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The Growth of Structure in the Universe

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1. Visible and Invisible Matter

a) Cosmological Deuterium

In cosmology we use as a reference value for the present cosmic density the so-called critical density,

$$\rho_{crit} = 3\,H_0^2/(8\,\pi\,G) = 2\times 10^{-29}\,h^2\,g\,cm^{-3},$$

$$H_0 = 100\,h\,(km/s/Mpc), \qquad \frac{1}{2} \gtrsim h \lesssim 1\,; \qquad (1)$$

determinations of H_0 are done by several authors, Sandage and Tammann (1976), de Vaucouleurs and Bollinger (1979), Aaronson et al. (1980). It is also convenient to use the ratios

$$\Omega_0 = \rho_0/\rho_{crit}, \qquad \Omega_{B_0} = \rho_{B_0}/\rho_{crit}, \ldots, \qquad (2)$$

for the total density and its partial components. (The subscript "0" refers here and below to the present epoch.) For $\Omega_0 \leqslant 1$, the Universe is spatially open or flat and energetically hyperbolic or parabolic - i.e. it will expand forever - while for $\Omega_0 > 1$ the Universe is spatially closed and energetically elliptic - i.e. it is bound to collapse. This beautiful interaction between energetics and geometry is born from General Relativity, as we will see.

Primordial nucleosynthesis, occurring at the end of the first three minutes (e.g. Weinberg 1972 and 1977) gives us a powerful theoretical tool to evaluate ρ_0 or - better - its baryonic component, ρ_{B_0}. At temperatures $T > 10^{10}$ K ($t < 1$ sec) weak interactions and β-decay keep the neutron to proton number density ratio to its equilibrium value,

$$\exp\{-\Delta m\,c^2/kT\},$$

where Δm is the mass difference between neutron and proton; meanwhile deuterium is formed and destroyed:

$$n + p \rightleftarrows D + \gamma\,.$$

As the temperature drops the weak interaction rate falls below the expansion rate ($\sim 1/t$): at $T = 10^{10}$ K the two rates are equal and the neutron to proton ratio freezes out; this ratio decreases then slightly further due to neutron decay. At 10^9 K deuterium is no longer destroyed, the bottleneck is broken and He^4 is formed; due to the absence of any stable nucleus at mass 5, all the nucleons end up in He^4. The abundance of the latter, Y, is therefore basically twice the abundance of neutrons. Therefore the cosmological abundance of He^4 depends essentially on the rate of the cosmological expansion during the nucleosynthesis, which is directly related to the energy density of the photons and the relativistic particles ($e^\pm$ and ν) at 10^9 K. The abundance of deuterium depends instead on the competition between formation and destruction in two body reactions: it is therefore sensitive to the nucleon density at nucleosynthesis, which is related to the present nucleon (or baryon) density. The above argument is made precise by detailed study of the time evolution of the abundance of the various nuclei by the numerical integration of the appropriate differential equations (e.g. Schramm and Wagoner 1977, Steigman 1979): in particular the amount of deuterium that survives decreases steeply as the present baryon abundance, ρ_{B_0} or $\Omega_{B_0} h^2$, increases.

The observed amount of deuterium (York and Rogerson 1976, Vidal-Madjar et al. 1977) is large, $\chi_D \cong 2\times 10^{-5}$. If it is of cosmological origin - which is not obvious since it might have been created and destroyed elsewhere; see the discussion by Greenstein (1980) - the implication that follows (Gott et al. 1974) is that the present baryon density is low, of the order of some units in 10^{-31} g/cm^3: hence

$$\Omega_{B_0} h^2 \cong 10^{-2}. \tag{3}$$

This takes into account all the baryonic matter that has been processed in the Big-Bang, irrespective of whether such matter at present is or not in a luminous form.
The standard Big-Bang scenario sketched above has been recently further exploited (Yang et al. 1979; see also Shvartsman 1969, Dolgov and Zel'dovich 1981) to set an upper limit on the number of lepton species, N_L. In this case, He^4 abundance must be used: in fact adding the associated neutrino flavors to the cosmic medium increases the energy density of relativistic particles and decreases the age of the Universe,

$$t \propto \varepsilon^{-1/2}.$$

Hence more neutrons are present and more He^4 is formed: from a limit $Y \gtrsim 0.25$, it is concluded that $N_L \gtrsim 3$; thus there should not be many more leptons beyond the known electron, the muon and the newly discovered tau. We will use this estimate later on (see, however, Stecker (1980) for a different point of view).

b) Luminous Matter

Cosmology has also straightforward observational arguments to evaluate the mean cosmic density. Zwicky in the 30's observed that in order to bind the Virgo Cluster it is necessary to have 500 times more mass than it is apparently there; this started an important line of research, that of the missing mass, wherein cosmologists try to discover whether or not in galaxies and their associations there is more mass than is directly responsible for the observed electromagnetic emission. The issue has become more compelling, already at the level of individual galaxies, after the remark by Ostriker and Peebles (1973) that the thin disks of spiral galaxies would not be stable against bar instability unless they were embedded in massive halos. Observational support of this theoretical speculation has been strong particularly from 21-cm observations showing constant rotational velocities of HI-clouds at large distances from galactic centers. For this and other reasons, we have now little doubts about the existence of the missing mass and we prefer to consider it hidden or dark and we rather speak of missing light.
A convenient tool for the investigation of this problem are mass-to-light ratios (in solar units, M/L); a recent review of this subject is given by Faber and Gallagher 1979. What we see there is an escalation of M/L from the small to the large systems: thus M/L is of the order of unity (or slightly larger) in the solar neighborhood, of the order of 10 for spiral galaxies, around 20 for ellipticals and SO's, of the order of 100 for binaries and small groups and finally of several hundreds for cluster of galaxies (on the latter issue see also Bahcall 1977 and Hoffman et al. 1980).
In particular it happens that M/L is large whenever evaluated by dynamical methods: thus for spiral galaxies, for instance, while M/L has acceptable values for the inner regions out to 20 kpc, when we move beyond 50 kpc the flatness of the rotation curves pushes M/L above 100. Clearly dark matter dominates there; incidentally its spatial distribution can be easily inferred: on the assumption of centrifugal equilibrium, $v^2 = G\,M(r)/r$, the constancy of the rotational velocity implies that $M(r) \propto r$ and hence that $\rho \propto r^{-2}$, which is reminiscent of isothermal spheres. Likewise, estimates of the mass of our own Milky Way from tidal effects on globular clusters or from globular cluster radial velocities place the mass above 10^{12} $M_\odot$ and M/L around 70. A value consistent with the latter can also be found from the dynamics of the Local Group, which, as we know, is dominated by M31 and the Milky Way: if the velocity of approach of the two galaxies arises from their mutual gravitational interaction, the total mass must be of the order of some units in 10^{12} $M_\odot$ and M/L consistently ranges up to 60.
Dynamical methods are also used to determine the mass of great clusters: most commonly the virial theorem; it yields values of M/L of several hundreds and thus much larger than the M/L's of the constituent galaxies. On the contrary the X-ray emission from the cluster cores (Lea et al. 1973, Cava-

liere and Fusco-Femiano 1976, Malina et al. 1978) accounts only for a small fraction, ~10%, of the virial mass.

We may now try to correlate mass-to-light ratios to masses and hence to representative densities: the standard technique multiplies M/L by an average luminosity function (Kirschner et al. 1979) and obtains a mass density. When we use the (M/L)'s of the solar neighborhood we obtain density limits in qualitative agreement with (3) above; the same is true when we estimate the mean density from the mass of hot gas in clusters of galaxies. On the contrary, the virial theorem, mass-to-light ratios of great clusters and the analysis of the correlation function send Ω_0 to values of the order of unity (Davis et al. 1978).

Thus observational cosmology suggests the view that most of the cosmic matter is in some hidden form, of which all we know is that it gravitates and that it is very likely dissipationless (Gunn 1978). If so, the fact that unseen matter is needed more at the larger scales, may be related to the fact that on the scales of galaxies ordinary baryonic matter did have the time to cool and sink in the potential wells; on the scales of cluster of galaxies instead, cooling times are longer that the age of the Universe and the separation between visible and invisible matter has not yet occurred.

2. Gravitational Instability in the Universe

a) The Jeans and the Silk Masses

Our Universe is homogeneous on the large scales (> 100 Mpc), but shows a considerable amount of clumpiness and structure at the small ones ($\gtrsim$ 10 Mpc). Among the main tasks of modern cosmology is the explanation of that degree of structure; we think it is a problem of following theoretically the evolution of this structure as it grows by self-gravitation from a slight perturbation in an initially uniform and expanding medium. This view meets serious difficulties on the mass scales of galaxies, as we will see, even if we postulate very "ad hoc" initial conditions. Thus, the situation is far from satisfactory; there are hopes however that we are close to a major breakthrough. In the sequel we will review the basic facts following Weinberg (1972).

We define our vocabulary starting from the elementary theory of Jeans instability: in a uniform (hence infinite) self-gravitating medium the evolution of a small, linearizable perturbation of all the quantities describing the fluid is studied via the equation of continuity, Euler's and Poisson's equation. We condense all this in a simple, second-order, partial differential equation for the density enhancement $(\delta\rho/\rho)$,

$$\left(\frac{\partial^2}{\partial t^2} - c_s^2 \nabla^2\right) \frac{\delta\rho}{\rho} = 4\pi G \rho \frac{\delta\rho}{\rho}, \tag{1}$$

where c_s is the sound speed. Clearly we seek a solution of the form

$$\frac{\delta\rho}{\rho} \propto \exp\{i(\mathbf{k} \cdot \mathbf{x} - \omega t)\}, \tag{2}$$

and we find the elementary dispersion relation

$$\omega^2 = k^2 c_s^2 - 4\pi G \rho. \tag{3}$$

The latter tells us that on the small wavelength side of the perturbation spectrum, we have genuine sound waves, $\omega^2 > 0$, while on the large wavelength side we have an instability, $\omega^2 < 0$, with two exponentially growing and decaying modes; in the limit $k \to 0$, the e-folding time is given by

$$\frac{1}{\tau} = \sqrt{4\pi G \rho}. \tag{4}$$

The separation between the two regimes occurs at a Jeans wavenumber

$$k_J = (4\pi G \rho / c_s^2)^{1/2}; \tag{5}$$

the corresponding wavelength is

$$\lambda_J = 2\pi / k_J. \tag{6}$$

In the sequel in order to apply the concept of gravitational instability to expanding cosmological models, where a length would not be an invariant quantity, we prefer to introduce a Jeans mass

$$M_J = \frac{4\pi}{3} \rho \left(\frac{\lambda_J}{2}\right)^3. \tag{7}$$

Furthermore it is convenient to modify and generalize (7) in two ways. Firstly, we want to make sure that ρ in (7) contains only the proper mass density of the constituent particles without any contribution from the internal energy; thus we write explicitly in place of ρ the product $n \times m_H$ of the number density times the mass of the individual particles, protons, say. In this way we can compare the behavior of a given rest mass under different conditions in the history of the Universe, regardless of the associated thermal energy. Secondly, we observe that (6) gives us a measure of the strength of the gravitational field and, in order to generalize (7) to the case of strong fields, we recall that in general relativity not only the rest-mass but any form of energy and pressure feel the gravitational field: we thus replace ρ in (5) by $(\varepsilon + p)/c^2$. The more general expression for the Jeans mass,

$$M_J = \frac{\pi}{6}\, n\, m_H \left[\frac{\pi c^2 c_s^2}{G(\varepsilon + p)}\right]^{3/2}, \tag{8}$$

is of interest in radiation dominated cases.

We can now apply (8) to various regimes of interest: thus, for instance, before hydrogen formation, $T > 4000$ K, the cosmic medium is a mixture of black-body radiation, $\varepsilon = a\,T^4$, $p = \varepsilon/3$, and non-relativistic protons, $\varepsilon = n \times m_H \times c^2$, $p = 0$. Then (8) yields

$$M_J \cong \eta^2 \left(1 + \eta\, \frac{kT}{m_H c^2}\right)^{-3} M_\odot, \tag{9}$$

where

$$\eta = \frac{4\,a\,T^3}{3\,n\,k}, \tag{10}$$

is the specific entropy or photon-to-baryon ratio. The latter is a large number, $10^8 \div 10^{10}$ (see also the recent estimates due to Olive et al. 1981, from nucleosynthesis and mass-to-light ratios). Then (9) has a high temperature limit

$$M_J \cong (m_H c^2/kT)^3/\eta\, M_\odot, \tag{11}$$

and a low temperature limit where it levels off at the very high value

$$M_J \cong \eta^2 M_\odot. \tag{12}$$

As hydrogen forms, $z_{dec} \cong 10^3 \gtrsim z_{eq}$, the picture changes substantially because radiation disappears from the budget

$$\varepsilon = n\, m_H c^2 + \frac{3}{2} n\,k\,T, \qquad p = n\,k\,T, \qquad c_s^2 = \frac{5}{3}\frac{kT}{m_H}, \tag{13}$$

then (8) yields

$$M_J \cong \frac{1}{2}\left(\frac{5\,k\,T}{G}\right)^{3/2} n^{-1/2} m_H^{-2}, \tag{14}$$

which starts as low as

$$M_J \cong 10\, \eta^{1/2} M_\odot \cong 10^5 \div 10^6 M_\odot$$

at z_{dec} and drops thereafter as $R^{-1.5}$. The above behavior of the Jeans mass is sketched together with the Jeans mass of the massive neutrinos in figure 4.

Comparing the Jeans mass with the typical mass of galaxies, $M_G \cong 10^{11}\, M_\odot$, we see that there are three regimes of interest. In the first phase, $M_G > M_J$, any oscillation involving a mass of the order M_G will grow due to self-gravity; in reality this growth would occur in a radiation dominated epoch which must be studied with a general-relativistic treatment (see sect. 2b). After this, there is a second phase where $M_G < M_J$ in which any perturbation on the scale of M_G oscillates at constant amplitude; whether or not a relativistic treatment is needed, depends on the value of η in the sense that, as before, a large amount of radiation cannot be described properly in Newtonian terms. Finally, there is a third and last phase $z < 10^3$, where $M_G > M_J$ and any matter perturbation on the scale M_G grows by self-gravitation in a matter dominated background.

The above discussion may not seem to relate very strongly to objects like galaxies, since it does not single out a mass of the order of M_G. Silk (1968), Peebles and Yu (1970) and Weinberg (1971) show instead that a mass around M_G comes very naturally into play when dissipation mechanisms are taken into account.

Dissipation arises as a consequence of the imperfect coupling between the photon and the baryon component of the cosmic medium; this is the case when the photon mean free path for collisions with the electrons

$$l_\gamma = (n_e \sigma_T)^{-1}, \qquad (15)$$

(where $\sigma_T = 2/3 \times 10^{-24}$ cm^2 is the Thomson cross-section) becomes long enough for the photon to random-walk out of the perturbation. Obviously this occurs increasingly as ionization decreases.
In first approximation the physics of this phenomenon may be studied by treating photons and baryons as a non-perfect fluid endowed with shear and bulk viscosity and heat conductivity (they are all proportional to l_γ). It is found that a sound wave of mass M will be damped at the end of ionization by a factor

$$\exp\{-(M_D/M)^{2/3}\}. \qquad (16)$$

Thus, the attenuation will be negligible if $M \gg M_D$ and will be substantial in the opposite case: recent estimates (Jones 1976) give for the Silk mass the expression

$$M_D \cong 10^{13} (\Omega_{B_0} h^2)^{-5/4} M_\odot. \qquad (17)$$

A fluctuations of the mass of a galaxy will suffer substantial damping and will unlikely survive to the matter dominated era.

b) The Influence of the Cosmic Expansion

The expansion of the Universe is conveniently described by the time evolution of the familiar scale factor R(t) of the Friedmann-Robertson-Walker (FRW) line element. In the simplest case R(t) obeys the Einstein field equation (see later)

$$\dot{R}/R = \sqrt{8\pi G \rho/3}. \qquad (1)$$

Thus, the expansion time scale is unconfortably close to the time scale for gravitational collapse given in (a.4). Since the formation of galaxies occurs in the expanding Universe, we are forced to generalize our theory of gravitational instability to the case of an expanding medium. This has been done by Lifshitz in 1946 in the full glory of general relativity; a much simpler yet very indicative Newtonian approach to this problem was given by Bonnor 1957; a recent review is given by Field 1975. On the other hand, when we consider sound waves

$$k \gg k_J,$$

frequencies are very large and the expansion of the Universe may be neglected.
Let us review briefly the description of the unperturbed model. The assumptions that are commonly accepted in cosmology are that the correct theory of gravity is general relativity where the gravitational field is the metric tensor.

$$ds^2 = g_{\alpha\beta}\, dx^\alpha\, dx^\beta, \qquad \alpha, \beta = 0, 1, 2, 3, \qquad (2)$$

and the dynamical equations of motion are Einstein's field equation,

$$G_{\alpha\beta} - \Lambda g_{\alpha\beta} = \frac{8\pi G}{c^4} T_{\alpha\beta}, \qquad (3)$$

$$T^\beta_{\alpha;\beta} = 0. \qquad (4)$$

We have introduced for complete generality the cosmological term, Λ; $G_{\alpha\beta}$ is the Einstein tensor and $T_{\alpha\beta}$ the energy-momentum tensor for which we use the perfect fluid expression,

$$T^{\alpha\beta} = (\varepsilon + p)\, u^\alpha u^\beta - p\, g^{\alpha\beta}, \quad u^\alpha = dx^\alpha/ds. \qquad (5)$$

Once we specialize the metric tensor (2) to the FRW form,

$$ds^2 = c^2 dt^2 - \frac{R^2(t)}{(1 + k r^2/4)^2} (dx^2 + dy^2 + dz^2), \qquad (6)$$

appropriate for dealing, in comoving coordinates ($u^0 = 1, u^k = 0$) with a uniform medium expanding isotropically, from (3) we obtain

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3c^2}(\varepsilon + 3p) + \frac{1}{3}\Lambda c^2, \qquad (7)$$

$$\left(\frac{\dot{R}}{R}\right)^2 + \frac{k c^2}{R^2} = \frac{8\pi G}{3c^2}\varepsilon + \frac{1}{3}\Lambda c^2. \qquad (8)$$

Equation (7) shows the deceleration of the cosmic expansion under self-gravity (to which pressure gives a contribution) and the accelerating role played by a positive Λ; (8), which generalizes (1), is a first integral of (7). Under the assumption (5), (4) yields an energy conservation equation,

$$\frac{d}{dt}(\varepsilon R^3) + p\frac{d}{dt}R^3 = 0. \tag{9}$$

Together with an equation of state, which we usually assume of the form

$$p = \gamma\,\varepsilon, \qquad \gamma = 0, \frac{1}{3}, 1, \tag{10}$$

to deal at least schematically with dust, radiation and a maximum stiffness fluid ($c_s = c$), we have now stated all the rules, equations (8), (9) and (10), which a cosmological model must obey.
For every particle component we have a continuity equation

$$(n\,u^a)_{;a} = 0, \tag{11}$$

which yields the conservation of total number in comoving volume:

$$n\,R^3 = \text{const}. \tag{12}$$

We now perturb the equilibrium solution given above by introducing small and linearizable changes in the thermodynamic quantities, $\delta n, \delta\varepsilon, \delta p, \delta u^a$ and in the gravitational field, $\delta g_{\alpha\beta}$; for the latter we limit ourselves without loss of generality to the gauge $\delta g_{0\alpha} = 0$.
Lifshitz' solution contains also radiative and rotational modes which decay away with the expansion of the Universe and which we will not consider. The compressional normal modes are found to obey a relatively simple equation in the long wavelength limit appropriate to study the gravitational instability (a straightforward derivation is given in Harrison 1967).
By perturbing the (00)-component of the field equations (3), we find

$$\frac{1}{R^2}\frac{d}{dt}R^2\frac{d}{dt}\delta g = \frac{8\pi G}{c^2}\left[1 + 3\left(\frac{c_s}{c}\right)^2\right]\delta\varepsilon, \tag{13}$$

where

$$\delta g = -(\delta g_{11} + \delta g_{22} + \delta g_{33}), \tag{14}$$

and the assumption of adiabaticity, $\delta p = c_s^2\,\delta\varepsilon$, has been used. By perturbing the (0)-component of (4) and the equation of continuity (11), we find for $\lambda \to \infty$, respectively

$$\frac{d}{dt}\frac{\delta\varepsilon}{\varepsilon + p} = \frac{1}{2}\frac{d}{dt}\delta g, \tag{15}$$

$$\frac{d}{dt}\frac{\delta n}{n} = \frac{1}{2}\frac{d}{dt}\delta g. \tag{16}$$

The comparison between the last two gives the result

$$\frac{\delta n}{n} = \frac{\delta\varepsilon}{\varepsilon + p}, \tag{17}$$

which is again a statement of the adiabaticity of the perturbation. If we replace in (13) δg and $\delta\varepsilon$ by (16) and (17), we end up with a single differential equation

$$\frac{1}{R^2}\frac{d}{dt}R^2\frac{d}{dt}\frac{\delta n}{n} - \frac{4\pi G}{c^2}(\varepsilon + p)\left(1 + 3\frac{c_s^2}{c^2}\right)\frac{\delta n}{n}. \tag{18}$$

This is the sought generalization of the limiting case of (a.1) for $\lambda \to \infty$; it takes into account the expansion and the strong gravitational fields (with the regeneration of the pressure) and is valid for any value of the curvature and of the cosmological constant.
The most important novelty of (18) is that it replaces the exponential growth of the Jeans instability with a much more modest law, tipically a power law. To see this, let us assume $k = 0$ for simplicity and let us look for a solution in the form

$$\frac{\delta n}{n} \propto t^a. \tag{19}$$

For the three values of γ in (10) we find the results of table 1, where $a_\pm$ give the growing and the damping mode, respectively.
The most important result we read in table 1 is that in an Einstein-de Sitter Universe

$$\Omega_0 = 1, \qquad \frac{1}{1+z} \propto t^{2/3}.$$

Table 1

	R	ε	$\frac{c_s}{c}$	α_+	α_-
$\gamma=0$	$t^{2/3}$	$\frac{c^2}{6\pi G t^2}$	0	$\frac{2}{3}$	-1
$\gamma=\frac{1}{3}$	$t^{1/2}$	$\frac{3c^2}{32\pi G t^2}$	$\frac{1}{\sqrt{3}}$	1	-1
$\gamma=1$	$t^{1/3}$	$\frac{c^2}{24\pi G t^2}$	1	$\frac{4}{3}$	-1

the growing mode amplifies in the linear regime according to the law

$$\frac{\delta n}{n} \propto t^{2/3} \propto \frac{1}{1+z} . \tag{20}$$

In particular, the amplification available between decoupling $1+z_{dec}=10^3$, and the present is just by a factor 10^3; thus a perturbation which enters non-linearity at the present ($\delta n/n=1$ at $1+z=1$) at decoupling had an amplitude $\delta n/n=10^{-3}$. The latter amplitude should be observed in the form of a small scale ($\gtrsim 1^0$) distortion in the microwave background: under the assumption of perfect adiabaticity, in fact

$$\left(\frac{\delta T}{T}\right)_{dec} = \frac{1}{3}\left(\frac{\delta n}{n}\right)_{dec} . \tag{21}$$

On the contrary (see Partridge 1980) the experimental limit, which is also the present limit of sensitivity of our detectors, is already down to

$$\frac{\delta T}{T} \gtrsim 10^{-4} , \tag{22}$$

and seems to indicate the existence of a conflict between theory and observations. There are ways out: the most obvious is that there might have been a reheating (early star formation) and consequent reionization of the cosmic medium. In that case further Thomson scattering would have blurred any imprint left in the microwave background by condensing galaxies or larger objects. Alternatively we must take into account the possibility that the coupling between matter and radiation fluctuations is minimum (as in isothermal fluctuations) and not maximum as in (21) (Davis and Boynton 1980): in that case the theoretical predictions for purely isothermal fluctuations are certainly below the present detection threshold, but an order of magnitude improvement of the current instrumental sensitivity will critically test the gravitational instability picture for galaxy formation.

Equation (18) is valid in all generality: let us consider the case of dust ($p=0$, $c_s=0$); it reduces to

$$\delta = \frac{\delta\rho}{\rho} ,$$

$$\left[\frac{1}{R^2}\frac{d}{dt}R^2\frac{d}{dt} - 4\pi G\rho\right]\delta = 0 , \tag{23}$$

and in this form analytic solutions are known for any value of Ω_0 (provided $\Lambda=0$). In particular $\Omega_0<1$ ($k=-1$) is of interest according to the considerations of section 1 for a low-density baryon cosmology. We plot in figure 1 the solution for the growing modes given in Weinberg 1972. Inspection of the figure gives us as a rule of the thumb the notion that in the $\Omega_0<1$ case the growing mode grows as in (20) only for $z \gtrsim \Omega_0^{-1}$, but levels off thereafter. Thus for $\Omega_0=0.1$ the total amplification available is only 100 (and not 1000 as in the Einstein-de Sitter case): temperature fluctuations in the microwave background of amplitude around 10^{-2} could be expected and the lack of their detection is disturbing (Boynton 1978).

A solution of this problem is given by Doroshkevich et al. (1974) (see also Gott 1979): they consider the case of perturbations obeying the Zel'dovich (1970) condition which is the assumption that a) all the perturbations are purely adiabatic and that b) they have the same amplitude $A=10^{-4}$ on all mass scales when they enter the horizon (see also Press 1980). These authors find that on scales smaller than the Silk mass ($\sim 10^{14}\ M_\odot$) all the fluctuations are damped as expected by photon viscosity, while on scales just larger than the Silk mass the fluctuations not only survive, but also undergo a two-order of magnitude amplification due to "velocity overshoot".

Press and Vishniac (1980) have however

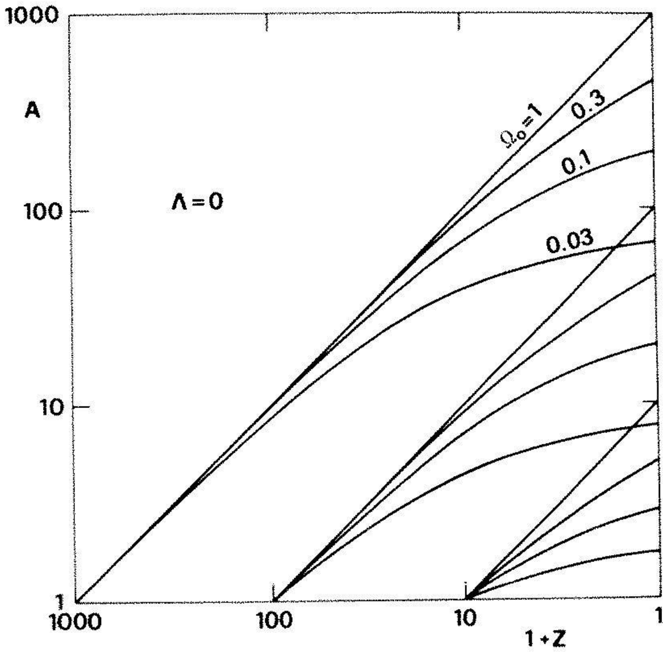


Fig. 1. Plot of the amplification of the growing modes in cosmological dust models with $\Lambda=0$ and $\Omega_0 \leqslant 1$ vs. redshift. In the Einstein-de Sitter ($\Omega_0=1$) case the straight diagonal lines apply: thus if $\delta\rho/\rho=1$ at the present the initial $\delta\rho/\rho$ was 10^{-3}, 10^{-2} and 10^{-1} at $1+z=10^3$, 10^2 and 10, respectively. For open models, $\Omega_0<1$, however, the amplification is reduced considerably with disturbing implications on the microwave background: however, a low-density model may be closed ($k=+1$) when $h \cong 1$ and $\Lambda>0$ because

$$k \propto (\Omega_0-1)+\Lambda c^2/(3\,H_0^2).$$

In that case the growing modes amplify better than in the $\Omega_0=1$ case, as shown in figure 2.

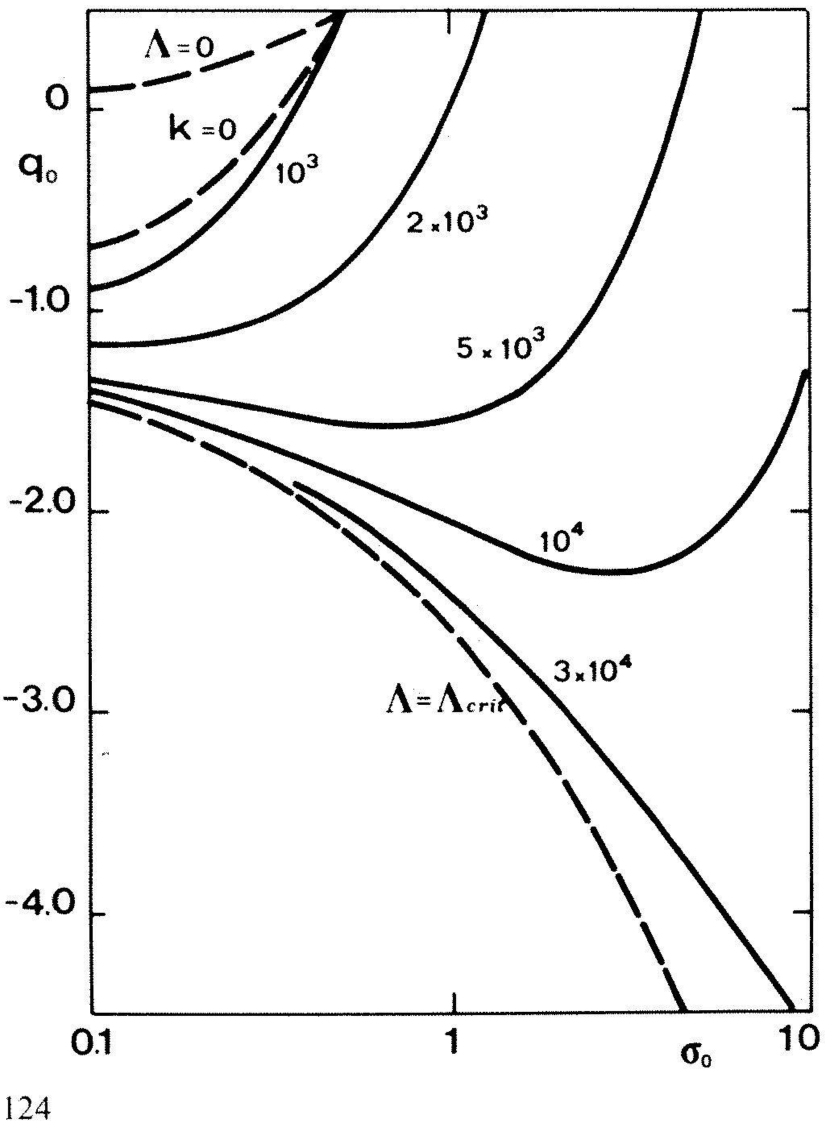


Fig. 2. Level curves of the amplification of the growing modes in a pure baryon Universe assuming that the linear growth starts at $z_{dec}=1000$. Broken lines define the constraints $k=0$, $\Lambda=0$ and $\Lambda=\Lambda_{crit}$. The labels on each curve define the total amplification available on that curve; thus the curve labelled 10^3 goes through the Einstein-de Sitter model $\sigma_0=q_0=0.5$. This plot shows that in the region of the plane where $k=+1$ and $\Lambda>\Lambda_{crit}$ the linear amplification may exceed considerably that of an Einstein-de Sitter model. These results must be compared with those given in figure 10 below.

shown that the effect is in reality a consequence of an inaccurate treatment of decoupling, i.e. of the assumption that it is instantaneous. An accurate treatment of hydrogen formation shows no sign of overshooting in agreement with previous results of Peebles and Yu (1970).

A possibility of removing the mentioned difficulties with the microwave background has been described by Occhionero et al. (1980); it is found investigating the dust solution of (23) after allowing, for complete generality, also for a non-vanishing Λ term. In that case, dust models form a biparametric set: the first parameter is the density parameter Ω_0 we met in (1a.2) and which we now replace by

$$\sigma_0 = \Omega_0/2\,, \tag{24}$$

in order to agree with the established conventions (e.g. McVittie 1965), while the second parameter is the deceleration parameter

$$q_0 = -(R\ddot{R}/\dot{R}^2)_0\,. \tag{25}$$

In terms of these

$$\frac{k\,c^2}{(H_0 R_0)^2} = 3\,\sigma_0 - q_0 - 1\,,$$
$$\frac{\Lambda\,c^2}{3\,H_0^2} = \sigma_0 - q_0\,; \tag{26}$$

clearly for $\Lambda = 0$ the (σ_0, q_0) parameter plane degenerates into a straight line.

In figure 2 we present some of the results: broken lines define the curves $k=0$, $\Lambda=0$ and $\Lambda=\Lambda_{crit}$. (It is known from the general theory of the FRW models that when $k=+1$ we must have $\Lambda>\Lambda_{crit}$ otherwise the cosmologic expansion reverts into a collapse.) Solid lines define the loci of points where the amplification of the density contrast between $z_{dec}=10^3$ and the present has a well defined value, which is the label on each curve. Thus we see that the interesting region on the plane satisfies both $k=+1$ and $\Lambda>\Lambda_{crit}$: indeed these growing modes amplify better than a factor 10^3 (which is the curve going through the Einstein-de Sitter model $\sigma_0 = q_0 = 0.5$).

In the framework of a low-density Universe, these consideration apply only if $H_0 = 100$ km/s/Mpc, $H_0^{-1} = 10\times10^9$ y and we must resort to the cosmological term for the age problem (see also figure 3). If so, the order of magnitude improvement of the growing mode amplification results partly from curvature and partly from the increased time span available for growth; indeed in the limiting case $\Lambda=\Lambda_{crit}$, the cosmic expansion is suppressed and power laws are again replaced by exponentials.

Let us now return to the era immediately before decoupling when baryons and photons are coupled by Thomson scattering and energetically equivalent: incidentally we may define the equivalence redshift between matter and radiation at

$$1 + z_{eq} = 4\times10^4\,\Omega_0\,h^2\,, \tag{27}$$

and say that for $z>z_{eq}$ radiation dominates while matter $(p=0)$ dominates afterwards, $z<z_{eq}$. The perturbations of the cosmic medium we have discussed above are adiabatic in the sense that radiation and matter are perturbed together and the ratio of photon to baryon number is kept constant. We have another fundamental mode of perturbation, however; the isothermal one, where only baryons are perturbed, but the background radiation is left unperturbed. In this case since the number of baryons per photon is changed we have an entropy perturbation. Mészàros (1974) addresses the question of whether given a completely uniform distribution of particles and radiation, can a perturbation of the particle distribution only grow. Under the assumption of flatness of space-time $(k=0)$ and, more importantly, of no interaction between the particles and the relativistic substratum, beside of course gravitation, the answer is that no growth is possible until the Universe is radiation dominated, but growth becomes possible thereafter.

3. High Density Universes

a) A Neutrino Dominated Universe

The experimental measure of the electron neutrino rest mass by Lubimov et al. (1980),

$$14\ \mathrm{eV} < m_{\nu e} c^2 < 46\ \mathrm{eV}\,, \tag{1}$$

makes it quite plausible that the sought hidden mass is in fact in the form of massive neutrinos, as it was suggested with considerable foresight by many authors. The conventional view holds instead that the unseen matter is ordinary baryonic matter of low luminosity such as dust, subluminous stars, black holes, rocks, etc.

Gershtein and Zel'dovich (1966) and Cowsik and McClelland (1972) compared the present known cosmic density in baryons with the theoretical cosmic density in neutrinos (see later) and derived an upper limit for the mass of the latter. Later on, Cowsik and McClelland (1973) assumed that massive neutrinos might dominate the gravitational dynamics of large clusters of galaxies and did build on this basis a simple model for the Coma cluster. Szalay and Marx (1976) called attention to the fact that density fluctuations in a primordial neutrino gas may initiate the formation of clusters of galaxies. An early review of neutrino cosmology is given by Bludman (1976) while Markov (1964) calls attention to degenerate massive neutrino superstars.

We will now examine the cosmological impact of the neutrino rest-mass as it has been studied by many authors (Zel'dovich et al. 1980, Bisnovatyi-Kogan et al. 1980, Schramm and Steigman 1980 and 1981, Bond et al. 1980, Klinkhamer and Norman 1981, Sato and Takahara 1981) who have come essentially to similar conclusions; it seems possible that we may solve at the same time the hidden mass problem of section 1 and the gravitational instability problem of section 2. In particular we will hold the view that the condensations of galactic or larger scale started out as massive neutrino condensations at $z \cong 10^4$; only after photon-baryon decoupling, $z \cong 10^3$, were the baryons capable of falling into the neutrino gravitational wells. The possibility that massive neutrinos are distributed like the galaxies is made plausible by the remark that the Universe does not possess a significant smooth component (Yahil et al. 1978).

The neutrinos we have around today in our Universe originated in the Big-Bang (we assume left-handed neutrinos of the Majorana type); at temperature in excess of 1 Mev ($\gg m_\nu$) all neutrinos of the three types (e, μ, τ) were in thermal equilibrium and an extremely relativistic (ER) Fermi distribution was established for each flavor (i)

$$\frac{dn_{\nu i}}{d^3q} = (g_i/h^3)\,[\exp(q\,c/kT) + 1]^{-1}\,. \tag{2}$$

We assume a vanishing chemical potential (Weinberg 1972) and $g_i = 2$ (as it is the case for Majorana neutrinos, while it would be $g_i = 4$ for Dirac neutrinos). As the temperature drops below 1 Mev the weak interaction rate falls below the expansion rate and thermal equilibrium is lost; however since both T and momentum fall like $1/R$, the distribution function (2) remains formally unchanged down to the non-relativistic (NR) region and the present. Clearly T has not the physical meaning of a temperature. By integration of (2) over momentum space we can relate the number density of neutrinos to the number density of photons:

$$n_{\nu i} = \frac{3}{4}\,\frac{1}{2}\,g_i\,n_\gamma = \frac{3}{4}\,n_\gamma\,, \tag{3}$$

where

$$n_{\gamma_0} \cong 400\,(T_{\gamma_0}/2.7\ \mathrm{K})^3\,. \tag{4}$$

As the temperature drops below the electron mass, electron-positron pairs annihilate and generate photons: it is known (Weinberg 1972) that

$$\frac{n_\gamma\,(T < 0.5\ \mathrm{MeV})}{n_\gamma\,(T > 0.5\ \mathrm{MeV})} = \frac{11}{4}\,; \tag{5}$$

hence the photon temperature jumps by a factor $(11/4)^{1/3}$ due to the electron-positron annihilation and remains higher by the same factor during the whole history of the Universe.

Thus the total number density of neutrinos now is given by

$$n_{\nu_0} = \sum_i n_{\nu i_0} = 3 \times \frac{3}{4} \times \frac{4}{11}\,n_{\gamma_0} \cong 300\,(T_{\gamma_0}/2.7\ \mathrm{K})^3\,. \tag{6}$$

Nowadays the neutrinos are non-relativistic; the associated mass density is the sum of their proper masses. Assigning each neutrino flavor the same average mass

$$m_\nu = m_{30}(m_\nu c^2/30\ \mathrm{eV}),$$
$$30\ \mathrm{eV} \cong 5\times 10^{-32}\ \mathrm{g}, \quad (7)$$

we end up with a present density in neutrinos which is very large

$$\rho_{\nu_0} = 2\times 10^{-29} m_{30}\,\mathrm{g\,cm^{-3}}. \quad (8)$$

When we compare this with the critical density (1a.1) we have:

$$\Omega_{\nu_0} = m_{30} h^{-2} \cong 1. \quad (9)$$

Thus there is a valid candidate for hidden matter of the density required in large systems; on the other hand, we must also hold that baryonic matter is scarce: we define here a new parameter

$$\varepsilon = \rho_{B_0}/\rho_{0\,\mathrm{tot}}, \quad (10)$$

which is small.
The cosmological model we want to explore now is one of high density

$$\Omega_{0\,\mathrm{tot}} = \Omega_{\nu_0} + \Omega_{B_0} \cong \Omega_{\nu_0} \cong 1,$$

where ordinary baryonic matter represents only a minor contamination (clearly very important for us!).
One question we must face immediately is whether neutrino rest-masses would affect the standard Big-Bang predictions: the answer given by Shapiro et al. (1980) and Dolgov and Zel'dovich (1981) is negative. Indeed although both left-handed and right-handed neutrinos could be present, right handed neutrinos would not be in equilibrium at 1 Mev, but would have decoupled much earlier ($kT \gg 100$ Mev) according to the Weinberg-Salam-Glashow theory of weak interactions.
Another issue that must be taken up if (9) is valid is whether massive neutrinos affect the theoretical estimates of the age of the Universe in a way that is still consistent with the ages derived from nucleocosmochronology and stellar evolution (see, e.g., Symbalisty et al. 1980). Nucleocosmochronology gives a lower limit of the order 10×10^9 y, which does not pose us any particular problem. However, the ages inferred from globular cluster stars (Iben 1974) are very large (more than 12 billion years) for standard helium abundances and may even exceed 20 billion years as the helium abundance is decreased: we must recall that the Hubble time

$$H_0^{-1} = 10\times 10^9 h^{-1}\,\mathrm{y},$$

is the upper limit to the age of a $\Lambda = 0$ FRW model valid when $\Omega_0 \to 0$; a high density model has a short age, $< 2/3 \times H_0^{-1}$.
The suggestion by Zel'dovich and Sunyaev (1980; see also Luminet and Schneider 1981) is to revitalize the cosmological term because a suitably chosen positive Λ-term, can make the age of the Universe arbitrarily long.
In figure 3 we give some numerical results: from the equation of motion (2b.8) we first evaluate numerically the age

$$t_0 = \int_0^{t_0} dt = \int_0^{R_0} dR/\dot{R},$$

as a function of the pair (σ_0, q_0) and then we plot on the (σ_0, q_0)-plane isochrones, loci of the points of the same age. We are interested in exemplificative ages of 12, 14, 16 and 18 billion years; the corresponding curves are conveniently parametrized by the dimensionless number $H_0 t_0$ which assumes the two sets of values a) 0.6, 0.7, 0.8 and 0.9 for $H_0 = 50$ km/s/Mpc and b) 1.2, 1.4, 1.6 and 1.8 for $H_0 = 100$ km/s/Mpc.
According to (9) Ω_0 ranges between 4 and 1 and σ_0 ranges between 2 and 0.5: in order to find a value for the Λ-term all we have to do is to find a value for q_0, (2b.26), by the intersections of the vertical lines $\sigma_0 = 2$ (labelled 50 to remind us the Hubble constant) and $\sigma_0 = 0.5$ (labelled 100) with the ages curves. These intersections occur at negative values for q_0, $\cong -2$. For a not unlikely intermediate value of H_0 (see, e.g., Van der Bergh 1981) and from (9), $\Omega_0 \cong 1$ for $m_{30} \cong 0.5$: to get an age $= 13$ billion years ($\cong H_0^{-1}$) we should look in figure 3 at the intersections between the vertical line $\sigma_0 = 0.5$ and the curve $H_0 t_0 = 1$ which has not been drawn to avoid further crowding of figure 3. Again the intersection yields a negative q_0, $\cong -1.5$. As before this implies that the Universe expansion is accelerating which formally calls for a positive Λ, (2b.7).

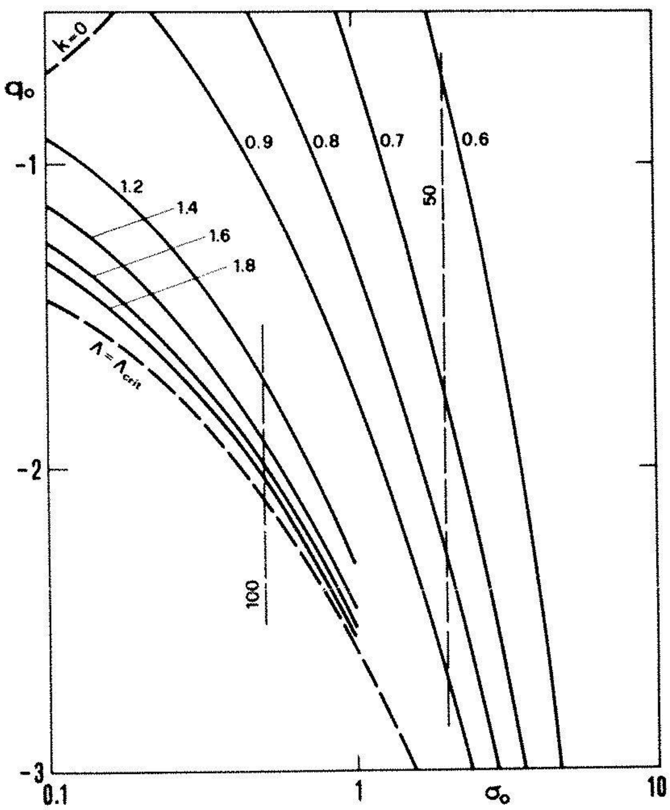


Fig. 3. Plot on the (σ_0, q_0)-plane of isochrones, loci of points where the cosmological models have the same age. We consider ages of 12, 14, 16 and 18 billion years. We label the curves on the graph by the values of $H_0 t_0$: for $H_0 = 50$ km/s/Mpc we have a first set of values $H_0 t_0 = 0.6$, 0.7, 0.8 and 0.9 and of corresponding curves; for $H_0 = 100$ km/s/Mpc we have a second set of values $H_0 t_0 = 1.2$, 1.4, 1.6 and 1.8 and of corresponding curves. Assuming $m_{30} = 1$, according to (3a.9) we have $\sigma_0 = 0.5$ for $h = 1$ or $\sigma_0 = 2$ for $h = 0.5$. The intersections of the vertical lines $\sigma_0 = 0.5$ (labelled 100) and $\sigma_0 = 2$ (labelled 50) with the corresponding age curves yield the values for q_0: in all cases negative value of q_0 are found (of the order -1 or -2). An acceleration of the cosmic expansion is implied, which means formally $\Lambda > 0$.

In the history of cosmology the Λ-term has seen many ups and downs (for a review see Petrosian 1974, and, more recently, Gunn and Tinsley 1975 and Tinsley 1977); from figure 3 we see that we are now far from the $\Lambda = \Lambda_{crit}$ curve which seemed interesting some years ago due to an apparent accumulation of quasar redshifts around 2 (the issue has now disappeared; Tytler 1981). Our difficulties with Λ stem from our failure to understand its physics, aside from an attempt by Zel'dovich (1968) to relate Λ to the quantum fluctuations of vacuum.

b) The Infall of Baryons onto Massive Neutrino Condensations

The growth of neutrino density fluctuations is of fundamental importance for the formation of the structure we observe in the Universe. To understand this we must remark that the growth of baryon fluctuations is inhibited by photon viscosity until $z_{dec} = 10^3$, while neutrinos decouple from equilibrium at $T = 1$ Mev and are collisionless ever since. This collisionless feature deserves some attention: the approaches to gravitational instability by Jeans, Lifshitz and Bonnor were all based on the hydrodynamic description of matter, that is on the assumption that the mean free path between particle collisions is small in comparison with any characteristic length of the problem. When we deal with neutrinos the opposite is true and we must resort instead to the distribution function and the evolution of its perturbations: we will quote here the results of the pioneering work of Gilbert (1966).

Under the Newtonian approximation, one describes the uniform cosmic dust by a collisionless Boltzmann equation, superposes the cosmic expansion and introduces a small perturbation in the distribution function.

The Fourier transform of the density contrast obeys a Volterra integral equation, which Gilbert (1966) studies numerically. He finds that the large wavelength density contrasts grow monotonically under self-gravitation and that the small wavelength density contrasts do not oscillate like sound waves, but first decrease due to Landau damping and eventually grow too. The separation between large and small wavelengths is given by a Jeans wavelength

$$\lambda_J = [\pi < v^2 > / 3 G \rho]^{1/2} \qquad (1)$$

which is built with a characteristic mean square particle velocity rather than with the speed of sound as (2a.5). The reason why small wavelength modes increase again after an initial Landau damping is that in (1) v^2 behaves as R^{-2} while ρ behaves as R^{-3}; the Jeans length thus increases as $R^{1/2}$ while the wavelength of any perturbation increases as R and eventually overtakes λ_J.

Stewart (1972) also adopts a kinetic theory rather than a hydrodynamical approach with the aim of studying the evolution of condensations of collisionless and massless neutrinos in a homogeneous isotropic FRW Universe. After generalizing the formalism to deal correctly with strong gravitational fields, he confirms the qualitative picture that emerges from the Newtonian analysis and in particular the Landau damping (see also Lynden-Bell 1967) of the short wavelength modes.

We must therefore compute the Jeans mass of the massive neutrinos: let us first set (somewhat conventionally) at

$$1 + z_{NR} \cong \frac{m_\nu c^2}{3(4/11)^{1/3} k T_{\gamma_0}} \cong 6 \times 10^4 m_{30}, \quad (2)$$

the redshift at which the neutrinos become non-relativistic in their adiabatic cooling. For $z \gg z_{NR}$ the neutrinos are considered to be extremely relativistic (ER), while they are considered to be fully non-relativistic (NR) for $z \ll z_{NR}$; strictly speaking in the middle neither approximation is true, but either is satisfactory for the purpose of obtaining order of magnitude estimates. The Jeans mass is a qualitative concept any way.

From (2.7) we have

$$M_J = \frac{\pi}{6} n m_\nu \lambda_J^3, \quad (3)$$

where λ_J is given in (1) and n is related to the distribution function by the generalization of (a.6) above:

$$n(z) \cong 300 \left(\frac{T_{\gamma_0}}{2.7 K}\right)^3 (1+z)^3. \quad (4)$$

For the ER regime, $v = c$, the Jeans mass coincides with the horizon mass and we find

$$M_{JER} \cong \frac{1}{16} m_p \left(\frac{m_p}{m_\nu}\right)^2 \left(\frac{m_\nu c^2}{k T_\gamma}\right)^3, \quad (5)$$

where

$$m_p = \left(\frac{\hbar c}{G}\right)^{1/2} \cong 2 \times 10^{-5} g,$$

is the Planck mass; this expression appears as a generalization of that found by Bisnovatyi-Kogan et al. (1980), which is interesting because it is constructed in terms of fundamental constants. Alternatively we may write for present purposes

$$M_{JER} \cong 2 \times 10^{29} \frac{m_{30}}{(1+z)^3} M_\odot; \quad (6)$$

this is the growing straight solid line in figure 4 which should extend only to $z = 6 \times 10^4 m_{30}$ but is used also to slightly lower redshifts.

For the NR regime, we find

$$< v^2 >^{1/2} \cong 6 \frac{1+z}{m_{30}} \text{ km/s}, \quad (7)$$

and in place of (5)

$$M_{JNR} \cong 20 m_p \left(\frac{m_p}{m_\nu}\right)^2 \left(\frac{k T_\gamma}{m_\nu c^2}\right)^{3/2}; \quad (8)$$

alternatively we write this as

$$M_{JNR} \cong 8 \times 10^8 \frac{(1+z)^{3/2}}{m_{30}^{7/2}} M_\odot, \quad (9)$$

which we plot as the decreasing straight solid line in figure 4. The two lines meet at a redshift

$$z_{max} \cong 3.5 \times 10^4 m_{30}, \quad (10)$$

where the Jeans mass attains its maximum

$$M_{J\,max} \cong 5.5 \times 10^{15} m_{30}^{-2}. \quad (11)$$

Strictly speaking this is a slight overestimate of the true value, which would result from a better approximation for the intermediate zone. As many authors have remarked it is especially gratifying to cosmology that (11) gives the order of magnitude of the mass of a cluster of a galaxies; in the scenario we are presenting here, however, the mass in baryons must be smaller than (11) by a factor ε, which may mean even two orders of magnitude.

Condensations on mass scales larger than M_{J_ν} can grow for $z > z_{NR}$, but not if they involve only neutrinos for an argument used in section 2a. Indeed the Universe is radiation dominated from the epoch when the neutrinos become non-relativistic, $z = z_{NR}$, until

$$1 + z_{EQ} \cong 4 \times 10^4 m_{30}, \quad (12)$$

(compare with (2b.27) which applies to baryonic matter-radiation equivalence), whereafter neutrinos take over and the Universe becomes dust dominated. During this phase, $z_{NR} > z > z_{EQ}$, a density perturbation involv-

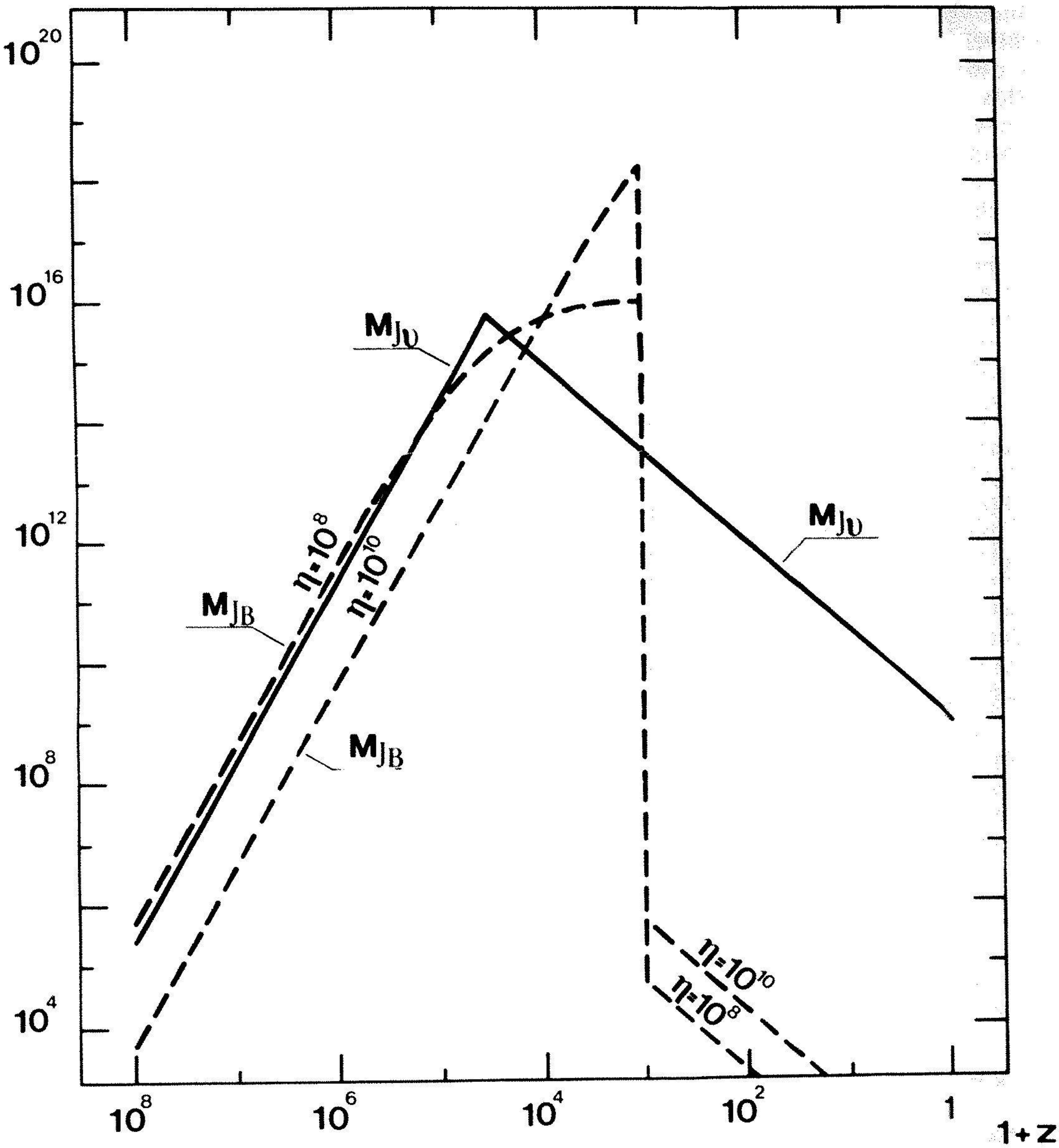


Fig. 4. Plot of the Jeans mass in solar masses in a massive neutrino Universe vs. redshift for $m_{30} = 1$. In the extreme relativistic regime (ER) the Jeans mass grows with time as shown by the straight solid line to the left, i.e. according to (3b.6), this expression being valid only for $z > z_{NR} = 6 \times 10^4$; in the non-relativistic (NR) region the Jeans mass decreases as shown by the straight solid line to the right, i.e. according to (3b.9). The intersection of the two straight lines gives an indication – perhaps slightly in excess – of the maximum of the Jeans mass: this occurs at

$$z_{max} \cong 3.5 \times 10^4 ,$$

which is $\gtrsim z_{NR}$ and close to the value z_{EQ} when the Universe starts being dominated by neutrinos rather than by radiation and the neutrino perturbations can start to grow. The maximum Jeans mass is of the order

$$M_{J\,max} \cong 5 \times 10^{15} \frac{1}{m_{30}^2} M_{\odot} .$$

Also shown by broken lines are the Jeans masses for a canonical baryon Universe evaluated with the formulae given in the section 2 for two values of the photon-to-baryon ratio, $\eta = 10^{10}$ and 10^8. The vertical drop at $z = 1000$ is a consequence of the fall of the sound speed due to hydrogen formation, assumed to occur instantaneously.

ing only the dust component cannot grow, but continues to expand with the substratum (Mészàros 1974).

For the Silk mass in baryons in a baryon-neutrino Universe, generalizing (2a.17), we have (Bond et al. 1980)

$$M_D \cong 10^{13} \Omega_{B_0}^{-1/2} \Omega_{\nu_0}^{-3/4} h^{-5/2} M_\odot ; \tag{13}$$

again in a qualitative sense, the latter reminds the correct range of masses.

In order to study the gravitational coupling between the baryon and the neutrino components we will limit ourselves here for semplicity to mass scales $\gtrsim M_{J\nu max}$ (which is the most interesting range due to the likely decrease of the fluctuation spectrum with mass) and to epoch $z < z_{max} \cong z_{EQ}$; by itself the baryonic content of such a perturbation ($\sim \varepsilon/(1-\varepsilon) M_{J\nu max}$) would undergo damped oscillations. The formalism of section 2b yields directly the differential equations for

$$\delta_\nu = \left(\frac{\delta\rho}{\rho}\right)_\nu , \qquad \delta_B = \left(\frac{\delta\rho}{\rho}\right)_B . \tag{13}$$

The physics we want to impose to our model is that δ_ν can grow in the interval $z_{EQ} > z > z_{dec} = 10^3$, where instead $\delta_B = 0$ due to photon viscosity:

$$\delta_\nu \propto t^{a_\pm} ,$$

$$a_\pm = \frac{1}{6} \left[-1 \pm \sqrt{25 - 24\varepsilon} \right] . \tag{14}$$

For $z < z_{dec}$, we have (Doroshkevich et al. 1980, Bond et al. 1980, Wasserman 1981)

$$\frac{1}{R^2} \frac{d}{dt} R^2 \frac{d}{dt} \delta_B - 4\pi G \rho \varepsilon \delta_B$$
$$= 4\pi G \rho (1-\varepsilon) \delta_\nu , \tag{15}$$

$$\frac{1}{R^2} \frac{d}{dt} R^2 \frac{d}{dt} \delta_\nu - 4\pi G \rho (1-\varepsilon) \delta_\nu$$
$$= 4\pi G \rho \varepsilon \delta_B . \tag{16}$$

These are valid in all generality for any pair of σ_0 and q_0. The power solution (14) is obtained from (16) setting $\delta_B = 0$ and using the Einstein-de Sitter approximation valid for high redshift.

The interesting remark that has been made is that, due to the inhomogeneous nature of (15), baryon perturbations can grow pulled into the neutrino gravitational wells even if $\delta_B = \dot{\delta}_B = 0$ at z_{dec} (Doroshkevich et al. 1980, Bond et al. 1980, Wasserman 1981). Therefore even if all the baryon perturbations have been damped by photon viscosity, we only need postulate an initial spectrum of neutrino perturbations earlier on at z_{NR} or $z_{max} \cong z_{EQ}$. An idea of the solution of (15) and (16) can be obtained in the limit $\varepsilon \to 0$ and for large redshifts (Einstein-de Sitter approximation): indicating with a subscript "1" a reference epoch when $\delta_B = \dot{\delta}_B = 0$ around z_{dec}, one has

$$\delta_\nu = \Delta_\nu (t/t_1)^{2/3} ,$$
$$\delta_B = \Delta_\nu \left[(t/t_1)^{2/3} + 2 (t_1/t)^{1/3} - 3 \right] , \tag{17}$$

where for simplicity only the growing mode of δ_ν has been considered.

From (17) we draw from the conclusion that δ_B and δ_ν lock together and attain the same values at the present time; it also appears that the baryon density amplification defined as

$$A = \delta_B (1+z=1) / \delta_B (1+z_1) , \tag{18}$$

is formally infinite. We are rather interested in using in the denominator of (18) the redshift of the later epoch when baryons last interact with the radiation background: let this be

$$1+z = (1+z_1) - \Delta z , \qquad \Delta z \ll (1+z_1) .$$

Then from (17) we find

$$\delta_B (1+z) = \frac{3}{4} \Delta_\nu \left(\frac{\Delta z}{1+z_1} \right)^2 , \tag{19}$$

which tells us that for $1+z_1 = 1000$, $\Delta z = 100$, $\delta_B (900) \cong 10^{-2} \Delta_\nu$. Thus an amplification of physical interest

$$A = \delta_B (1+z=1) / \delta_B (1+z)$$
$$\cong \delta_\nu (1) / (10^{-2} \Delta_\nu) , \tag{20}$$

would result larger than the amplification of the neutrino modes by a couple of orders of magnitudes.

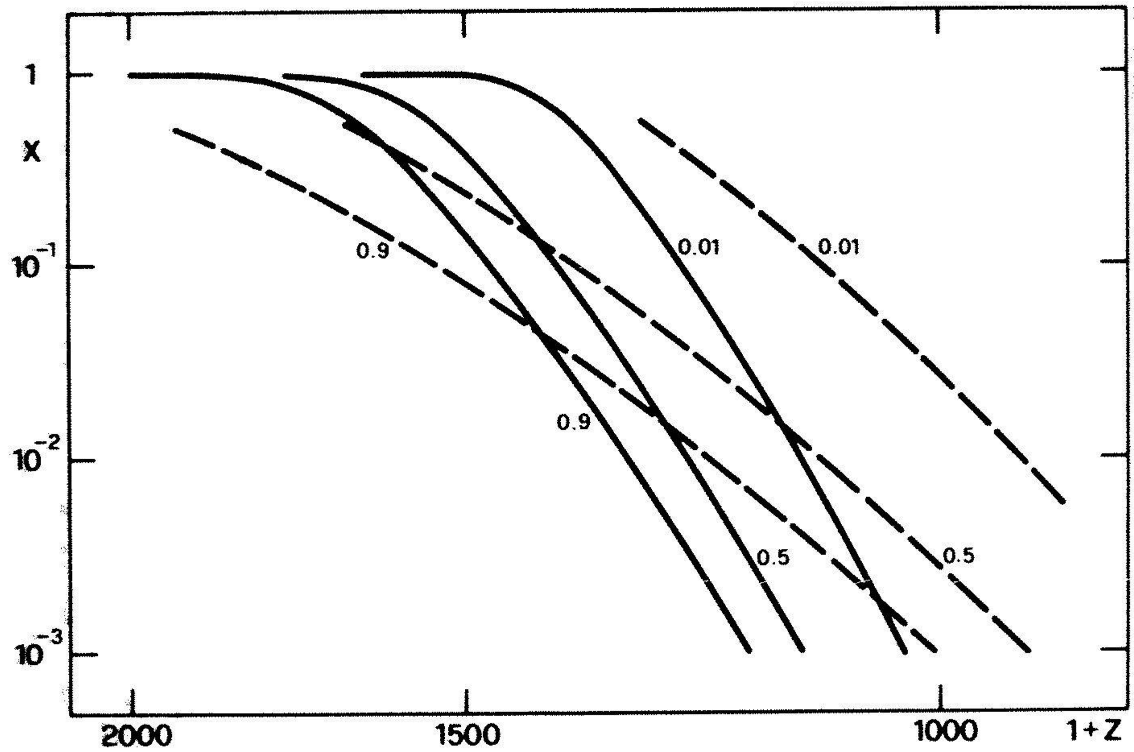


Fig. 5. Plot of the ionization fraction

$$x = \frac{n_p}{n_p + n_H},$$

vs. redshift for various values of ε, defined in (3a.10). Full lines are evaluated via the Saha equation (e.g. Peebles 1971). As expected, the smaller ε, the later the ionization drop occurs: for $\varepsilon = 0.01$, x drops very rapidly only for $z \gtrsim z_{dec} = 1000$. This is the basis for the approximation used in the text to let the baryons fall freely in the interval $z_{dec} + \Delta z > z > z_{dec}$, $\Delta z = 100$. For comparison, for the same values of ε also the approximate formula by Sunyaev and Zel'dovich (1970 and 1980) is shown by broken lines: in this case ionization lasts much longer than indicated by the Saha equation.

In fact, decoupling between baryons and photons is not an instantaneous process but lasts a certain time. The corresponding thickness in redshift is of the order of several hundreds as we see in figure 5 where we plot the degree of ionization vs. redshift as a function of ε evaluated either via the Saha equation (see, e.g., Peebles 1971) or via a more elaborate treatment due to Sunyaev and Zel'dovich (1970 and 1980).

Although during this phase the coupling between baryons and photons is described correctly only by the method of Peebles and Yu (1970; for more recent work see Silk and Wilson 1980), an order of magnitude information can be extracted from (15) and (16) as well. For this purpose we assume that baryons start falling freely, i.e. obeying (15) and (16), at $z_1 = z_{dec} + \Delta z, \Delta z = 100$ with $\delta_B(z_1) = \dot{\delta}_B(z_1) = 0$. We then place conventionally at $z_{dec} = 10^3$ the end of any interaction between baryons and photons having in mind to evaluate an upper limit to the perturbations on the microwave background via

$$\left(\frac{\delta T}{T}\right)_{dec,max} = \frac{1}{3}\,\delta_B(z_{dec}). \tag{21}$$

We have integrated numerically (15) and (16) on the $(\sigma_0 - q_0)$ plane and for various values of ε with the purpose of understanding what sort of modifications have been generated with respect to the results of figure 2 by the two fluid nature of the background model.

In figure 6 we study high density models: full lines give the growth of δ_B (normalized to unity at the present) vs. redshift, broken lines give the growth of δ_ν. Broken and full lines merge together. For ε we consider a very low value $\varepsilon = 0.01$, and an intermediate one, $\varepsilon = 0.5$; it turns out that the dependence on ε is minor in the range of physical interest (which excludes $\varepsilon \to 1$). For $\varepsilon = 0.01$ the first

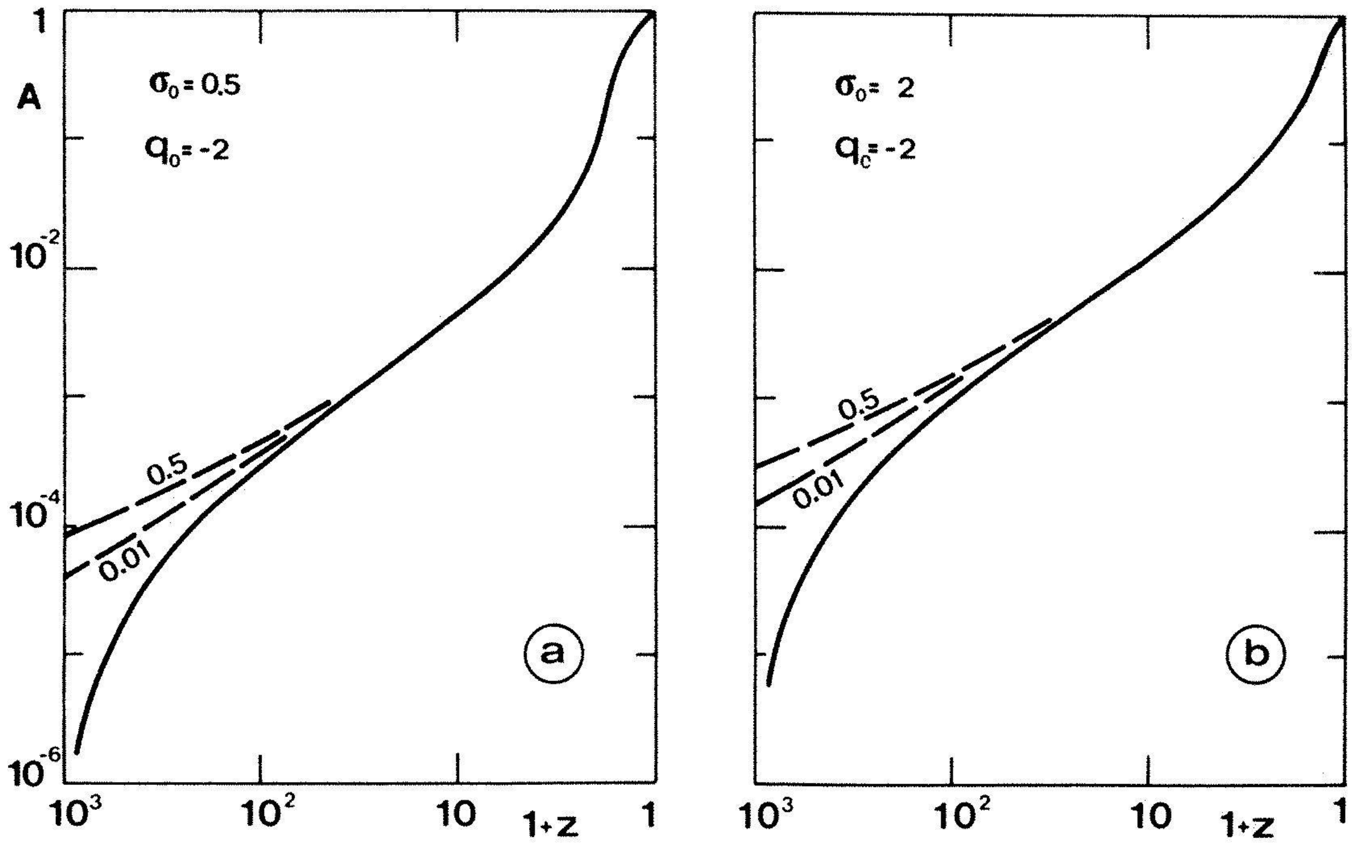


Fig. 6. Amplification of the growing modes of baryons and neutrinos in high-density Universe models: $\sigma_0 = 0.5$, $q_0 = -2$ in a); $\sigma_0 = 2$, $q_0 = -2$ in b). Full lines refer to baryons, dashed lines to neutrinos; we show the cases $\varepsilon = 0.01$ and $\varepsilon = 0.5$. When baryon density contrasts are normalized to unity at the present, the baryon curves are undistinguishable from each other, so that we plot only one of them. This means that the total amplification available to baryons is not very sensitive to the actual value of ε, as long as the latter is $\varepsilon \ll 1$, but depends mainly on the nature of model, i.e. the coexistence of two self-gravitating fluids, one of which is able to begin its gravitational condensation very early, at $z \cong z_{EQ} \cong 10^4 > z_{dec} = 1000$. Baryon fluctuations in high density models are shown to amplify by six orders of magnitude. As far as the neutrinos are concerned instead, their amplification depends strongly on ε; for small ε the baryons do not matter and neutrino fluctuations amplify as in (2b.20). On the contrary for larger ε neutrino self-gravity is weak and neutrino fluctuations grow little for $z_{EQ} > z > z_{dec}$.

part of the neutrino growth is virtually that of an Einstein-de Sitter model (2b.20); for lower z the neutrino growth rate goes to what is expected from figure 2. For $\varepsilon = 0.5$ the neutrino condensations build up very slowly in the beginning because 50% of the total matter content (all the baryons) is unable to condense before z_{dec}; later on, the neutrino and the baryon growth curve are undistinguishable from each other and from the curves valid for $\varepsilon = 0.01$. We have in fact drawn only one curve for the baryons.

The exit from linearity for the baryons and the neutrinos occurs simultaneously; the amplification available to δ_B is of six orders of magnitude.

In Figure 7a we examine the paradigmatic Einstein-de Sitter case, where δ_B amplifies by five orders of magnitude between z_{dec} and the present - rather than three as in the pure baryon model - due to the presence of the neutrinos, provided they are a major component of the total density ($\varepsilon < 0.5$). In figure 7b we show that this is basically true even in a low density Universe, though to a lesser extent than in high density models of figure 6.

The dependence of our results from ε is shown in figure 8 where the exemplificative models of figures 6 and 7 are studied in the range $0 < \varepsilon < 1$: we plot the baryon and the neutrino amplification vs. ε and we consider the two cases, that the exit from linearity occurs a) at the present, $1+z=1$, or b) at $1+z=5$. For baryons the dependence on ε is insignificant, for neutrinos it becomes relevant only for $\varepsilon \to 1$, which is not interesting. When we assume that linear growth applies all the way to the present, then the total amplification depends to a certain

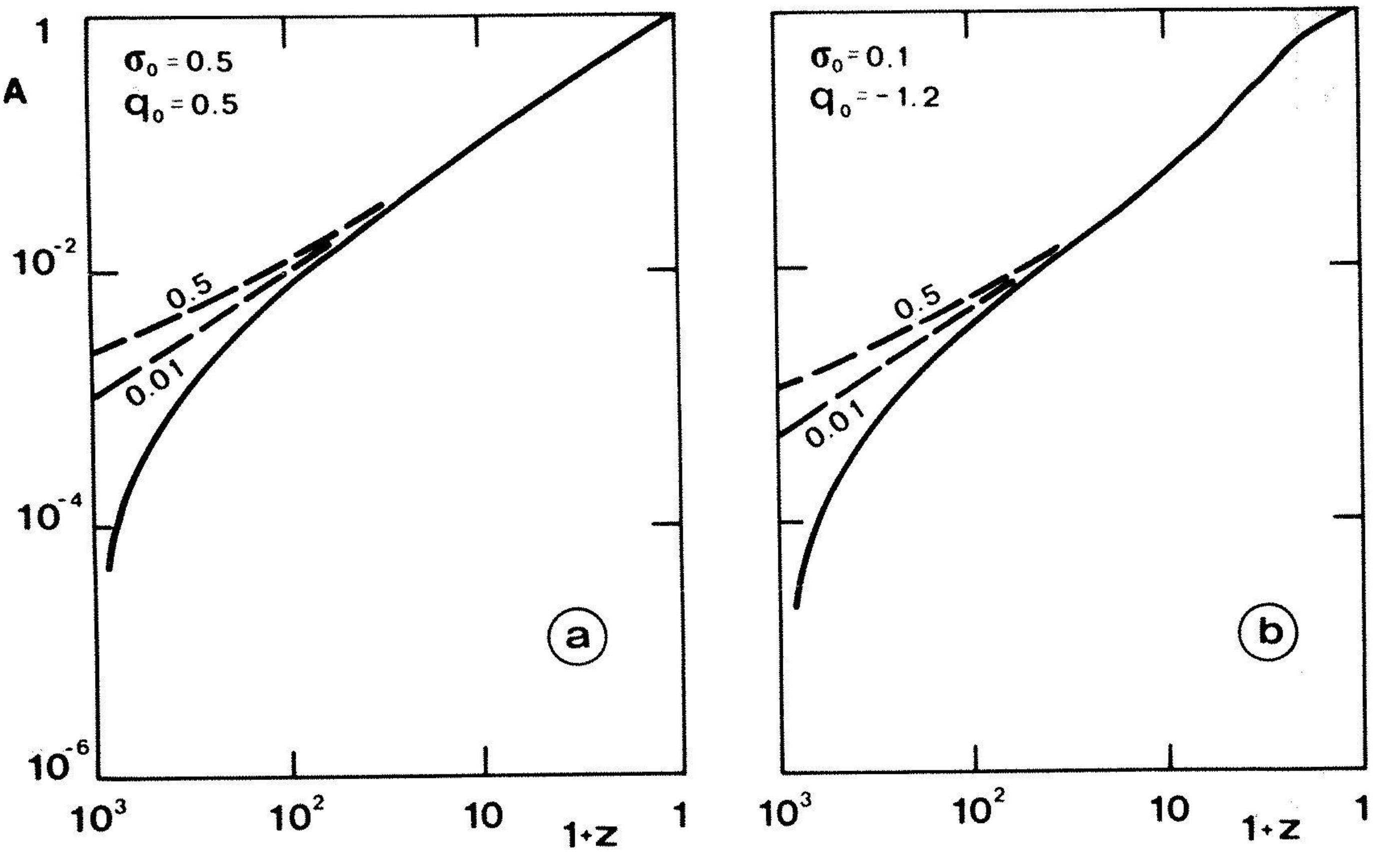


Fig. 7. The same as in figure 6 for the Einstein-de Sitter case in a) and for a low density model in b); in the latter case q_0, taken from the age curves of figure 3, yields an age of the order of 15 billion years. Basically the same comments of figure 6 apply: due to the early locking of the baryon to the neutrino fluctuation, even when the total density is low the amplification of the baryon fluctuations exceeds four orders of magnitude, thus justifying the lack of detection in the microwave background of the condensation's footprints.

extent on the actual values of σ_0 and q_0; but when non-linearity is attained earlier at $1+z=5$, which is physically more interesting, the amplifications available are largely independent of σ_0 and q_0 (since the linear growth is interrupted during the Einstein-de Sitter phase).

Depending on whether the condition $\delta_\nu = 1$ is reached at $1+z=1$ or at $1+z=5$, we may evaluate $\delta_\nu(z_{EQ})$ by combining the growth resulting from the numerical integration of (15) and (16) with the initial growth given by (14) between z_{EQ} and z_{dec}. In figure 9 we plot $\delta_\nu(z_{EQ})$ vs. ε: in the interesting region $\varepsilon \ll 1$, the initial neutrino amplitudes are below 10^{-4}.

In figure 10 we present our results for the amplifications at the present on the whole $(\sigma_0 - q_0)$ plane for $\varepsilon = 0.01$; the level curves we obtain there should be compared with those of figure 2, where the case of a one-fluid Universe is examined. A couple of orders of magnitude are gained everywhere; when we translate this in upper limits for the microwave background temperature fluctuations at z_{dec} according to (17), which we plot in figure 11, we see that we are many orders of magnitude below the present detectability. Even for $\delta_B(1+z=5)=1$, we read from figure 8 that $(\delta T/T)_{dec} < \delta_B(z_{dec})/3 \cong 10^{-5}$.

As we have seen baryon-neutrino, high-density cosmological models hold great potentialities for the linear growth of baryon density enhancements and make less remote the understanding of the condensed structures we see. Progress has not been equally fast in the realm of non-linear condensations, but some results in agreement with the above considerations are already available.

The study of the (non-linear) evolution of spherically symmetric condensations may be of interest for modelling clusters of galaxies (Occhionero et al. 1981a and b, and references therein). During the formation of such a condensation the matter which accumulates at the center is swept away from the space around the condensation itself, so

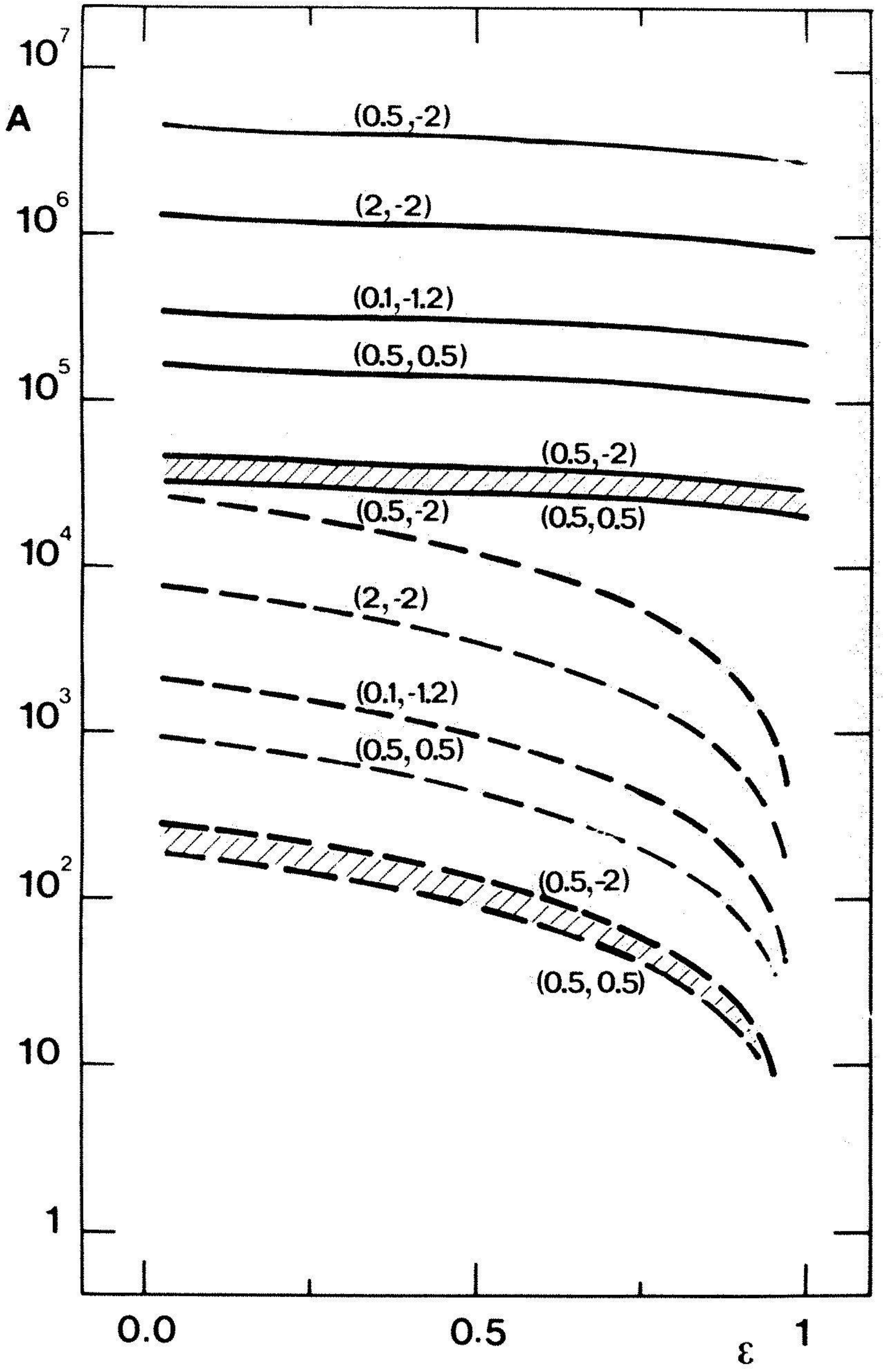
Fig. 8. Plot of the baryon and of the neutrino mode amplification vs. ε; full lines refer to baryons, broken lines to neutrinos. In one set of curves, exit from linearity is assumed to occur at the present and the amplifications are defined as

$\delta_B(1)/\delta_B(1000)$,
$\delta_\nu(1)/\delta_\nu(1000)$;

in this case each curve is labelled by the σ_0 and q_0 to which it refers. In the second set of curves, exit from linearity is assumed to occur at $1+z=5$; in this case the amplifications are defined instead as

$\delta_B(5)/\delta_B(1000)$,
$\delta_\nu(5)/\delta_\nu(1000)$,

and their spread is confined within the dashed regions bounded by the indicated pairs of σ_0 and q_0.



that two things occur simultaneously: a density enhancement at the center and a density deficit in a spherical concentric shell. For such an empty shell a theoretical dimension is given by

$$L = 10(\Omega_0 h^2)^{-1/3} m_{15}^{1/3}\,\mathrm{Mpc}, \qquad (22)$$

where m_{15} is the mass in units of $10^{15}\,M_\odot$. This must be confronted with the observations which call for $L=50$ Mpc (Kirschner et al. 1981).

A low density solution, $\Omega_0 h^2 = 0.01$, is certainly possible; it requires a very strong initial density contrast because the binding condition (Sunyaev 1971) is

$$\left(\frac{\delta\rho}{\rho}\right)_1 \gtrsim \frac{1}{\Omega_0}\,\frac{1}{1+z_1}, \qquad (23)$$

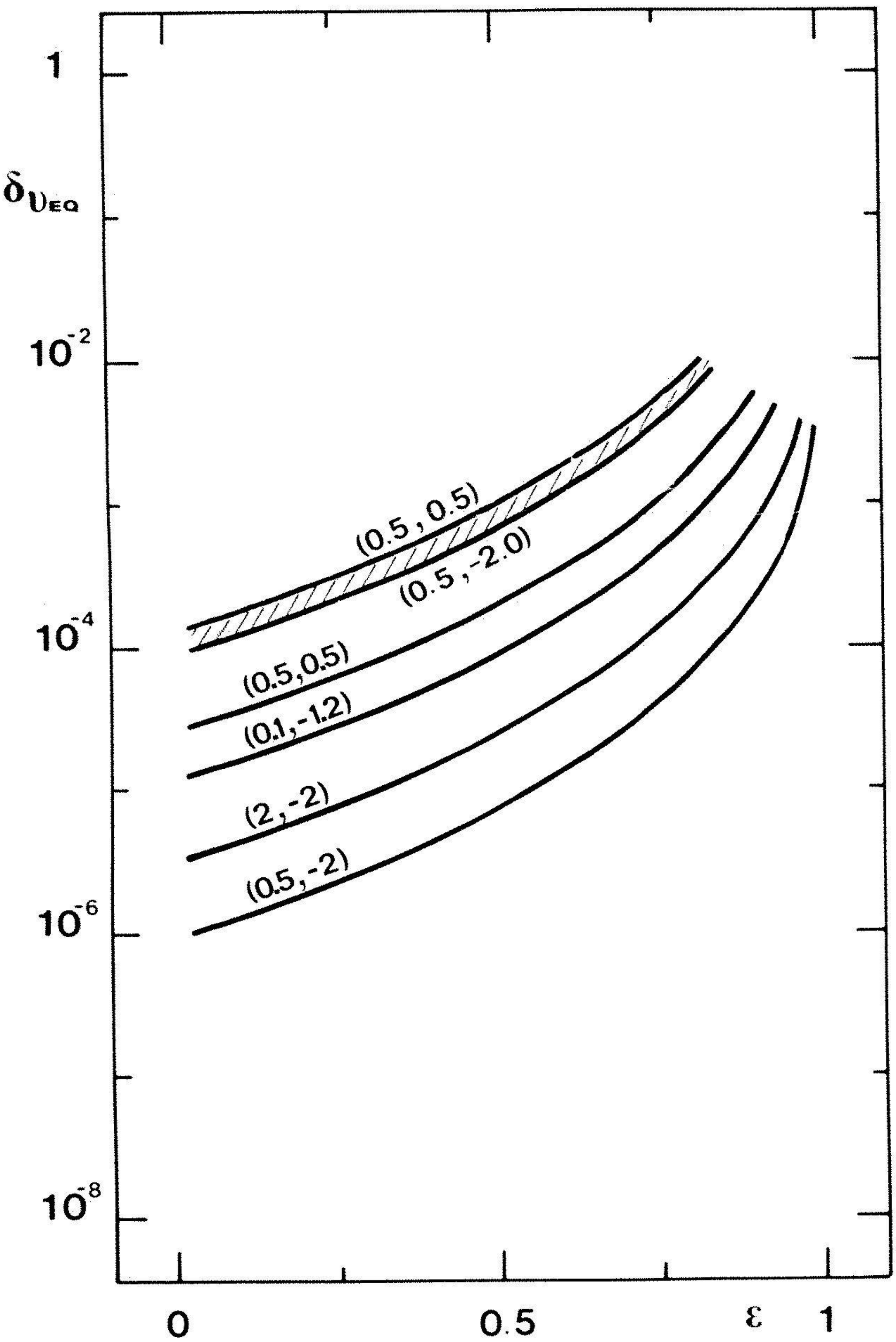


Fig. 9. Amplitude of the neutrino fluctuation at z_{EQ} when exit from linearity occurs at the present (the four lower curves) and when exit from linearity occurs at $1+z=5$ (curves within the dashed region). Labels define the pairs $\sigma_0 - q_0$ of the unperturbed cosmological models. For $\varepsilon \ll 1$ these initial amplitudes of the order of 10^{-4}, as in Zel'dovich's assumption, are enough to guarantee entrance into the non-linear growth well before the present; on the other hand, when $\varepsilon \lesssim 1$, larger initial amplitudes are required because neutrino self-gravity is weaker.

where z_1 may be assumed again to be 1000. This implies an excess binding energy per unit mass

$$b = \left(\frac{\delta\rho}{\rho}\right)_1 = \frac{\delta W}{W}, \tag{24}$$

for which (23) translates into

$$B = b(1+z_1) \gtrsim \frac{1}{\Omega_0} = 10^2.$$

In a high density cosmological model, $\Omega_0 h^2 \simeq 1$, the dimension of the cavity is right if one assumes $m_{15} = 100$ or $10^{17}\ M_\odot$ in such a condensation. This mass cannot be in baryons and may be in massive neutrinos; when spread over a sphere of 50 Mpc, it amounts to the density of 2×10^{-29} g/cm^3, in agreement with (a.8). From an energetic point of view the binding condition is much weaker than (23) and may be formulated as

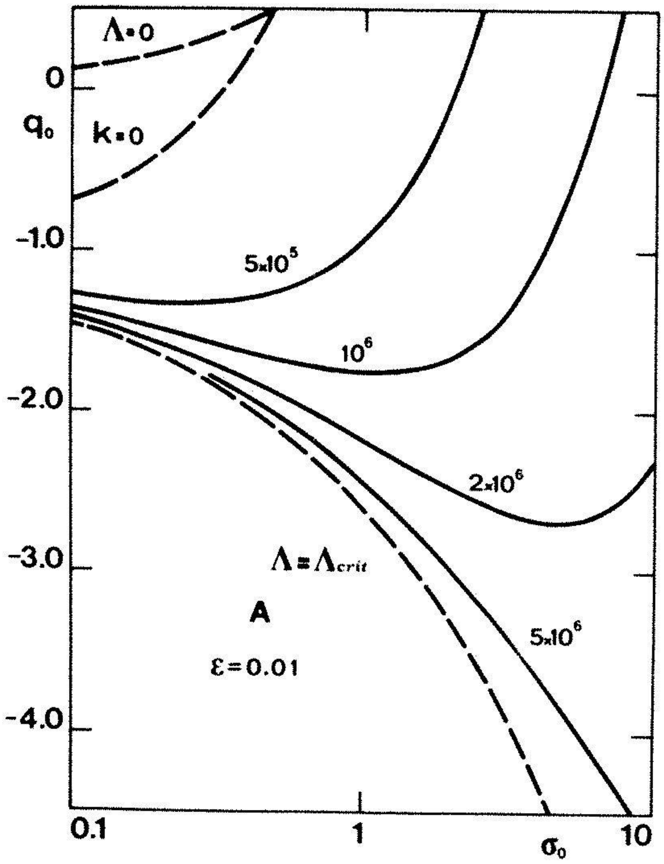


Fig. 10. Level curves of the amplification of the baryon growing modes in a baryon-neutrino Universe with $\varepsilon = 0.01$. The results given here must be compared with those given in figure 2: broken lines have the same meaning; the label on each solid line is the amplification evaluated again as

$$A = \delta_B(1)/\delta_B(1000);$$

it is recalled that $\delta_B(1100) = \dot{\delta}_B(1100) = 0$. The qualitative trend of the solid lines here is the same as in figure 2; from a quantitative point of view, however, there is a gain of two orders of magnitude due to the gravitational coupling between the two fluids.

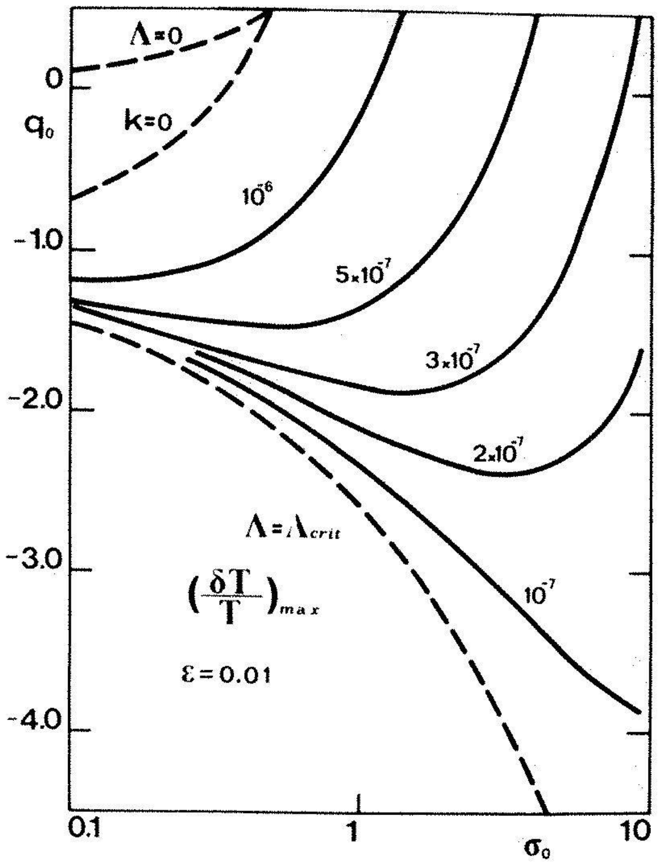


Fig. 11. Level curves for an upper limit on the temperature fluctuations in the microwave background evaluated under the adiabatic assumption given in (3b.21). As in figure 10, $\varepsilon = 0.01$. Labels on each curve define the expected $(\delta T/T)_{max}$ for perturbations that enter non-linearity only at the present. If more realistically we assume that this occurs at a redshift of the order of 5 or 10, the linear growth is reduced by not more than two orders of magnitude; the expected temperature fluctuations, increased by the same amount, may remain under the present detectability.

$$B \geq B_{crit} = \{3\,[\sigma_0^2(\sigma_0 - q_0)]^{1/3} - (3\,\sigma_0 - q_0 - 1)\}/(2\,\sigma_0)\,.$$

For example for $\sigma_0 = 0.5$, $q_0 = -2$, $B_{crit} = 0.065$ and for $\sigma_0 = 2$, $q_0 = -2$, $B_{crit} = 0.140$. We give in figure 12 density profiles for spherical condensations developing from $1 + z = 1000$ with $B = B_{crit}$ in high density cosmological models. We underline that the structure of each condensation is fully non-linear by the present. By contrast in the standard open model, $\Omega_0 = 0.01$, the choice $B = 0.1$ implies $(\delta\rho/\rho)_l = B/(1 + z_l) = 10^{-4}$ and a linear growth only by a factor 10. In these conditions the density excess at the center would amount to an inconspicuous 10^{-3}; of the same order of magnitude would result the depth of the surrounding hole.

Abstract

Several authors have pointed out that massive neutrino condensations may trigger the formation of baryonic matter condensations in cosmology, probably on the scale of clusters of galaxies. We review their work and we give new results on the linear growth of baryon condensations from decoupling onwards under the influence of self-gravitation and the gravitational coupling to pre-

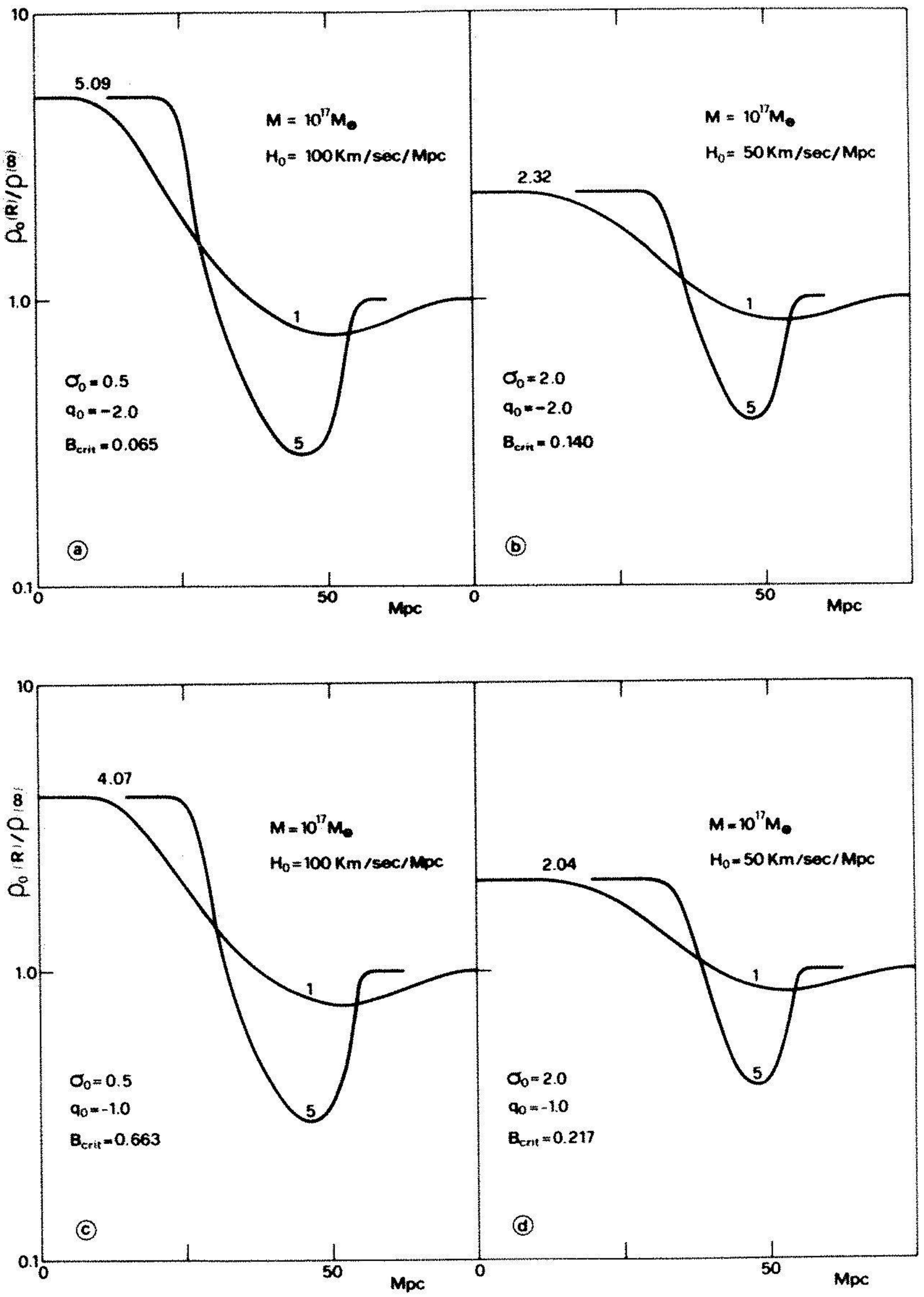


Fig. 12. Plot of the present density profiles in units of the asymptotic density vs. radius in spherical condensations developing in high density cosmological models. The mean density is assumed to be 2×10^{-29} g/cm^3; this translates in $\sigma_0 = 0.5$ for $H_0 = 100$ km/s/Mpc (a and c) and $\sigma_0 = 2.0$ for $H_0 = 50$ km/s/Mpc (b and d). An unusually large total mass of 10^{17} $M_\odot$ is involved, mostly in neutrinos with only a small fraction ε in baryons. Each condensation is marginally bound at the center ($B = B_{crit}$). The other details of the energetics are specified in the references quoted in the text; thus the labels 1 and 5 attached to the curves define an integer n with which we parametrize our models. The evolution is fully nonlinear; cavities are shown to develop around each condensation with dimensions of the order of 50 Mpc.

existing neutrino condensations. We parametrize our work by the ratio of the present density in baryons to the present total density; such a number is likely to be small. We also allow for a positive cosmological constant, which – as it has been suggested – may be needed if the cosmic density in neutrinos is around the closure value.

As it was already known, we find that the fractional baryon density enhancements reach quickly the level of the fractional neutrino density enhancements and remain locked to the latter thereafter. Secondly, in agreement with previous work of ours, we find that at low redshift the linear growth of the condensations (of either component) is stronger in that region of the parameter space where curvature is positive and the cosmological constant exceeds the critical value. If this really applies to our Universe, the latter argument may further justify the lack of detection of small scale fluctuations in the microwave background or, at least, help push down their theoretical upper limit.

Finally we give some results on the formation of non-linear condensations with spherical symmetry; the motivation for this work lies in the observation of large scale voids (linear dimensions of the order of 100 Mpc). A high density Universe is again preferred because in a low density model similar condensations would not have reached non-linear growth.

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