# The application of prestressing at composite steel plate girder bridges co-operation with the overlying reinforced concrete slab

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## L'emploi de la précontrainte aux ponts à tablier solidaire des maîtresses-poutres

## Die Anwendung der Vorspannung auf Brücken mit Verbundträgern

## The application of prestressing at composite steel plate girder bridges co-operating with the overlying reinforced concrete slab

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Previous to the construction of the "Árpád" bridge (1) large-scale experiments were made with different types of light-weight deck constructions. All the tested types were based on an intensive co-operation between supporting steel ribs and covering plate and the overlying concrete filling or coating. First of all it was stated that sufficient co-operation might be secured between a steel plate and the overlying concrete layer by fairly simple means i.e. by steel hooks or stirrups, etc. affording an increase in the stiffness of 30-300 %. Furthermore the transversal distribution of point loads and actual stresses, set up in the single rib elements was considered of primary importance. As a result of the experiments it was concluded upon that the co-operative width of the R. C. slab stressed by the point-load is not constant but it is varying in proportion to the rigidity of the slab's support : e.g. where the steel-ribs (purlins) are directly supported by the cross girders this width is minimum and in the middle of the purlin-span it is maximum. Results were in fair conformity with the calculations when a rhomboidal area was taken into account as co-operating width. The maximum extension in transversal direction does not exceed two or three neighbouring bays according to the experiments and thus the width may be expressed by the equation (fig. 1):

$$S^2 = l\left(1 - \frac{x}{4\,a}\right).$$

(1) Ch. Széchy, *L'Ossature Métallique*, octobre 1947.

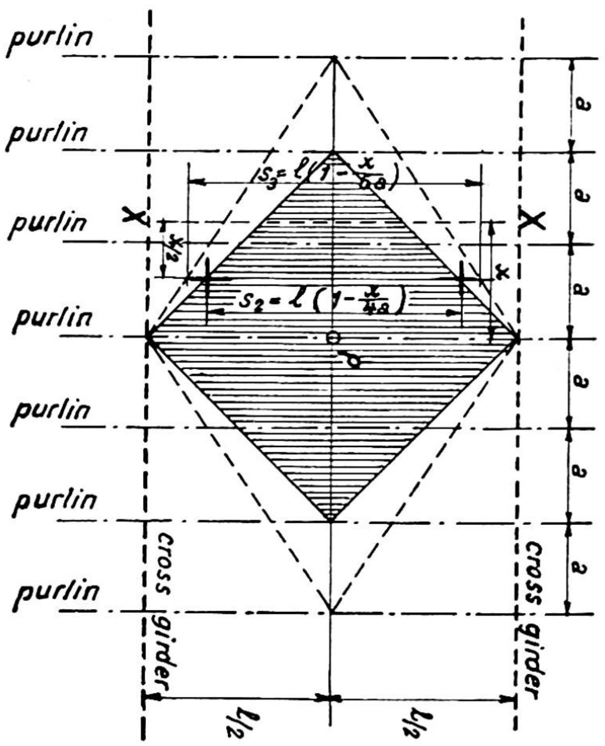



Fig. 1.

In the other direction the peaks of this surface will be situated at the cross-girders.

Very interesting observations were made with executed examples as to transversal load distribution. Figure 2 represents the test loading of a 16.50 m (33′) span grillage-beam bridge. As it is shown transversal load distribution was calculated in different ways.

1. The actual elasticity of load transmitting cross girders (cross beam + R. C. slab) was taken into account by their actual moment of inertia (Leonhardt).

2. The action of the cross beams would incur a transversal flexion of the longitudinal beams but this is greatly counteracted by the great torsional (grillage) resistance of the latter one. This resistance is revealed along the whole length of the longitudinal beams. Thus we are much nearer to the reality when assuming that the tangents at the end sections of the cross beams remain horizontal (fixed).

3. Infinitely rigid cross beams are assumed i.e. the transversal deflection line is linear.

Actual test loadings have proved that the behaviour of such grillage-beam constructions is much nearer to assumption 2 and 3 than to 1. (See fig. 2 with $E_1 = 400$ t/cm².) We have tried to make some formulae regarding the torsional resistance of grillage beams.

The effect of the torsional resistance of the main girders exerted upon the transversal load-distribution was computed by the well-known equation system of deformation set up by the unit forces acting upon the statically determinate ground-systems

$$\Sigma a_{im} X_i + a_{om} = 0$$

where

$a_{im}$ denotes the displacement set up by $X_i=1$ unit force at a place $m$ of the determinate structure, if $i=1, 2, \ldots$ and $m= 1, 2, \ldots$;

$a_{om}$ denotes the displacement set up by the actual external loading at $m$ if $m=1, 2, \ldots$.

The calculation of the grillage-beam system presented below is solved only by approximation for sake of simplicity based on the following assumptions (fig. 3):

1. Only one cross beam of constant rigidity ($J_k$) was taken into account in the middle of the span. The transversal load distributing effect of several cross-beams or that of a co-operating slab was considered partly by the increase of the moment of inertia of the single central cross-beam ($d \cdot J_k$) and partly by the decrease of the length of torsion ($h$). The values of $d$ may be taken according to Leonhardt (1) to 1, 1.6, 2 in the case of two, three and four, or five or more cross-beams; while the value of $h$ is equal-according to the results of test-loadings in the case of a co-operating slab to the distance ($\lambda$) of the cross-beams themselves.

2. The grillage beam was assumed to be built up symmetrically i.e. that the distance ($b$) between the main girders is the same and the moment of inertia (J), the torsional coefficient ($J_c$) of the inner main girders is also identical, while as to the extreme girders their moment of inertia ($J'$) and their torsional coefficient ($J_c'$) are conforming to the ratio:

$$J=\varepsilon \cdot J' \quad \text{and} \quad J_c=\eta \cdot J_c'.$$

The torsional coefficient may be computed in general at the first approximation, by the formula:

$$J_c=\frac{1}{3}\Sigma s^3 \cdot m$$

where $s$ and $m$ are denoting the depth (smaller dimension) and respectively the width of the quadrangular elements of the cross section.

3. The main girders were regarded infinitely rigid against torsion at their supports, but with the assumption that they can freely move in their own plane i.e. that the bending stiffness of the cross beams applied over the support is infinitely big, but their torsional stiffness is practically negligible.

4. The $X_i$ unknown quantities of the equation system are denoting bending moments acting upon the supported or fixed sections of the cross-beams and respectively torsional moments for the main girders. The computation is reduced to the computation of a continuous beam on elastic supports, where the elastic supports (main girders) are rigid against bending and torsion.

The terms used in the determinate structure with one single cross beam in the middle are the following (see fig. 3):

(1) See: A. F. Leonhardt, *Vereinfachte Trägerrostberechnung*, W. Ernst u. Sohn, 1940.

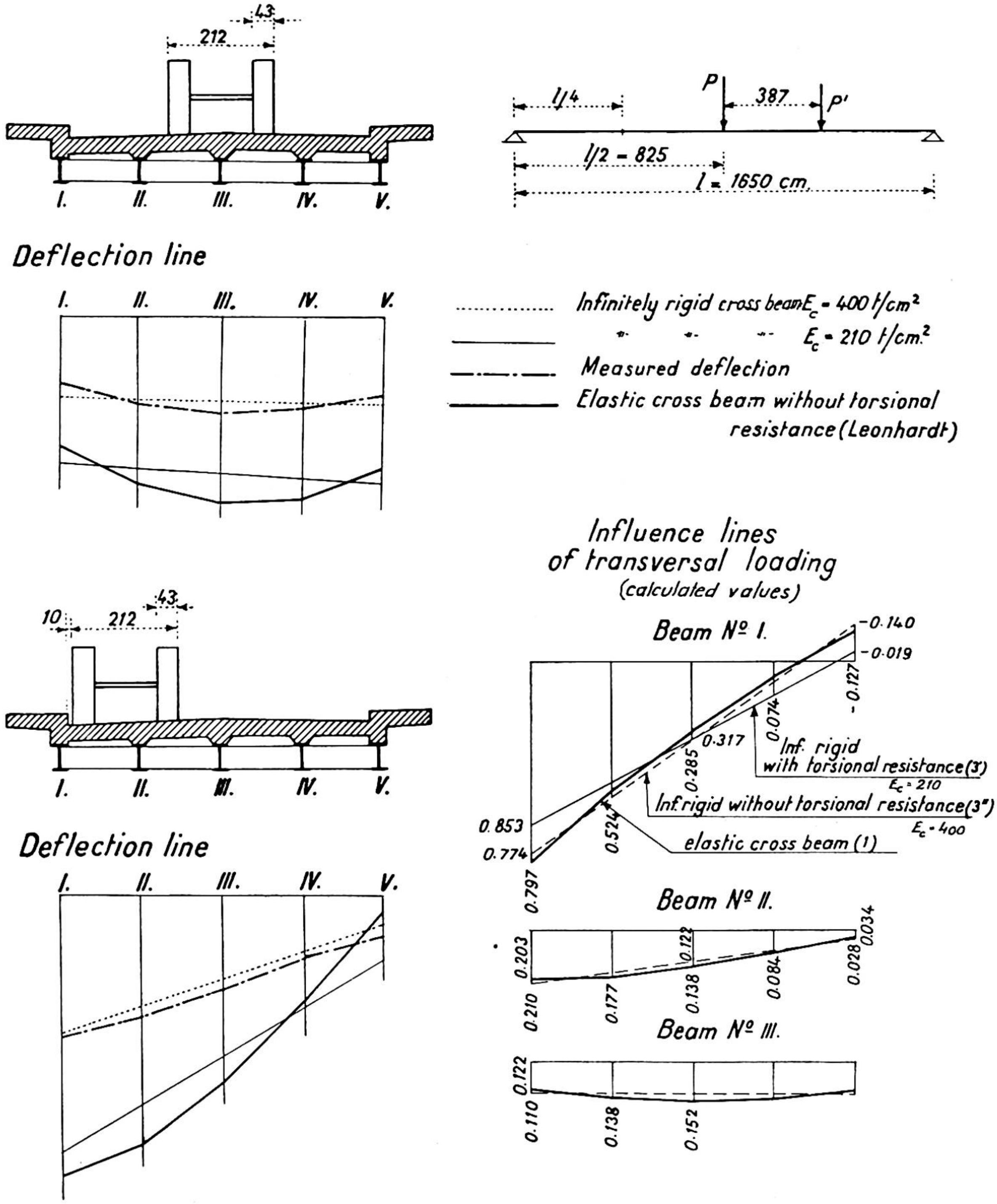



Fig. 2.

$e$ : vertical displacement of the inner main girder in the middle of the main girder due to the effect of the unit force acting at the place of the central cross beam.

In the case of a freely supported beam :

$$e = \frac{l^3}{48\,EJ} \,;$$

$e' = \varepsilon \cdot e$ as above but for the extreme girders ;

$v = \dfrac{b^3}{6\,EJ_k}$ the flexibility of the cross beam ;

| | | | ⓐ + Ⓣ | | ⓑ + Ⓣ | | ⓒ + Ⓣ | |
|---|---|---|---|---|---|---|---|---|
| | | | $\sigma_c$ kg/cm² | $\sigma_s$ kg/cm² | $\sigma_c$ kg/cm² | $\sigma_s$ kg/cm² | $\sigma_c$ kg/cm² | $\sigma_s$ kg/cm² |
| Beam № I. | Section | Ⓐ | 114·1 | 1140 | 57·7 | 2775 | 96·3 | 1728 |
| | –"– | Ⓑ | 66·3 | 1168 | 38·1 | 1978 | 67·1 | 1194 |
| Beam № II. | –"– | Ⓐ | 59·1 | 586 | 25·4 | 2862 | 48·6 | 1276 |
| | –"– | Ⓑ | 44·8 | 1242 | 28·0 | 2380 | 45·4 | 1191 |

See fig. 2.

ⓐ Loading: Camber (prestressing) + dead weight (one central support)

ⓑ " Dead weight: joist + concrete (without support)

ⓒ " " " " " (staging over entire span)

Ⓣ Surfacing + Live load

TABLE I

$z = \dfrac{e}{v}$ stiffness characteristic of grillage system with regard to the inner main girder.

In the case of a freely supported beam :

$$z = \left(\frac{l}{2b}\right)^3 \cdot \frac{J_k}{J} ;$$

$z' = \varepsilon \cdot z$ as above but for the extreme girders ;

$\vartheta = \dfrac{1}{4} \dfrac{h}{G \cdot J_c}$ angle of torsion on the inner girder due to the effect of a unit torsional moment acting in the middle ;

$\vartheta' = \eta \vartheta$ the same for the extreme girder ;

$u = \vartheta b^2$ torsional rigidity of inner girder ;

$u' = \eta b^2$ torsional rigidity of extreme girder ;

$G =$ modulus of torsion approximatively equal for steel and for concrete : 0.385 E ;

$w = \dfrac{u}{v}$ characteristic of torsional rigidity of grillage beam as regards to inner girder ;

$w' = \eta w$ as above but for extreme girder.

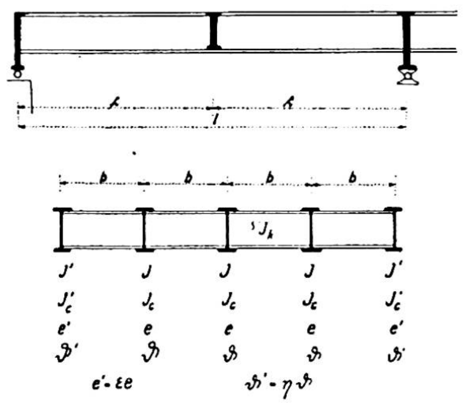
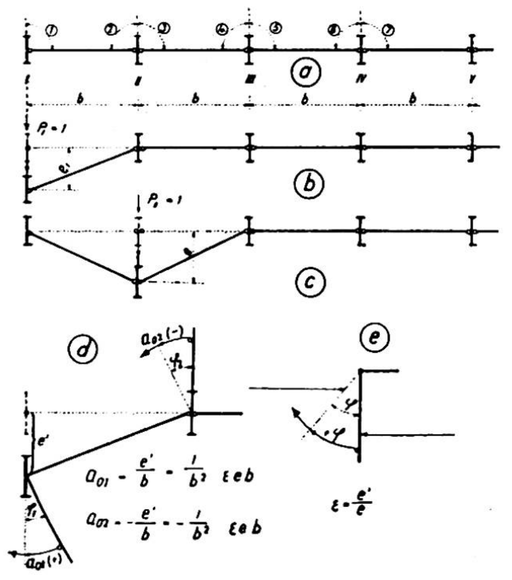
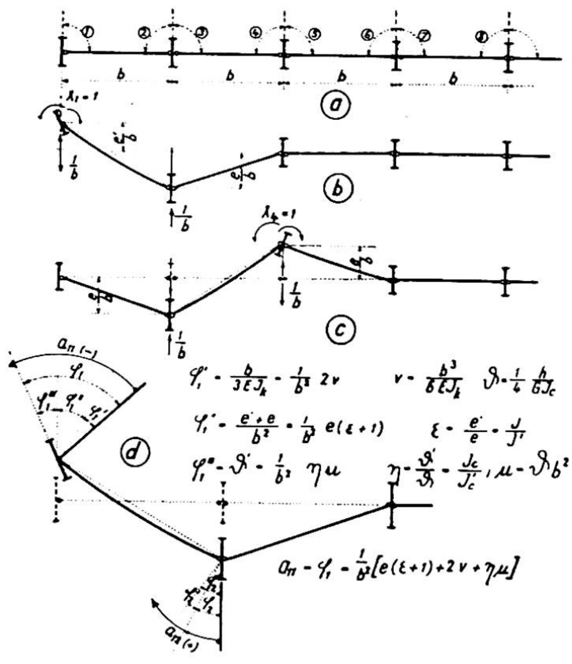


Fig. 3 (left, above).

Fig. 4 (left, below).

Fig. 5 (right, below).





With these designations and with the sign rule indicated on figure 4 and 5 we get the loading quantities and coefficients in the equation system after simplification (omitting the multiplicator $\frac{1}{b^2}$ in all members):

*a*) Quantities for the cases of figure 4 supposing that a unit concentric force is acting upon the main girder at the place of the cross beam:

Due to a force $P_1 = 1^t$:

$a_{01} = \varepsilon \cdot e \cdot b$ (respectively $\varepsilon \cdot z \cdot b$);

$a_{02} = -\varepsilon \cdot c \cdot b$ (respectively $-\varepsilon \cdot z \cdot b$).

Due to a force $P_4 = 1^t$:

$a_{01} = -e \cdot b$ (respectively $-zb$);

$a_{02} = e \cdot b$ (respectively $zb$);

$a_{03} = c \cdot b$ (respectively $zb$);

$a_{04} = -e \cdot b$ (respectively $-zb$).

*b*) Coefficients for the cases on figure 5 if at the places of the redundant joints $X_i = 1$ unit moments are acting upon the determinate structure:

For a moment $X_1 = 1$:

$$
\begin{array}{lll}
a_{11} = -[e(\varepsilon+1)+2v+\eta u] & \text{resp.} & -[z(\varepsilon+1)+2+\eta w]; \\
a_{12} = e(\varepsilon+1)-v & \text{resp.} & z(\varepsilon+1)-1; \\
a_{13} = e & \text{resp.} & z; \\
a_{14} = -e & \text{resp.} & -z.
\end{array}
$$

For a moment $X_4 = 1$:

$$
\begin{array}{lll}
a_{41} = -e & \text{resp.} & -z; \\
a_{42} = e & \text{resp.} & z; \\
a_{43} = 2e-v & \text{resp.} & 2z-1; \\
a_{44} = -(2e+2v+u) & \text{resp.} & -2(z+1)+w; \\
a_{45} = -(e-u) & \text{resp.} & -(z-w); \\
a_{46} = e & \text{resp.} & z.
\end{array}
$$

From the moments $X_i$ got from the solution of the equations we can derive first the reactions and acting forces of the continuous beam on elastic supports and afterwards we can get the ordinates of the load distribution influence lines.

In the case of an infinitely rigid cross beam a simple equation may be derived for the $q$ ordinates of the load distribution influence line. With the designations of figure 6 the load distribution ordinates for the main girder $k$ at the perpendicular of the extreme girders 1 and $n$, if $k = 1, 2, \ldots, n$

$$
q_{1k-nk} = \frac{\varepsilon}{2+(n-2)\varepsilon} \pm \frac{3\,[n-(2k-1)]\,\varepsilon}{(n-2)(n-3)\,\varepsilon+6\,(n-1)+c}
$$

where $n$ : number of main girders,
$c$ : a surplus from torsion

$$
c = \frac{12}{(n-1)}\,\frac{\varepsilon}{\eta}\,\frac{e}{n}\,[2+(n-2)\,\eta]
$$

and if $k = 1$, the value of $\varepsilon = 1$ is to be substituted in the numerator of the equation.

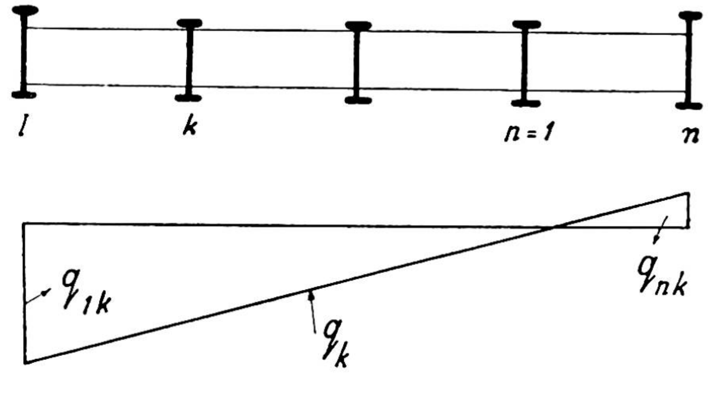



Fig. 6.

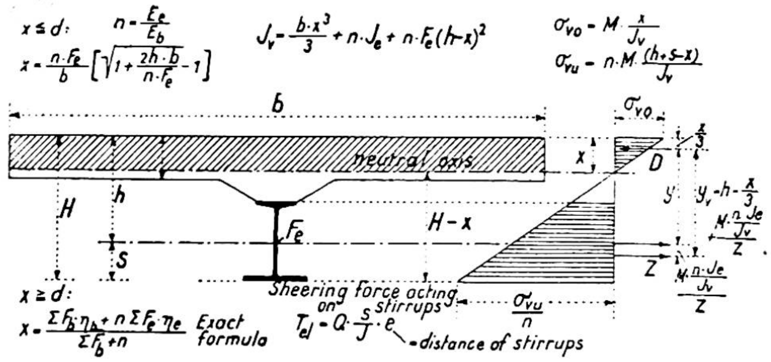



Fig. 7.

If $e' = e$ and $\vartheta' = \vartheta$ i.e. $\varepsilon = \eta = 1$

$$q_{1k-nk} = \frac{1}{n} \pm \frac{3\,[n - (2\,k - 1)]}{n\,(n+1) + c}$$

where $c = 12\ \dfrac{e}{n}\ \dfrac{n}{n-1}$ .

If $c = 0$ and $\varepsilon = 1$ i.e. if we do not take torsion into account the above equation is transformed to the known formula :

$$q_{1k-nk} = \frac{1}{n} \pm \frac{3\,[n - (2\,k - 1)]}{n\,(n+1)} \,.$$

Owing to the great advantages of the composite girders they find more and more widespread applications everywhere. The most frequent type is consisting of standard steel-joists bound by welded stirrups and hooks into the overlying R. C. slab, which is stressed consequently once transversely as a bent plate and longitudinally as the upper (compression) chord of the composite-girder (fig. 7). This double utilisation of the concrete results a considerable saving in steel consumption and a still greater one in scaffolding and in construction time. A further advantage is a possible reutilisation of the recuperated steel deck girders (cross-girders, purlins, etc.) of blown up steel bridges.

The execution of these bridges has inspired us to introduce a simple prestressing which affords further economy. Namely if the steel joists are just placed on the abutments without any staging or any temporary intermediate support they will have to carry alone the weight of the concrete slab thus securing composite action only against live-loading, whereas the steel joist itself has suffered a heavy overstressing from the dead weight of the fresh concrete. In order to avoid this overstressing it was necessary to provide at least for a temporary central support or for a light temporary supporting staging all over the span. This would secure composite action against dead load too. When arrived at that, we went a bit further and studied the possibility of raising slightly the joists in the middle and when tying them down to the abutments even a certain prestressing might be easily obtained introducing compression into the lower flange and tension to the upper flange (fig. 8). In this position the concrete is poured and when hardened, the temporary support removed. It is evident that the concrete slab will overtake a greater part of the stresses and will materially

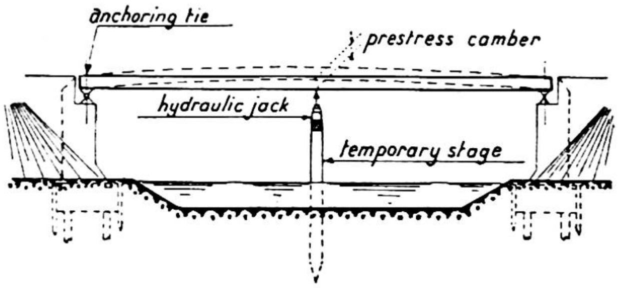



Fig. 8.

relieve the steel joist. For a given example table I is showing comparative numerical data of the stresses in concrete and in steel with and without prestressing and we can see that with prestressing the stress in concrete is 114.1 kg/cm$^2$ (1,630 lb/sq.in) and 1,140 kg/cm$^2$ (16,300 lb/sq.in) in steel, whereas without it, the stress in concrete is decreased to 57.7 kg/cm$^2$ (815 lb/sq.in) but in steel it is increased to 2,775 kg/cm$^2$ (40,000 lb/sq.in). Bearing in mind that the depth of the concrete slab is determined rather by its transversal than by its longitudinal rôle and never can be reduced below a certain practical dimension (15 cm = 6″) prestressing is always advantageous and by very simple means feasible.

**Résumé**

Essais effectués sur poutres composées travaillant solidairement avec le tablier en béton armé.

Résultats des essais de charge de ce type de pont pour lequel les maîtresses-poutres sont soumises à torsion. Formules approchées pour le calcul de ces effets.

Précontrainte simple de ces ponts par un appui central mobile.

Résultats économiques.

**Zusammenfassung**

Ergebnisse der Versuche an zusammengesetzten Trägern mit spezieller Berücksichtigung der Veränderlichkeit des Zusammenwirkens mit der Eisenbetonplatte.

Resultate der Versuchsbelastungen von Verbundträgerbrücken, die die Querverteilung der Lasten infolge der Torsionsteifigkeit der Längsträger zeigen. Näherungsformeln für die Berechnung dieser Effekte.

Einfache Vorspannung der Verbundträgerbrücke durch eine verstellbare Mittelstütze.

Wirtschaftliche Erwägungen.

**Summary**

Results of the experiments made with composite-girders as regards the variation of co-operative width of R. C. slabs.

Test loading results of composite-girder bridges showing the trans-

versal load distributing effect of the torsional resistance of longitudinal beams. Approximative formulae for the computation of these effects.

Simple prestressing of the composite girder beam bridges by means of an adjustable central support.

Economic results.