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Evidence for the Big Bang

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Cosmic Evolution

Hubble's discovery in 1929 of the linear distance-redshift relation of external galaxies was widely accepted as proof for an expanding universe, which must have begun in a singularity (Big Bang). However, the postulation in 1948 of a steady-state universe by H. Bondi, T. Gold, and F. Hoyle suggested that the Big Bang could be avoided: even an expanding universe can in principle be infinitely old and expand forever, provided that mass is continuously created. In this way the steady-state universe fulfills what has been called the 'perfect cosmological principle', viz. it appears the same to any fundamental observer in the universe at *any* time. Only during the last 20 years overwhelming evidence has become available that the universe as a whole is in fact evolving, thus discriminating the steady-state theory and restoring the faith in a Big Bang. This evidence is indeed a vivid illustration of the enormous progress observational cosmology has made during the last decades.

While in the following the emphasis lies on the presently available evidence for an expanding and cooling universe, i.e. for a hot Big Bang, it must be remembered that there is in addition an increasing set of data, which require cosmic evolution with time. An outstanding example for this is the increasing (co-moving) space density of quasars out to redshifts of $z \approx 2$ (Schmidt and Green 1980, 1981) and the cutoff of quasars at higher redshifts (Sandage 1972). This cutoff at $z \approx 4$ is most probably not a selection bias, but it seems now to be a real effect (Osmer 1982). A cutoff is also required by the fact that quasars brighter than $m_B \approx 20^m$ account already for a large fraction of the X-ray background (Setti and Woltjer 1979; Setti 1981).

Another striking result is the narrow colour range of first-ranked (elliptical) cluster galaxies, after they are reduced to a fiducial redshift $z = \text{const}$. Because elliptical galaxies must become redder with time due to the evolution of their individual stars, the near constancy of their colours can only be explained if they were formed during a relatively short epoch (Sandage 1973; cf. Fig. 1). A simple geometrical consideration shows that the number of extragalactic objects brighter than m, N(m), increases for fainter magnitudes as

$$\log N(m) \propto 0.6\, m\,. \qquad (1)$$

(If, for simplicity, radio fluxes are expressed in magnitudes, the relation holds here also for radio sources.) The redshift effect (K-correction) can only flatten the relation in eq. 1. But for several objects steeper relations are observed, which can only mean that these objects were brighter and/or more frequent at larger distances, i.e. at earlier epochs. Such steep count rates are observed for quasars (e.g. Green and Schmidt 1978), X-ray sources (Schwartz 1980), high-flux density radio sources (e.g. Longair 1978), as well as normal galaxies at faint levels (Kron 1980a, 1980b). It should be remarked here that the discussion of the slope of the count rates at large distances is, of course, in addition to the K-correction further complicated by the problem of the exact volume sampled, which depends on the (unknown) space curvature, or – what is essentially the same – on the deceleration parameter q_0 (cf. Fig. 2).

In addition to first-ranked cluster galaxies, which clearly have secular colour changes for variable z, also other galaxies are suspected to have surprisingly strong colour evolution. This is quite well established for radio galaxies, which are very blue at redshifts of $0.4 \lesssim z \lesssim 0.7$ (van der Laan, Katgert and de Ruiter 1980; van der Laan 1981). The suggested increasing blueness at large redshifts

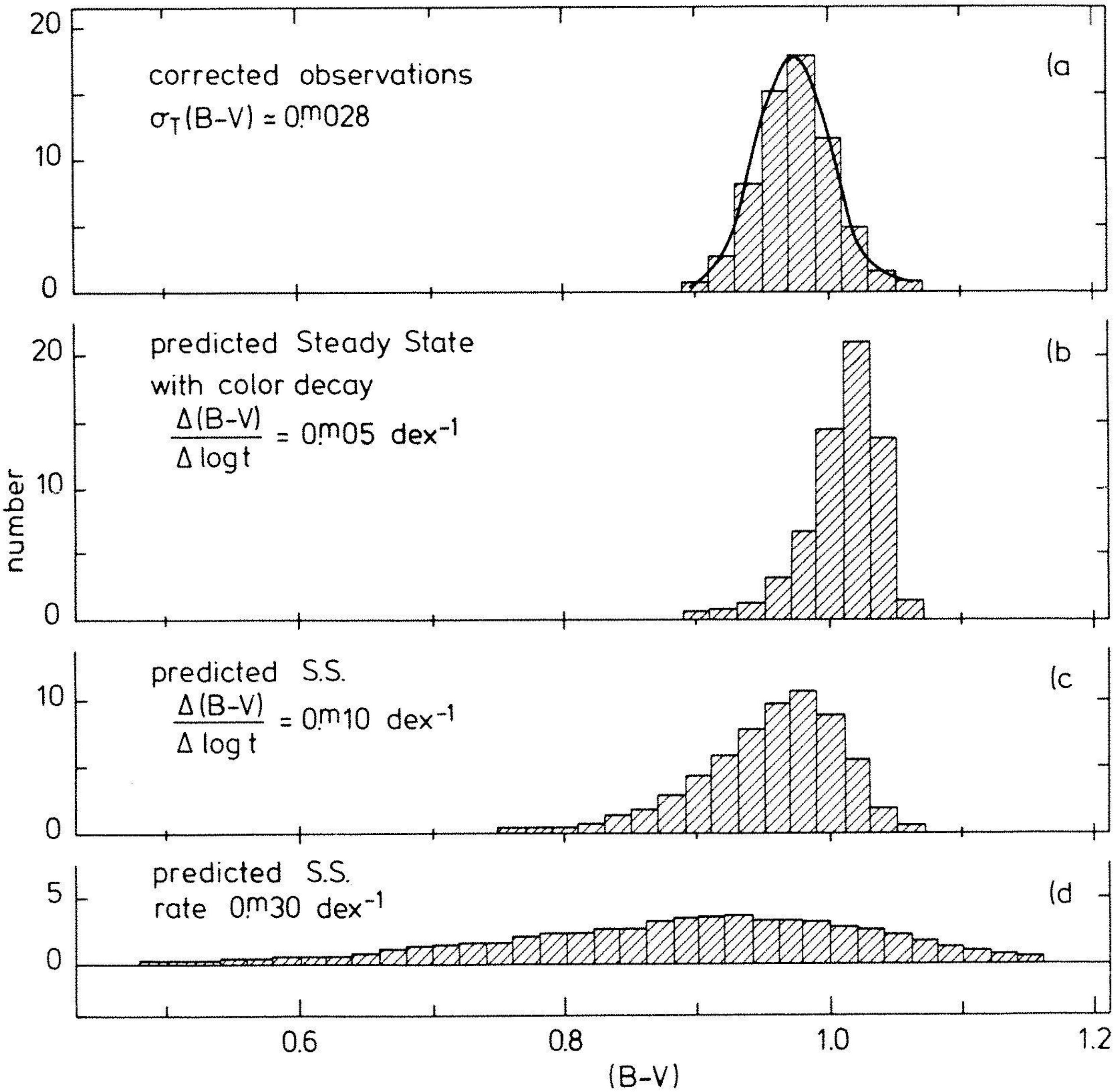


Fig. 1. The observed colour distribution of first-ranked cluster galaxies (upper panel). Note the narrow range in (B-V)-colours. The lower panels show the expected colour distribution, if galaxies were formed with constant rate (steady-state situation). Three different cases of colour evolution are assumed. Considering that the assumed colour evolution is improbably small in case b), first-ranked cluster galaxies must have had a preferred formation epoch in the past. (Adapted from Sandage 1973.)

($z > 0.2$–0.3) of field galaxies (Turner 1980) and of some cluster galaxies (Butcher, Oemler, and Wells 1980; Spinrad 1980; Oke 1980) may still need further confirmation (cf. also Buser 1981).

These short and incomplete remarks on evolutionary effects in the universe already show that it cannot be questioned anymore, that the universe has not always been the same, and that it is in a constant state of evolution.

However, it is the present goal not only to demonstrate that the universe reveals several aging effects, but rather that it partakes of a general expansion since a singularity in the past. The overwhelming evidence for this expansion rests mainly on four independent arguments, which shall be discussed here in turn: (1) the redshift-magnitude relation of standard candles, (2) the cosmic microwave background radiation (CMB), (3) the nucleo-

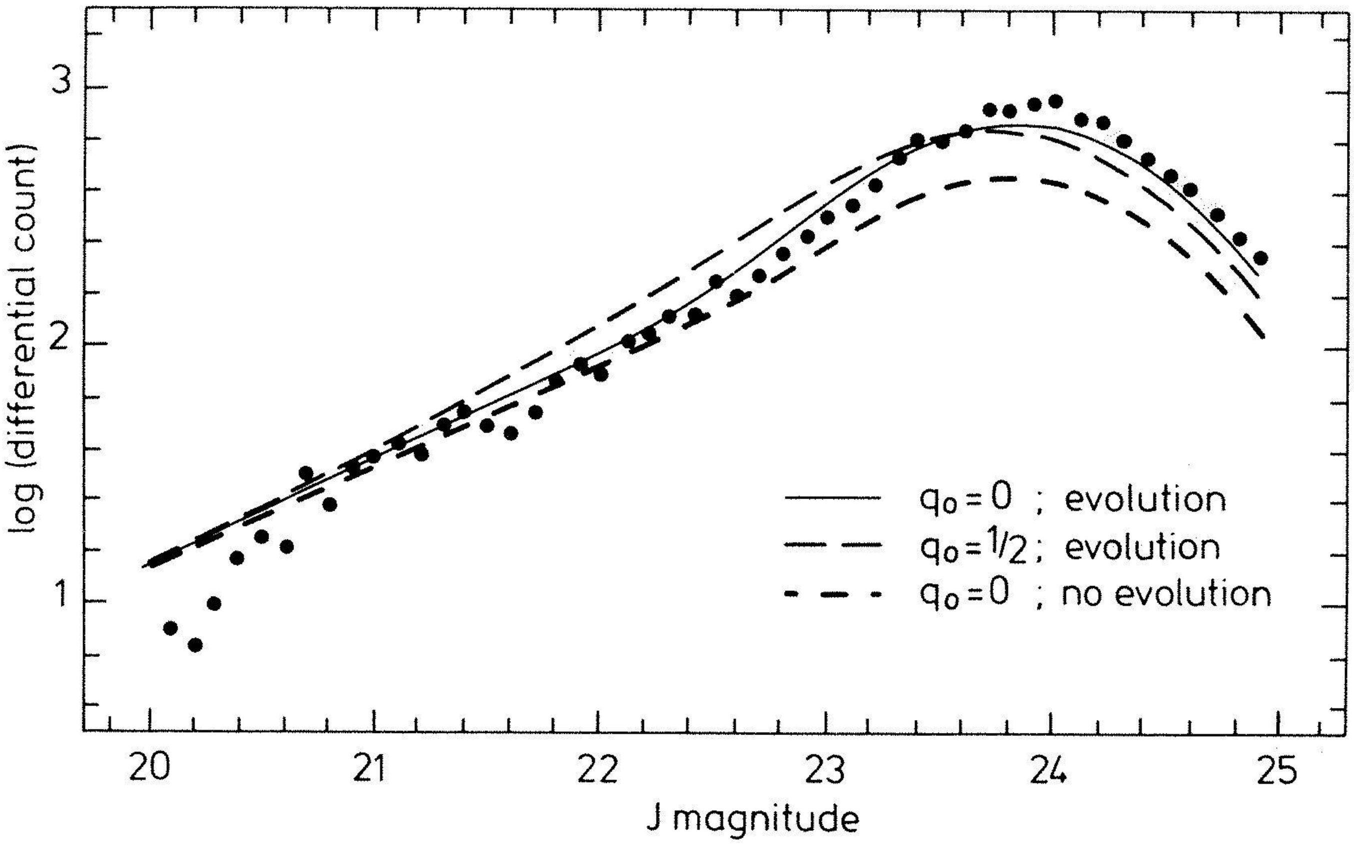


Fig. 2. The counts of galaxies (on a logarithmic scale) at very faint (J-) magnitudes. It is clear that the models with no evolution give poor fits to the counts, – whatever (reasonable) value of the deceleration parameter q_0 is assumed. (Adapted from Kron, 1980a.)

synthesis of the lightest elements, and (4) the agreement of the time scales of the expansion and of the oldest objects in our Galaxy.

The Redshift-Magnitude Relation

It is well known that in the Friedmann model (with zero cosmological constant), which is the most simple and therefore exceptionally attractive model of an expanding universe, the apparent bolometric magnitude m_{bol} of an object with absolute bolometric magnitude M_{bol} is a function only of

$$m_{bol} = f(M_{bol}, H_0, z, q_0) . \quad (2)$$

H_0 is the present value of the Hubble constant measuring the expansion rate. Generally $H = \dot{R}/R$, where R is the cosmic scale factor. The redshift z is defined by $z = \lambda/\lambda_0$, where λ_0 is the rest wavelength of a given spectral line. The present value of the deceleration parameter is defined by $q_0 = \ddot{R}_0/R_0 H_0^2$; it measures the total deceleration of the expansion due to matter and energy in the universe. In a matter-dominated universe q_0 is related to the mean mass density ρ_0 in a simple way:

$$q_0 = \frac{4\pi G \rho_0}{3 H_0^2} \quad (3)$$

Also the space curvature is determined by q_0: if $0 \leq q_0 \leq 1/2$ space is negatively curved (the gravitational binding energy in a representative volume is smaller than its kinetic expansion energy, and the universe will expand forever); if $q_0 = 1/2$ space is flat (the expansion of the universe will stop only in the infinite future); and if $q_0 > 1/2$ space is positively curved (the potential energy exceeds the kinetic energy, and the universe will eventually collaps). (For a rigorous derivation of these correlations see Sandage 1961).

For standard candles with M = const and small redshifts ($cz \ll 1$), which are observed in heterochromatic magnitudes, – for instance in red magnitudes m_R, – equation 2 becomes:

$$m_R = 5 \log cz + \text{const}. \quad (4)$$

Here the constant term contains (besides of M_R and H_0) the K-correction, which allows for the photometric redshift effect of stretching the energy distribution curve of the emitter and shifting it through the respective filter band, – in the present example through the R-band (cf. Sandage 1975b).

In Figure 3 the log cz – versus – m_R relation ('Hubble diagram') for the brightest galaxy in various clusters of galaxies is shown. The small scatter ($\sigma = 0^m3$) in the diagram proves that these first-ranked cluster galaxies are indeed good standard candles. The most remarkable fact, however, is that these galaxies define a line of slope 5, as predicted by the theory in equation 4. The implied linear relation between recession velocity and distance has two fundamental properties: (1) it requires a singularity in the past (Big Bang), and (2) a linearly expanding universe is the only one which can accomodate an unlimited number of fundamental observers having the same aspect of the universe. With other words: all non-linearly expanding universes would violate the Copernican Principle.

The most distant first-ranked cluster galaxies in Figure 3 with $z \approx 0.5$ (corresponding very roughly to recession velocities of 120,000 km s^{-1}) deviate somewhat from the straight line with slope 5. This is expected, because the effect of q_0 is neglected in equation 4: looking very far out in space (and back in time) one sees galaxies, which had little time to be decelerated and which lie therefore too high in the Hubble diagram. One has hoped for a long time that from the exact position of very distant galaxies in the Hubble diagram one could just determine the value of q_0.

One knows now that this hope is unrealistic, because first-ranked cluster galaxies are not perfect standard candles over cosmic epochs: they do suffer luminosity evolution, as discussed in Section 1, and this effect is accentuated by dynamical interactions within their respective clusters.

While first-ranked cluster galaxies establish the linear expansion out to the largest redshifts, less luminous, but locally more numerous standard candles are important to put limits on any deviations from an ideal Hubble flow. Such additional standard candles are brightest galaxies in small clusters and groups of galaxies (Sandage 1975a), the mean luminosity of the 10 or 5 brightest cluster galaxies (Weedman 1976), the maximum luminosity of supernovae of type I in elliptical galaxies (where they do not suffer absorption in their parent galaxies) (Tammann 1982; Fig. 4), as well as very local groups of galaxies with known individual distances (Tammann, Sandage and Yahil 1980). These various data show that the deviations from a pure Hubble flow are surprisingly small on all scales: the random velocities in the line of sight of field galaxies lie at the detection limit, which means for nearby galaxies $\lesssim 100$ km s^{-1}, and the upper limit for any streaming velocity our Galaxy is involved in (probably together with a very large volume of supercluster size) is 600 km s^{-1}. The latter value is derived from a small dipole anisotropy of the cosmic background radiation (Smoot 1980; Boughn, Cheng and Wilkinson 1981; cf. Section 3). In view of the large density fluctuations, which in fact are strong enough to have built up gravitationally bound clusters within an expanding universe, it is one of the most surprising facts of cosmology that the (density enhanced) peculiar motions of field galaxies are so small.

Against overwhelming evidence it is still occasionally put forward that the observed extragalactic redshifts are due to other effects than the Doppler effect. To the extent that these alternative interpretations involve 'non-physical' photon propagation, it should be stressed that in addition to the redshift-magnitude relation of standard candles there exists a redshift-angular diameter relation of standard rods. The latter relation for the diameters of first-ranked cluster galaxies (Fig. 5) and for whole clusters of galaxies (Fig. 6) independently support the linear expansion of the universe. The reader is referred to the original literature (Sandage 1961; 1975b) for the correct definition of isophotal diameters; it is these diameters which must be used for galaxies because their metric diameters are ill defined. In the case of galaxy clusters it is in principle possible to measure metric diameters, but

Fig. 3. The Hubble diagram of first-ranked cluster galaxies (adapted from Kristian, Sandage and Westphal 1978). The red magnitudes of the abscissa are corrected for galactic absorption, the K-correction, for the richness of the parent cluster and its Bautz-Morgan class. The Bautz-Morgan class is determined by the relative brightness of the first-ranked galaxy in comparison to that of the other cluster galaxies.

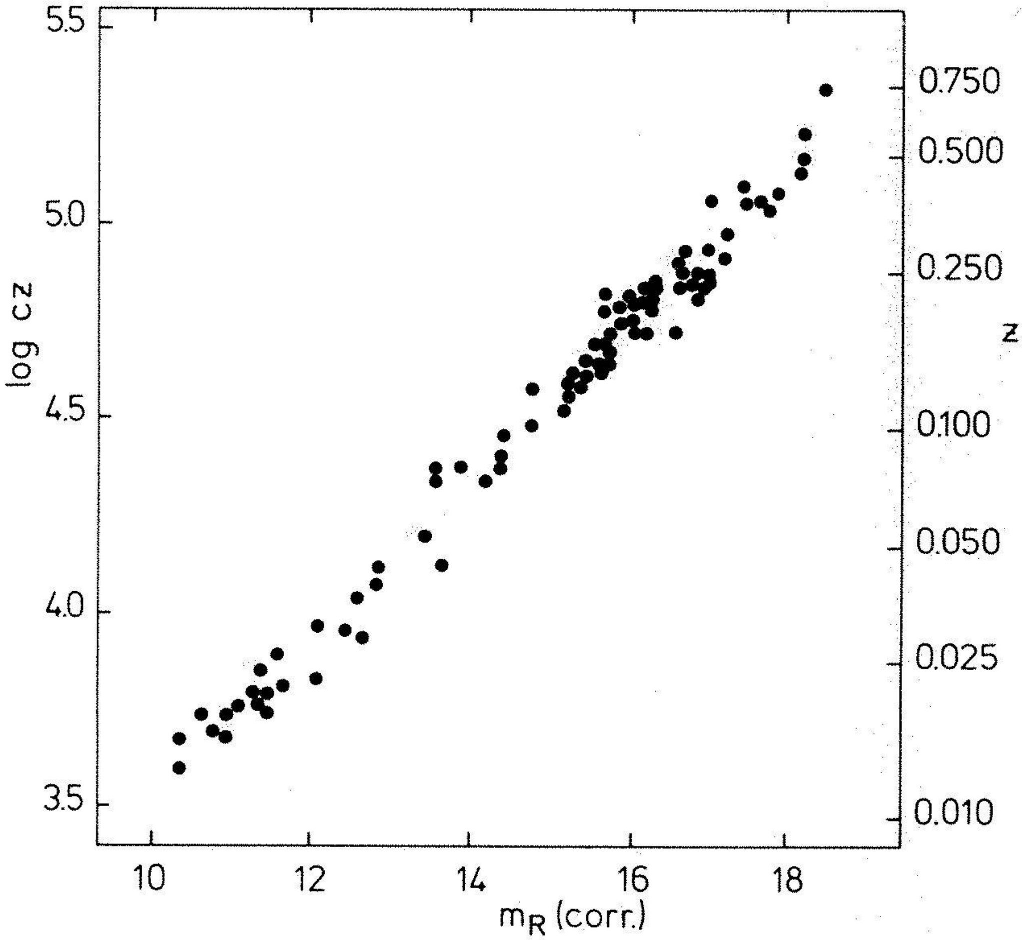
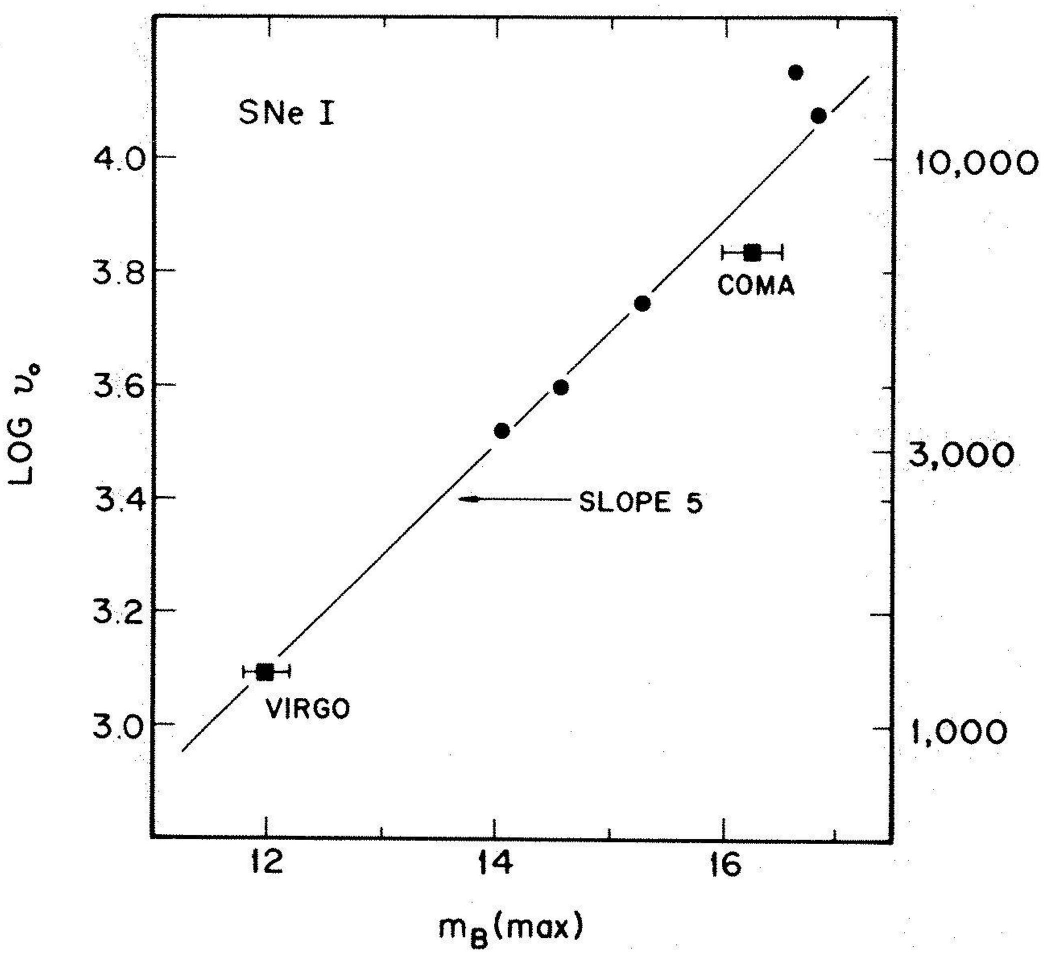


Fig. 4. The Hubble diagram of supernovae of type I in absorption - free elliptical galaxies at maximum blue light. The square for the Virgo cluster is the mean of 6 SNe, that for the Coma cluster is the mean of 5 SNe.



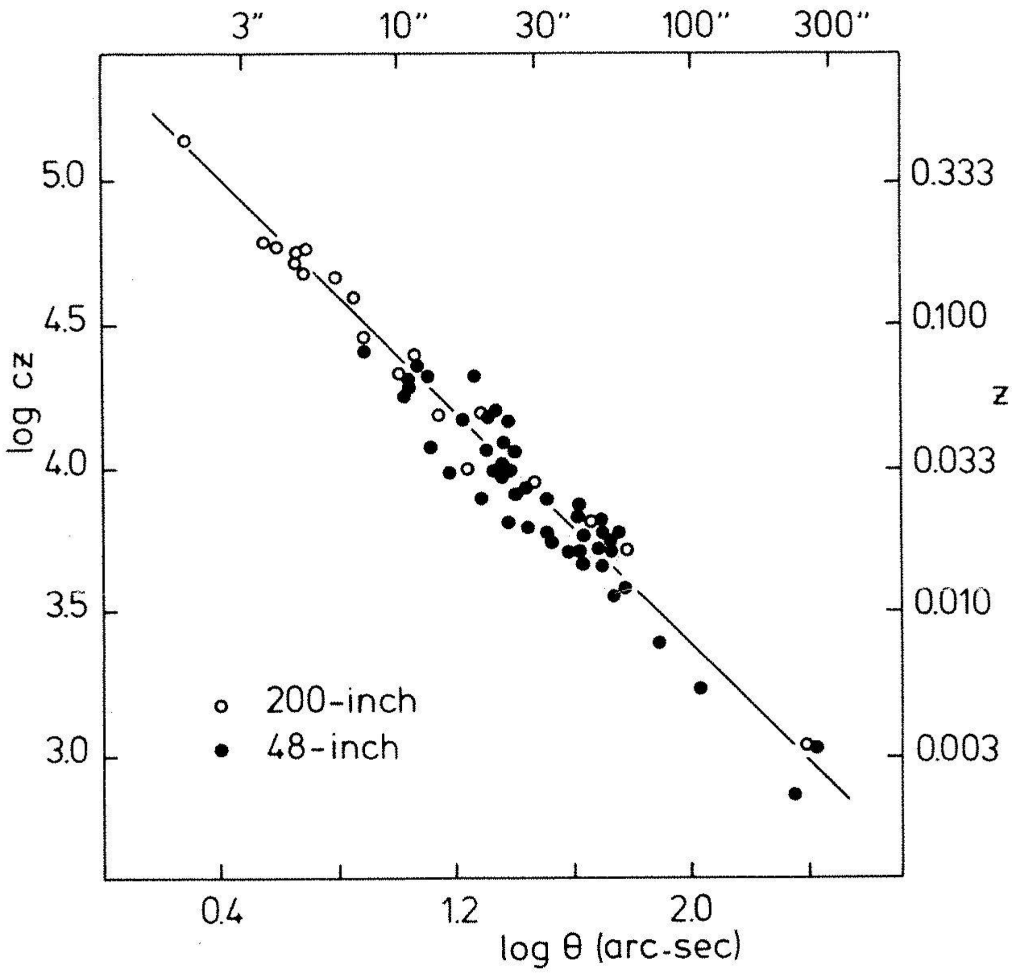


Fig. 5. The redshift-diameter relation of first-ranked cluster galaxies (adapted from Sandage 1972a). The abscissa shows the logarithm of the angular isophotal diameter. The different symbols refer to different telescopes.

their unambiguous, distance-independent definition poses also formidable difficulties.

The Cosmic Microwave Background Radiation (CMB)

The ability of a theory to make predictions, which are subsequently validated by observations, is considered to be the ultimate scientific test of any theory. It is therefore of the highest significance that the CMB radiation, first detected by Penzias and Wilson in 1965, had been predicted already in 1946 and subsequent years by Gamov and his collaborators for an expanding, hot Big Bang universe.

At early epochs the universe was opaque, and particles and photons had the same (high) temperature. About 650,000 years after the Big Bang (at redshifts of $z \approx 1100$ and a temperature of $T \approx 3000$ K) the universe had sufficiently cooled that protons could 're'-combine with free electrons to form neutral hydrogen, and the universe became transparent. From then on photons cooled adiabatically with the expansion of the universe, such that their present temperature is [note that $T_0 \propto R/R_0 = (z+1)$; cf. Sunyaev and Zel'dovich 1980]:

$$T_0 = T_{z=1100}/1101 = 2.7\ \mathrm{K}. \tag{5}$$

Indeed the best determinations of the blackbody temperature give 2.7–3.0 K (e.g. Wilkinson 1980; Richards 1980; Weiss 1980).

All attempts to explain alternatively the CMB by large numbers of unresolved objects have failed mainly because of three reasons:

(1) The observed spectrum of the CMB is nearly perfectly Planckian (Weiss 1980). No individual known objects, even with redshift effects taken into account, can match such a spectrum;

(2) At microwave wavelengths, i.e. $\lambda = 0.1$–6 cm, the temperature fluctuations over angular scales of 2′ to 2° are less than $\Delta T/T = 10^{-3}$ to 10^{-5} (Boynton 1978; Partridge 1980). To account for this degree of isotropy it would require an excessive num-

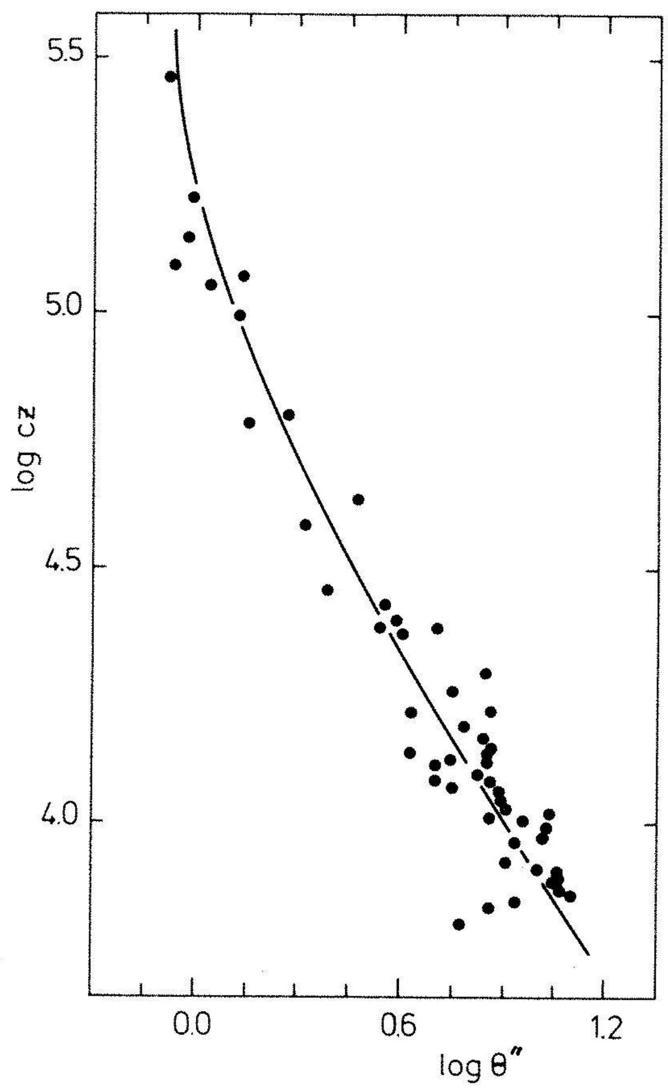


Fig. 6. The redshift-diameter relation for galaxy clusters (adapted from Bruzual and Spinrad 1978). Because of the large distances reached, deviations from an intuitively expected straight line become apparent. The full-drawn line is calculated for a linearly expanding universe and an assumed value of $q_0 = 0.5$. The fit, however, does not give a reliable determination of q_0 because of conceivable evolutionary diameter effects of clusters, due to their dynamically evolution.

ber of individual sources (cf. Longair 1978); and

(3) The number density of CMB photons is $\sim 400\ cm^{-3}$, which corresponds to an energy density of ~ 0.25 eV cm^{-3}. This is at least 95% of the present total mean energy density in form of radiation (Longair 1978). In spite of its low temperature the energy content of the CMB radiation is therefore very high.

While the CMB has widely been accepted as the final proof for a hot Big Bang universe, it should be added that Rees (1980) has argued that a different origin of the CMB cannot totally be excluded: if a hypothetical first generation of stars had formed at a very early epoch ($z > 600$) and generated enough energy, then their radiation could have been thermalized by a pregalactic medium, of which one can possibly postulate that it were opaque at epochs $z > 100$ due to dust and molecules (cf. also Woltjer 1981).

The Nucleosynthesis of the Lightest Elements

Wherever the ^{4}Helium abundance can be measured, i.e. in young and old stars, in the interstellar medium of our Galaxy, in the Magellanic Clouds and other galaxies, and even in metal-poor galaxies, one finds that its fractional abundance (in mass) is *at least* $Y = 0.24 \pm 0.02$ (Peimbert and Torres-Peimbert 1976; Greenstein 1980). This is a highly significant result which can hardly be explained by the assumption that this large amount of ^{4}He were processed in stars from hydrogen. It can be shown, that if this were the case the energy released would result in higher galaxian surface brightnesses than actually observed (Burbidge 1958; Hoyle and Tayler 1964).

The only way out of this dilemma is to assume that the ^{4}He abundance is primordial, which is also supported by other observational arguments (Greenstein 1980). Upon detailed calculations of the early phases of a standard hot Big Bang model, it turns out that the production of ^{4}He with $Y \approx 0.24$ is a nearly unavoidable consequence (cf. Weinberg 1972). The quantitative agreement of the calculated and the observed value of Y gives strong support to the adopted model.

The ^{4}He argument is further strengthened by the light isotopes ^{2}D, ^{3}He, and ^{7}Li (David and Reeves 1978; Audouze 1981). In particular it came as a great surprise that deuterium, which is effectively distructed in stars, is among the 12 most frequent isotopes in the interstellar medium (cf. Laurent, Vidal-Madjar and York 1979). Its origin must be primordial, and it turns out that it is indeed easily produced in the Big Bang model, as well as ^{3}He and ^{7}Li.

The nucleosynthesis of the light elements occurred about 220 sec after the Big Bang at a temperature of $T \approx 9 \cdot 10^8$ K. While the yield of ^{4}He is quite insensitive to other parameters, sufficient amounts of the other light isotopes were produced only, if the mean baryonic mass density was relatively low. The allowed range of the baryonic density at that epoch ($z = 3.6 \cdot 10^8$) can be fixed to within a factor of ~ 1.5; if this density is scaled down to the present epoch ($z = 0$) one finds a present baryonic density of $\rho_{b,0} = (1.9\text{–}3.8) \cdot 10^{-31}$ g cm^{-3} (Yang, Schramm, Steigman, and Rood 1979), which gives for the deceleration parameter through equation 3:

$$0.02 < q_0 < 0.04 .$$

This, and the above-mentioned small peculiar motions of field galaxies (Yahil, Sandage, and Tammann 1978) are presently the strongest arguments for an open universe. If the universe should actually be closed, the closing matter is probably not in baryons, and it is not clumped like the visible galaxies. It has been speculated therefore that the closing matter consists of neutrinos with non-zero mass ($m_\nu \approx 30$ eV). In that case it remains to be explained, however, why at most a fraction of all neutrinos cluster like galaxies.

The Time Scale of the Universe

If the present expansion rate, i.e. the Hubble constant H_0, is known as well as the deceleration parameter q_0, it is clear that the expansion age of the universe (the 'Friedmann time') can be determined. Obviously this age must be larger than the age of the oldest known objects.

The oldest objects, ever found in our Galaxy, are the globular clusters. Their age is conventionally determined from the absolute magnitude of the stars, which are just leaving the main-sequence (cf. Demarque and McClure 1977). The enormous observational difficulty of the method is, that this turn-off point lies at very faint magnitudes, and that unavoidable errors in the magnitudes and colours (!) have a large effect on the age. Moreover the result depends on the position of the fiducial main-sequence, which is gouverned by the adopted He abundance, and the (obviously variable) He content cannot be directly observed for the cool globular cluster stars.

In view of this difficulty a recent important progress must be mentioned: it is possible to fit the colour-magnitude diagrams of globular clusters by means of their (relatively bright!) horizontal-giant branches. The zero-point calibration of the luminosities is provided by the now understood period-luminosity relation of RR Lyrae variables (Sandage 1982). The resulting colour (temperature)-luminosity diagrams of individual globular clusters can then be fitted to existing model calculations of stellar evolution (Iben and Rood 1970; Demarque and McClure 1977), allowing for the known differences in metal content and almost independently of the He content. The result is for all globular clusters a uniform age of $(17 \pm 2) \cdot 10^9$ years (Sandage 1981). The near equality of the ages of globular clusters, even for those with widely different metal content, is in perfect agreement with the expectation, that the globular cluster system was formed during the relatively short collaps time of our early Galaxy.

The ages derived from evolutionary stellar models depend sensitively on the assumed constancy of the gravitational constant G. It is therefore important that the G-independent ages of the radioactive elements (particularly for the $^{187}Re/^{187}Os$ ratio) yield a time since the beginning of stellar nucleosynthesis of $(13\text{–}22) \cdot 10^9$ years (Luck, Birck and Allegre 1980), – where the relatively wide error range reflects the uncertainty of the necessary assumptions on the stellar birth rate function in our Galaxy.

The minimum age of the universe from two totally different methods is therefore probably $\geq 17 \cdot 10^9$ years, and almost certainly $> 15 \cdot 10^9$ years. How does this compare with the expansion age?

The Hubble constant H_0 has been determined to be 50 km s^{-1} Mpc^{-1} (Sandage and Tammann 1976). It has since been pointed out (Tammann, Yahil, and Sandage 1979) that some assumptions going into this determination (viz. mainly the reliability of the morphological luminosity classification of spiral galaxies) are not as rigid as originally assumed. On the other hand, values as high as $H_0 = 100$ have occasionally been proposed. However, these very high values depend either on distance indicators, whose usefulness cannot be proven independently, or are dominated by selection effects, which are known to stellar astronomers as 'Malmquist bias'. This bias makes that distant objects must be intrinsically brighter (or larger) than their nearby counterparts in order to enter the same magnitude-limited (or angular-diameter-limited) catalogue; it always leads to an underestimate of large distances and hence to an overestimate of H_0.

A new way to H_0 may resolve the discrepancy. Through Cepheid variables as primary distance indicators one knows the absolute magnitude of the brightest red supergiants in a number of galaxies. This absolute magnitude is well defined, because these stars have a sharp upper luminosity cutoff, and it is a very stable distance indicator, as judged from the widely different calibrating galaxies. Using these red supergiants as distance indicators one can obtain distances to a number of galaxies, including IC 4182 and NGC 4214. The latter two galaxies have produced each a well observed supernova of type I, from which one can derive a mean absolute magnitude at maximum light of $M_B(max) = -19^m.65$ (Sandage and Tammann 1981).

In Figure 4 it was shown already that supernovae of type I are very good standard candles (banning also any appreciable effect of the Malmquist bias). If one assumes $H_0 = 100$, the straight line in Fig. 4 is best represented by

$$M_B(max) = -18^m.22 + 5 \log H_0/100. \qquad (6)$$

Inserting into this equation the above value of $-19^m.65$ gives

$$H_0 = 52 \pm \sim 10 \text{ km s}^{-1} \text{ Mpc}^{-1}.$$

This result for H_0 must represent the truly cosmic value, beyond any local peculiar motions, because equation 6 is defined by supernovae out to recession velocities of ~ 10000 km s^{-1} (Sandage and Tammann 1981).

In the absence of any deceleration ($q_0 = 0$) the expansion age is $H_0^{-1} = (19 \pm 4) \cdot 10^9$ years. This age is decreased by any amount of deceleration (for a tabulation of the Friedmann time in function of q_0 see Sandage 1961a). With $q_0 = 0.05$, which is indicated by the baryonic matter in the universe (cf. Section 4), the Friedmann time becomes $(16 \pm 3) \cdot 10^9$ years. If there should be still enough, yet undiscovered matter to close the universe ($q_0 = 1/2$), the Friedmann time is reduced to $(13 \pm 3) \cdot 10^9$ years.

In any case the near agreement of the minimum age of the universe and its expansion age is stunning, and it is hard to see how this could only be an accident.

In this short presentation only Friedmann cosmologies were considered. Many other fancy and more complicated cosmologies have been put forward. But at present just the simplicity of the Friedmann Big Bang models and the fact that they are in perfect agreement with all available observations, makes them exceptionally attractive.

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