ε_{Ma} as functions of α compared with test values of $\varepsilon_M{}^v$. $\nu = 3.5$.

Résumé

L'Institut de Construction des Bâtiments et des Ponts de l'Ecole Royale Polytechnique à Stockholm poursuit actuellement une étude sur les influences dynamiques exercées par les charges mobiles sur les ponts. La première partie de cette étude traite le cas d'une charge concentrée unique qui roule à une vitesse constante le long d'une poutre à section uniforme. En ce cas, on peut démontrer que l'augmentation dynamique ε est complètement déterminée par deux quantités définies comme suit :

$$\alpha = \frac{\text{vitesse de la charge}}{2 \times \text{fréquence naturelle de la poutre} \times \text{longueur de la poutre}}$$

$$\nu = \frac{\text{masse de la charge}}{\text{masse de la poutre}}$$

Au moyen d'une méthode imaginée par M. le Professeur Inglis, qui a été légèrement modifiée et complétée par l'auteur du présent rapport, on a exprimé ε par une fonction de α et ν. La figure 3 montre l'augmentation dynamique maximum des moments fléchissants.

On a constaté une bonne concordance entre les résultats des calculs théoriques et ceux des essais effectués sur modèle, ainsi qu'il ressort des figures 5 et 6.

Zusammenfassung

Im Institut für Hochbau und Brückenbau an der Kgl. Technischen Hochschule in Stockholm wird gegenwärtig eine Untersuchung über die dynamischen Einflüsse der beweglichen Lasten auf Brücken durchgeführt. Deren erster Teil behandelt den Fall einer Einzellast, die sich mit konstanter

Geschwindigkeit längs eines Trägers von gleichbleibendem Querschnitt bewegt. Es zeigt sich, dass der dynamische Zuschlag ε in diesem Falle durch zwei Grössen vollständig bestimmt ist, die wie folgt definiert werden :

$$\alpha = \frac{\text{Geschwindigkeit der Last}}{2 \times \text{Eigenschwingungszahl des Trägers} \times \text{Länge des Trägers}}$$

$$\nu = \frac{\text{Masse der Last}}{\text{Masse des Trägers}}$$

Mit Hilfe eines von Prof. Inglis angegebenen Verfahrens, das vom Verfasser etwas abgeändert und ergänzt wurde, kann ε als Funktion von α und ν ausgedrückt werden. Abb. 3 zeigt die Höchstwerte des dynamischen Zuschlags zu den Biegungsmomenten.

Bei Modellversuchen wurde weitgehende Uebereinstimmung zwischen den theoretisch errechneten Werten und den Versuchsergebnissen festgestellt, siehe Abb. 5 und 6.

Summary

An investigation into the dynamic influences of moving loads on bridges is being made at the Institution of Structural Engineering and Bridge Building, Royal Institute of Technology, Stockholm, Sweden. The first part of this investigation deals with the case where a single concentrated load moves at a constant speed along a girder of uniform section. In this case, it can be shown that the dynamic increment ε is completely determined by two quantities defined as follows :

$$\alpha = \frac{\text{velocity of load}}{2 \times \text{natural frequency of girder} \times \text{length of girder}}$$

$$\nu = \frac{\text{mass of load}}{\text{mass of girder}}$$

By means of a method due to Prof. Inglis, which has been slightly modified and amplified by the Author, ε has been expressed by a function of α and ν. Fig. 3 shows the maximum dynamic increment in bending moment.

The results of model tests were found to be in good agreement with the theoretical results (see figs. 5 and 6).

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