# Thermal expansion, compressibility and volumetric changes of quartz obtained by single crystal dilatometry to 700 °C and 3.5 kilobar (0.35 GPa)

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# Thermal expansion, compressibility and volumetric changes of quartz obtained by single crystal dilatometry to 700 °C and 3.5 kilobar (0.35 GPa)

by *Urs Raz*[1,2], *Sven Girsperger*[1] and *Alan Bruce Thompson*[1]

**Abstract**

A dilatometer has been developed to measure volumetric changes of oriented single crystals simultaneously as a function of pressure and temperature to 0.6 GPa and 900 °C. Argon gas is used as the pressure medium inside an externally heated hydrothermal cold-seal rod-bomb. The present apparatus has been used to measure thermal expansion of quartz in the two principal crystallographic directions as a function of pressure to 0.35 GPa, 700 °C. Thermal expansion for directions parallel to a(x) and c(z) increases with increasing temperature, and increases very sharply close to the low-high-quartz transition. High-quartz shows negative thermal expansion. The linear compressibility of low-quartz is greater in the direction parallel to c(z) than it is parallel to a(x), at least to 0.35 GPa. The measured data have been fitted to an Equation Of State (EOS) and can be used to determine the volume and directional length changes of single crystals of quartz in piezothermometric devices, in other industrial applications, as well as in measurement sensors in natural volcanic and hydrothermal systems.

*Keywords:* quartz, thermal expansion, compressibility, dilatometer, equation of state.

## 1. Introduction

For the calculation of phase equilibria involving extrapolation over large ranges in pressure and temperature relevant to conditions inside the Earth and other planets, thermodynamic data need to be measured as accurately as possible. Unfortunately their measurement is a tedious and technically demanding task especially at elevated pressures and temperatures. Consequently the data bases for thermal properties of minerals (heat of formation, specific heat, latent heat), as well as thermoelastic properties (density, reactive volume), have grown quite slowly.

In recent years micro-scale specimens in diamond anvils and solid media cells mounted in synchrotron beams for x-ray diffraction work, have been used to determine crystal structures and sometimes volumetric properties of minerals at elevated temperatures and pressure. While techniques for crystal structure determinations have certainly improved in recent years (see D'Amour et al., 1979; Levien et al., 1980; Glinneman et al., 1992; Angel et al., 1997; Ogata et al., 1987), pressure and temperature control in such devices is not very precise (see Miletich, 2000). Furthermore in such microscopic devices, directional thermal expansion and compression data must be obtained from unit cell measurements and only small scale samples (<10 μm) can be used. Measurement techniques on meso-scale samples (mm to cm) have advanced very little since the pioneering work of P.W. Bridgman (1948 a and b, 1949). Our study presents a new dilatometer for direct measurement of length change of single crystals at elevated pressure and temperature, with corresponding data for meso-scale crystals of quartz.

While our method imposes limitations in terms of specimen type (need for large and disorder-free single crystals) and P-T-range (correspondingly spacious experimental chamber required), our type of experiment may be run with straightfor-

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ward basic hydrothermal pressure-temperature equipment (cold seal rod-bomb). In addition, it provides superior accuracy and resolution for volumetric measurements, because it permits direct determination of directional length change ($\alpha$ - directional expansion, $\beta$ - directional compressibility) as a function of simultaneous changes in pressure and temperature on crystals of industrial size.

## 2. Previous volumetric data for quartz as functions of pressure and temperature

The structure of low-quartz can be viewed as a distortion of that of high-quartz (HEANEY et al., 1994, p. 6). The low-quartz structure (with a space group of $P3_121$ for left-handed and $P3_221$ for right-handed twins) consists of two sets of chains of $SiO_4$ tetrahedra forming spirals parallel to the c-axis (HEANEY et al., 1994, p. 9; HEMLEY et al., 1994, p. 43).

A recent comparison of the P–V equations of state for various silica polymorphs at room temperature (near 300 K, see HEMLEY et al., 1994, p. 49, Fig. 4) summarises for low-quartz the measured continuous decrease in molar volume with increasing pressure[1].

The thermal expansion ($\alpha$) for most silica polymorphs has only been measured at ambient pressure (1 bar).

Three simultaneous mechanisms permit the quartz structure to decrease in volume with increasing pressure, (1) distortion of tetrahedra in response to changes in bond angle, (2) decrease in bond length, but mainly (3) rotation of linked tetrahedra (JORGENSEN, 1978). Changes in the Si–O–Si angle and the tetrahedral tilt angle control the thermal expansion, whereas smaller changes in the Si–O–Si angle and the tetrahedral distortion control isothermal compression (D'AMOUR et al., 1979; LEVIEN et al., 1980).

### 2.1. VOLUMETRIC DATA MEASURED IN SITU AT P AND T

Traditionally, hydrothermal experimentation techniques rely on quenching before samples can be examined. In quenching methods, starting materials (solids and liquids) are hermetically enclosed in a chemically inert container and taken to pressure P and temperature T. The selected P-T-conditions are maintained for a duration deemed long enough for the expected reaction to fully take place, typically hours to months. Subsequently, the sample is returned to ambient conditions as rapidly as possible in order to 'freeze' the phases that were stable at P and T, thus allowing the experimenter to remove and examine the products of the reaction. Obviously this procedure is unsuitable for the measurement of ther-

[1] Because the symbol $\alpha$ is often used to denote the thermal expansion coefficient, as well as the low temperature form of quartz ($\alpha$-quartz) we generally refer to this form as *low-quartz* here.

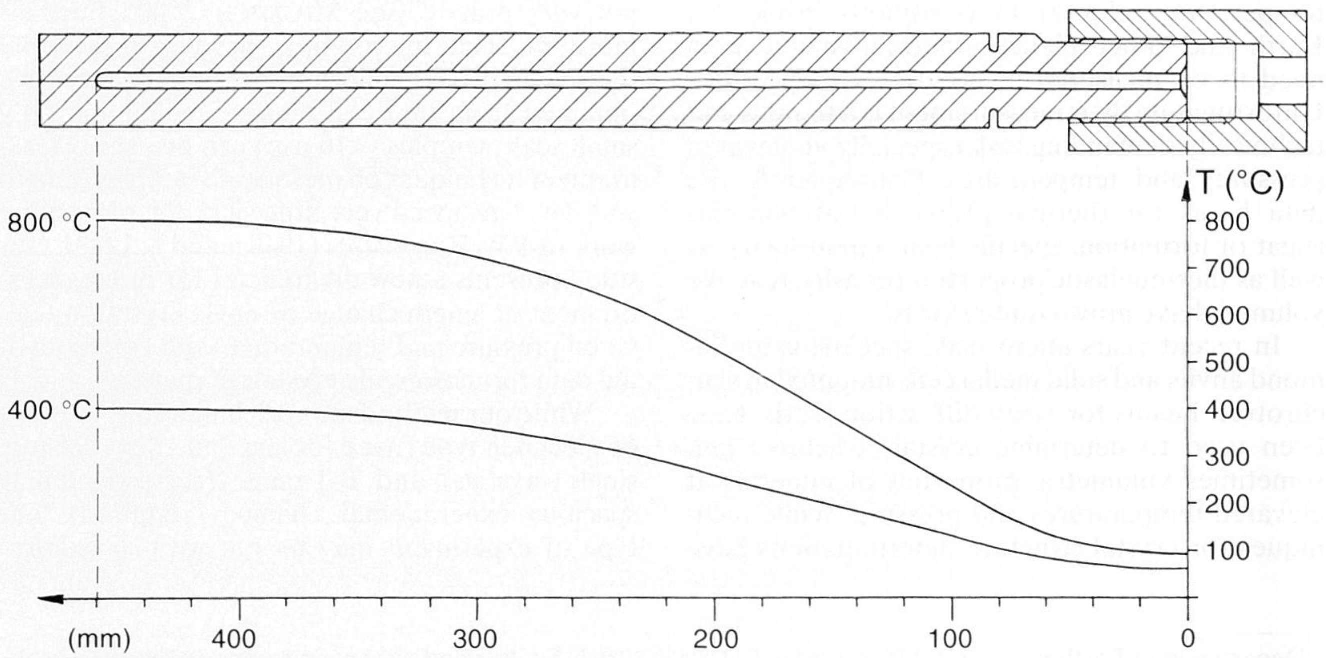


*Fig. 1* Axial temperature gradient in pressure vessel at 400 and 800 °C at 1 bar. Samples are placed near the closed end of the bore hole where a thermal gradient of less than 1 °C over 20 mm is attained.

modynamic data at P and T. The properties in question are not quenchable and must be studied in situ at P and T. We therefore needed to develop procedures to perform relative and absolute metric measurements inside an experimental chamber at P and T.

Apparatus constraints, such as diminishing material strength at increasing temperatures, finite overall hardness and tensile strength, limit our type of measurement to a maximum of about 15 kbar (1.5 GPa) and 1000 °C in internally heated pressure vessels. Higher gas pressures are extremely difficult to seal routinely and gas is required as the pressurizing medium to ensure isostatic shear-free conditions. Externally heated vessels (cold seal rod-bombs), as used in this study, are limited to about 7 kbar and 900 °C. However, in the cold seal type apparatus the metal rapidly loses strength with rising temperature. It is thus unsuitable to follow normal geothermal gradients where temperature and pressure increase simultaneously (the tensile strength of alloys used for cold seal bombs drops off sharply at approx. 500 °C).

Our apparatus allows both pressure and temperature scans; i.e. compressibility with increasing temperature and thermal expansion with increasing pressure may be measured directly. In terms of pressure and temperature, the scan range and direction (up or down P and T) be may be chosen without restriction. This feature proved to be extremely helpful for the volumetric and kinetic study of phase transitions (experiments of this type will be discussed in another paper).

In comparison, micro volume diamond anvil cells do offer a vast pressure range up to 300 GPa, but usually at room temperature. Thus such experimentation still lacks a reliable and accurate pressure scale covering this P-T range, especially when the diamond cells are laser-heated to attain elevated temperatures. Data obtained by anvil cells still need to be considered to have limited metric accuracy and resolution, especially when extended to higher temperatures either by internal or external heating.

## 3. Design of apparatus and control of temperature and pressure

A conventional cold seal type externally heated pressure vessel has been modified for this study. In order to achieve a reasonably large isothermal zone, the bomb design was changed from the routinely used length of 250 mm to 500 mm. This, together with appropriately distributed electric heating power of the furnace resulted in an axial thermal gradient of less than 1 °C over 20 mm at the stationary hot spot for all temperatures up to 900 °C (Fig. 1).

The thermal gradient was established in a special cold seal type externally heated pressure vessel containing six equally spaced thermocouples at pressures up to 6 kbar. It turned out to be independent of pressure down to ambient conditions. It was found however, that the location of the smallest thermal gradient shifted with changing heating and cooling rate. Therefore, the location of the bombs within the furnace had to be shifted according to the actual rate of temperature change chosen for a given experiment. The radial temperature gradient is assumed to be negligible. The pressurizing medium was Argon gas of 99.999% purity (technical term: Argon 5.0).

Argon gas from commercial bottles at a maximum pressure of 200 bar was pressurized in two steps up to a maximum of 6.5 kbar. A commercial air driven two-stage membrane pump was used up to 3.0 kbar, whereas higher pressures were attained with a 1:10 intensifier driven by a small piston water pump on the low pressure side. For safety, and economic reasons, the dead volume (= pressurized gas volume) of the entire system was kept as small as practical by using either 1/16" capillary tubing (ID = 0.1 mm) or 1/8" tubing (ID = 0.5 mm). The typical dead volume in a bomb set up for experimentation was 1.5 to 2 $cm^3$.

Pressures were routinely measured with strain-gauge type and piezo-electric transducers having a resolution of 200 mbar and 20 mbar at 7 kbar at full scale, respectively. This is to say that, at 200 or 20 mbar, the signal started to disappear in amplifier electronic noise. These pressure readings were compared to Bourdon type gauges (manufactured by Heise) with a resolution of 5 bar (= 1 division on the dial). Hysteresis of the beryllium alloy Bourdon tubes was remarkably low (5 to 10 bar maximum even after long duration, days to weeks, near full-scale excursions above 6 kbar). The gauges were calibrated against secondary standards (I-II phase transition of ammonium fluoride (KANEDA et al., 1971), low-high-quartz transition). A primary dead weight tester calibration was not available. Scale linearity between the calibration points is assumed. Pressures are considered accurate to +/–15 bar.

Temperatures were measured with type-K Inconel sheathed thermocouples inside the bomb by means of brazed pressure feed-throughs. Hot junctions were kept fully isolated from the ground to avoid inductive noise stemming mainly from the 50 Hz AC power line feeding the heating coils. A conventional water–ice bath in a Dewar flask served as the cold junction. Thermocouples were calibrated against a type-S Pt-Rh laboratory standard which itself was verified at the Swiss Federal Institute of Standards in Bern. Comparison of the working type-K thermocouples with the standard type-S thermocouple was performed in a special calibration furnace under computer control. Temperatures are considered accurate to +/–2 °C, whereas temperature resolution is below 0.5 °C.

Furnace temperatures were kept to the set value by means of PID (Proportional Integral Differential) controllers driving two AC-line half-wave thyristors. This technique was preferred to phase-angle control in order to prevent injection of spurious noise into the AC line, which interferes unfavorably with the measuring computers. Set values were changed under computer control by adding a time dependent offset voltage to the

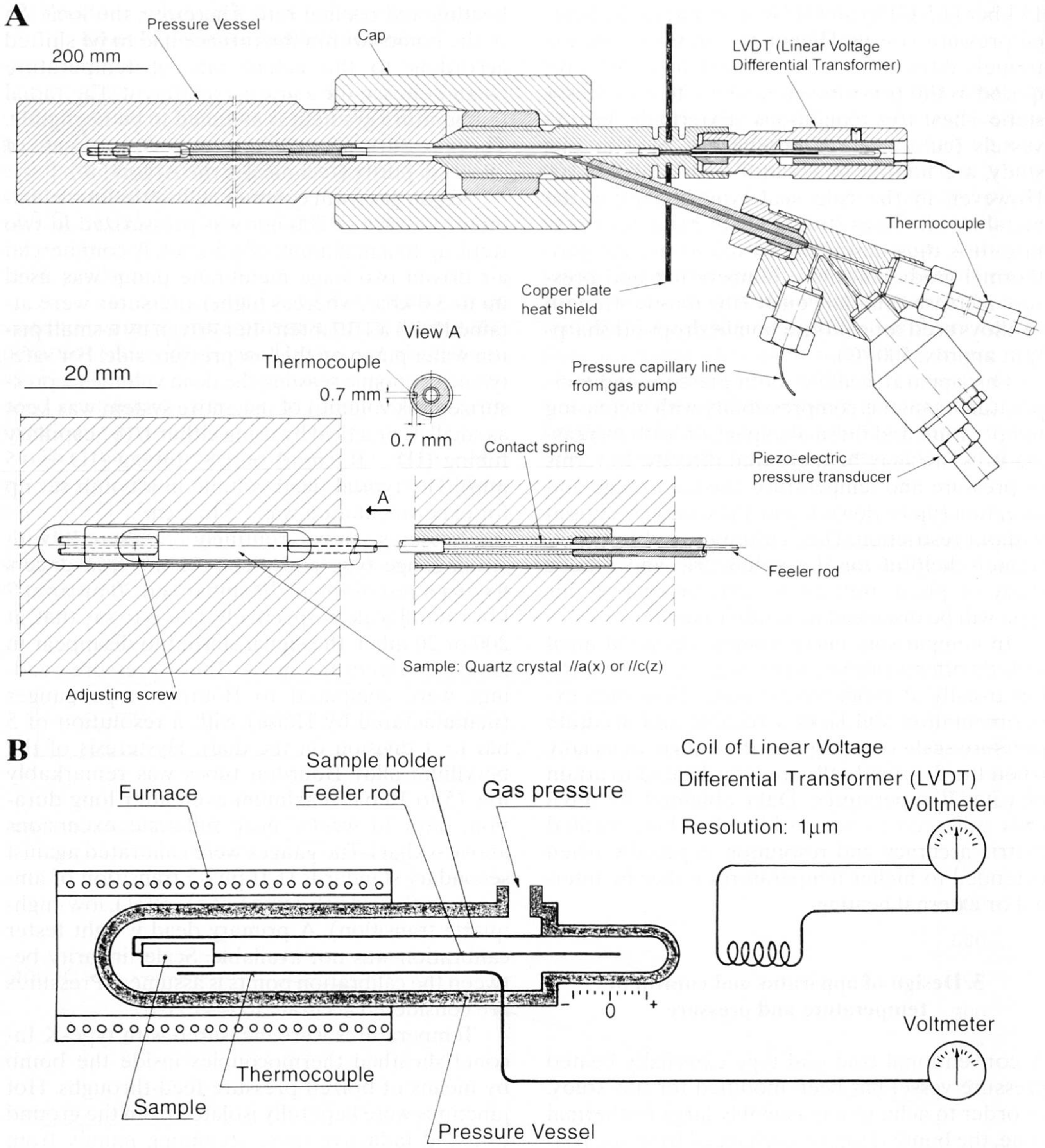


*Fig. 2* (A) Engineering drawing of pressure vessel with dilatometer, LVDT, thermocouple and piezo-electric pressure transducer. (B) Schematic layout of dilatometer. Both feeder rod and sample holder are manufactured from the same batch of AIAI 316 stainless-steel in order to fully compensate relative thermo-elastic response.

controller's sensing thermocouple (also called generation of temperature "ramps").

Pressures were set by manually-operated shut-off valves. Thermally induced pressure changes followed the coefficient (δp/δT)V for Argon (isochoric temperature induced pressure change). In a later addition to the apparatus, a computer controlled bleed valve allowed us to decrease pressure in steps of approx. 2 to 5 bar starting at 6.5 kbar.

The dilatometer option (Fig. 2) in the pressure vessels was made up in a conventional way by a

*Table 3* The interpolated data set for the PVT properties of quartz single crystals, given as molar volume (in cm³), as $dV/V_0$ in percent, $(dl_x/dl_{0x})$ and $(dl_z/dl_{0z})$ in percent; Pressures given in bars, temperatures in degrees Celsius.

| Pressure (bar) | Temp. (°C) | (V(P,T) (cm³) | $dV/V_0$ (%) | dlx/dlx0 (%) | dlz/dlz0 (%) |
|---|---|---|---|---|---|
| 3000 | 25 | 22.5163 | –0.7567 | –0.2890 | –0.1806 |
| 3000 | 100 | 22.5881 | –0.4405 | –0.1673 | –0.1065 |
| 3000 | 200 | 22.6834 | –0.0202 | –0.0081 | –0.0040 |
| 3000 | 300 | 22.7827 | 0.4174 | 0.1541 | 0.1086 |
| 3000 | 400 | 22.8985 | 0.9276 | 0.3478 | 0.2292 |
| 3000 | 500 | 23.0469 | 1.5818 | 0.5929 | 0.3879 |
| 3000 | 520 | 23.0798 | 1.7268 | 0.6487 | 0.4197 |
| 3000 | 540 | 23.1202 | 1.9049 | 0.7210 | 0.4512 |
| 3000 | 560 | 23.1622 | 2.0899 | 0.7933 | 0.4892 |
| 3000 | 580 | 23.2007 | 2.2598 | 0.8640 | 0.5154 |
| 3000 | 600 | 23.2479 | 2.4679 | 0.9393 | 0.5697 |
| 3000 | 610 | 23.2775 | 2.5984 | 0.9887 | 0.5993 |
| 3000 | 620 | 23.3092 | 2.738 | 1.0424 | 0.6291 |
| 3000 | 630 | 23.3481 | 2.9093 | 1.1105 | 0.6612 |
| 3000 | 640 | 23.3930 | 3.1073 | 1.1911 | 0.6943 |
| 3000 | 650 | 23.4511 | 3.3632 | 1.3000 | 0.7273 |
| 2600 | 25 | 22.5407 | –0.6494 | –0.2466 | –0.1576 |
| 2600 | 100 | 22.6115 | –0.3370 | –0.1270 | –0.0834 |
| 2600 | 200 | 22.7077 | 0.0869 | 0.0333 | 0.0203 |
| 2600 | 300 | 22.8087 | 0.5320 | 0.1986 | 0.1339 |
| 2600 | 400 | 22.9258 | 1.0481 | 0.3942 | 0.2561 |
| 2600 | 500 | 23.0780 | 1.7189 | 0.6466 | 0.4161 |
| 2600 | 520 | 23.1117 | 1.8677 | 0.7042 | 0.4480 |
| 2600 | 540 | 23.1540 | 2.0538 | 0.7787 | 0.4828 |
| 2600 | 560 | 23.1973 | 2.2449 | 0.8533 | 0.5221 |
| 2600 | 580 | 23.2371 | 2.4203 | 0.9269 | 0.5477 |
| 2600 | 600 | 23.2909 | 2.6573 | 1.0135 | 0.6076 |
| 2600 | 610 | 23.3230 | 2.7987 | 1.0667 | 0.6402 |
| 2600 | 620 | 23.3594 | 2.9594 | 1.1291 | 0.6732 |
| 2600 | 630 | 23.4043 | 3.1572 | 1.2096 | 0.7062 |
| 2600 | 640 | 23.4663 | 3.4307 | 1.3271 | 0.7391 |
| 2600 | 650 | 23.6089 | 4.0590 | 1.5987 | 0.8099 |
| 2400 | 25 | 22.5513 | –0.6023 | –0.2283 | –0.1469 |
| 2400 | 100 | 22.6230 | –0.2864 | –0.1072 | –0.0723 |
| 2400 | 200 | 22.7199 | 0.1407 | 0.0341 | 0.0324 |
| 2400 | 300 | 22.8209 | 0.5857 | 0.2190 | 0.1466 |
| 2400 | 400 | 22.9397 | 1.1095 | 0.4185 | 0.2683 |
| 2400 | 500 | 23.0937 | 1.7884 | 0.6731 | 0.4318 |
| 2400 | 520 | 23.1284 | 1.9410 | 0.7321 | 0.4646 |
| 2400 | 540 | 23.1711 | 2.1292 | 0.8080 | 0.4986 |
| 2400 | 560 | 23.2152 | 2.3236 | 0.8840 | 0.5382 |
| 2400 | 580 | 23.2572 | 2.5088 | 0.9614 | 0.5658 |
| 2400 | 600 | 23.3136 | 2.7575 | 1.0522 | 0.6287 |
| 2400 | 610 | 23.3476 | 2.9073 | 1.1084 | 0.6634 |
| 2400 | 620 | 23.3874 | 3.0826 | 1.1768 | 0.6986 |
| 2400 | 630 | 23.4397 | 3.3132 | 1.2722 | 0.7338 |
| 2400 | 640 | 23.6333 | 4.1667 | 1.6118 | 0.8882 |
| 2400 | 650 | 23.6335 | 4.1676 | 1.6100 | 0.8927 |

**All Tables** including Table 1 and 2 are available in electronic form from our website: http://e-collection.ethbib.ethz.ch/show?type=bericht&nr=184

*Table 3* (continued)

| Pressure (bar) | Temp. (°C) | (V(P,T) (cm³) | $dV/V_0$ (%) | dlx/dlx0 (%) | dlz/dlz0 (%) |
|---|---|---|---|---|---|
| 2200 | 25 | 22.5631 | –0.5506 | –0.2087 | –0.1342 |
| 2200 | 100 | 22.6348 | –0.2344 | –0.0874 | –0.0598 |
| 2200 | 200 | 22.7325 | 0.1963 | 0.0756 | 0.0450 |
| 2200 | 300 | 22.8354 | 0.6497 | 0.2445 | 0.1593 |
| 2200 | 400 | 22.9535 | 1.1700 | 0.4419 | 0.2818 |
| 2200 | 500 | 23.1087 | 1.8541 | 0.6979 | 0.4472 |
| 2200 | 520 | 23.1439 | 2.0092 | 0.7577 | 0.4808 |
| 2200 | 540 | 23.1882 | 2.2047 | 0.8366 | 0.5158 |
| 2200 | 560 | 23.2339 | 2.4062 | 0.9155 | 0.5566 |
| 2200 | 580 | 23.2776 | 2.5988 | 0.9962 | 0.5848 |
| 2200 | 600 | 23.3378 | 2.8642 | 1.0939 | 0.6501 |
| 2200 | 610 | 23.3745 | 3.0260 | 1.1546 | 0.6875 |
| 2200 | 620 | 23.4184 | 3.2192 | 1.2302 | 0.7257 |
| 2200 | 630 | 23.4823 | 3.5011 | 1.3491 | 0.7639 |
| 2200 | 640 | 23.6430 | 4.2095 | 1.6261 | 0.9013 |
| 2200 | 650 | 23.6436 | 4.2121 | 1.6255 | 0.9050 |
| 2000 | 25 | 22.5743 | –0.5010 | –0.1900 | –0.1218 |
| 2000 | 100 | 22.6464 | –0.1833 | –0.0683 | –0.0468 |
| 2000 | 200 | 22.7434 | 0.2530 | 0.0971 | 0.0586 |
| 2000 | 300 | 22.8475 | 0.7030 | 0.2636 | 0.1742 |
| 2000 | 400 | 22.9669 | 1.2293 | 0.4628 | 0.2988 |
| 2000 | 500 | 23.1250 | 1.9259 | 0.7253 | 0.4633 |
| 2000 | 520 | 23.1608 | 2.0840 | 0.7863 | 0.4973 |
| 2000 | 540 | 23.2061 | 2.2838 | 0.8672 | 0.5326 |
| 2000 | 560 | 23.2530 | 2.4904 | 0.9480 | 0.5745 |
| 2000 | 580 | 23.2989 | 2.6925 | 1.0326 | 0.6041 |
| 2000 | 600 | 23.3626 | 2.9734 | 1.1360 | 0.6731 |
| 2000 | 610 | 23.4028 | 3.1505 | 1.2023 | 0.7142 |
| 2000 | 620 | 23.4548 | 3.3798 | 1.2934 | 0.7566 |
| 2000 | 630 | 23.6410 | 4.2006 | 1.6177 | 0.9094 |
| 2000 | 640 | 23.6494 | 4.2375 | 1.6341 | 0.9125 |
| 2000 | 650 | 23.6528 | 4.2523 | 1.6398 | 0.9155 |
| 1800 | 25 | 22.5860 | –0.4495 | –0.1709 | –0.1084 |
| 1800 | 100 | 22.6575 | –0.1346 | –0.0502 | –0.0343 |
| 1800 | 200 | 22.7566 | 0.3024 | 0.1149 | 0.0723 |
| 1800 | 300 | 22.8616 | 0.7650 | 0.2874 | 0.1883 |
| 1800 | 400 | 22.9826 | 1.2985 | 0.4883 | 0.3164 |
| 1800 | 500 | 23.1411 | 1.9972 | 0.7523 | 0.4797 |
| 1800 | 520 | 23.1781 | 2.1601 | 0.8151 | 0.5148 |
| 1800 | 540 | 23.2247 | 2.3654 | 0.8984 | 0.5506 |
| 1800 | 560 | 23.2730 | 2.5785 | 0.9816 | 0.5939 |
| 1800 | 580 | 23.3207 | 2.7889 | 1.0699 | 0.6242 |
| 1800 | 600 | 23.3886 | 3.0880 | 1.1805 | 0.6965 |
| 1800 | 610 | 23.4318 | 3.2784 | 1.2550 | 0.7341 |
| 1800 | 620 | 23.4755 | 3.4710 | 1.3304 | 0.7718 |
| 1800 | 630 | 23.6193 | 4.1048 | 1.6014 | 0.8490 |
| 1800 | 640 | 23.6531 | 4.2537 | 1.6463 | 0.9040 |
| 1800 | 650 | 23.6601 | 4.2847 | 1.6501 | 0.9265 |

sample holder and a spring loaded feeler-rod, both manufactured from the same material (AISI 316 stainless-steel) in order to cancel out their relative thermo-elastic response.

### 4. Measurement procedures

Movement of the feeler rod was sensed electromagnetically through the walls of a piece of pressure tubing located inside the coil of a conventional LVDT (Linear Voltage Differential Transformer). Stainless steel tubing of AISI 304 or 316 grades turned out to dampen the LVTD signal the least. With this arrangement, resolution was below 1 μm and limited only by electronic amplifier noise. The LVDT's amplified signal was calibrated with a micrometer allowing readings down to 1μm. This technique of taking metric length measurements inside a pressurized container without the

*Table 3* (continued)

| Pressure (bar) | Temp. (°C) | (V(P,T) (cm$^3$) | dV/V$_0$ (%) | dlx/dlx0 (%) | dlz/dlz0 (%) |
|---|---|---|---|---|---|
| 1600 | 25 | 22.5978 | –0.3976 | –0.1512 | –0.0957 |
| 1600 | 100 | 22.6697 | –0.0806 | –0.0298 | –0.0210 |
| 1600 | 200 | 22.7689 | 0.3566 | 0.1350 | 0.0862 |
| 1600 | 300 | 22.8745 | 0.8219 | 0.3086 | 0.2025 |
| 1600 | 400 | 22.9961 | 1.3579 | 0.5102 | 0.3315 |
| 1600 | 500 | 23.1579 | 2.0710 | 0.7805 | 0.4961 |
| 1600 | 520 | 23.1957 | 2.2376 | 0.8447 | 0.5320 |
| 1600 | 540 | 23.2434 | 2.4482 | 0.9299 | 0.5691 |
| 1600 | 560 | 23.2928 | 2.6656 | 1.0150 | 0.6128 |
| 1600 | 580 | 23.3436 | 2.8896 | 1.1085 | 0.6439 |
| 1600 | 600 | 23.4182 | 3.2187 | 1.2316 | 0.7224 |
| 1600 | 610 | 23.4665 | 3.4315 | 1.3159 | 0.7622 |
| 1600 | 620 | 23.6538 | 4.2567 | 1.6365 | 0.9263 |
| 1600 | 630 | 23.6611 | 4.2889 | 1.6504 | 0.9299 |
| 1600 | 640 | 23.6660 | 4.3107 | 1.6592 | 0.9335 |
| 1600 | 650 | 23.6679 | 4.3192 | 1.6618 | 0.9366 |
| 1400 | 25 | 22.6088 | –0.349 | –0.1335 | –0.0824 |
| 1400 | 100 | 22.6811 | –0.0304 | –0.0114 | –0.0076 |
| 1400 | 200 | 22.7815 | 0.4119 | 0.1560 | 0.0994 |
| 1400 | 300 | 22.8872 | 0.8779 | 0.3295 | 0.2164 |
| 1400 | 400 | 23.0103 | 1.4208 | 0.5336 | 0.3470 |
| 1400 | 500 | 23.1735 | 2.1401 | 0.8065 | 0.5123 |
| 1400 | 520 | 23.2121 | 2.3099 | 0.8716 | 0.5495 |
| 1400 | 540 | 23.2614 | 2.5273 | 0.9603 | 0.5862 |
| 1400 | 560 | 23.3129 | 2.7544 | 1.0489 | 0.6323 |
| 1400 | 580 | 23.3671 | 2.9934 | 1.1486 | 0.6676 |
| 1400 | 600 | 23.4488 | 3.3533 | 1.2834 | 0.7506 |
| 1400 | 610 | 23.4960 | 3.5615 | 1.3637 | 0.7937 |
| 1400 | 620 | 23.6514 | 4.2462 | 1.6240 | 0.9410 |
| 1400 | 630 | 23.6700 | 4.3281 | 1.6631 | 0.9426 |
| 1400 | 640 | 23.6740 | 4.3459 | 1.6710 | 0.9442 |
| 1400 | 650 | 23.6748 | 4.3495 | 1.6715 | 0.9466 |
| 1200 | 25 | 22.6210 | –0.2951 | –0.1128 | –0.0698 |
| 1200 | 100 | 22.6936 | 0.0246 | 0.0093 | 0.0060 |
| 1200 | 200 | 22.7937 | 0.4660 | 0.1755 | 0.1143 |
| 1200 | 300 | 22.9003 | 0.9358 | 0.3511 | 0.2308 |
| 1200 | 400 | 23.0248 | 1.4847 | 0.5575 | 0.3625 |
| 1200 | 500 | 23.1914 | 2.2188 | 0.8365 | 0.5299 |
| 1200 | 520 | 23.2311 | 2.3936 | 0.9038 | 0.5675 |
| 1200 | 540 | 23.2818 | 2.6172 | 0.9953 | 0.6046 |
| 1200 | 560 | 23.3350 | 2.8515 | 1.0868 | 0.6519 |
| 1200 | 580 | 23.3932 | 3.1082 | 1.1932 | 0.6910 |
| 1200 | 600 | 23.4854 | 3.5148 | 1.3462 | 0.7831 |
| 1200 | 610 | 23.6692 | 4.3247 | 1.6569 | 0.9517 |
| 1200 | 620 | 23.6748 | 4.3496 | 1.6685 | 0.9527 |
| 1200 | 630 | 23.6780 | 4.3635 | 1.6748 | 0.9536 |
| 1200 | 640 | 23.6822 | 4.3822 | 1.6835 | 0.9545 |
| 1200 | 650 | 23.6830 | 4.3856 | 1.6840 | 0.9567 |

*Table 3* (continued)

| Pressure (bar) | Temp. (°C) | (V(P,T) (cm$^3$) | dV/V$_0$ (%) | dlx/dlx0 (%) | dlz/dlz0 (%) |
|---|---|---|---|---|---|
| 1000 | 25 | 22.6339 | –0.2383 | –0.0913 | –0.0559 |
| 1000 | 100 | 22.7057 | 0.0779 | 0.0291 | 0.0197 |
| 1000 | 200 | 22.8059 | 0.5195 | 0.1953 | 0.1280 |
| 1000 | 300 | 22.9135 | 0.9940 | 0.3726 | 0.2456 |
| 1000 | 400 | 23.0397 | 1.5501 | 0.5821 | 0.3781 |
| 1000 | 500 | 23.2087 | 2.2950 | 0.8659 | 0.5462 |
| 1000 | 520 | 23.2491 | 2.4733 | 0.9347 | 0.5842 |
| 1000 | 540 | 23.3024 | 2.7078 | 1.0302 | 0.6239 |
| 1000 | 560 | 23.3579 | 2.9528 | 1.1257 | 0.6735 |
| 1000 | 580 | 23.4211 | 3.2314 | 1.2416 | 0.7149 |
| 1000 | 600 | 23.5306 | 3.7140 | 1.4241 | 0.8219 |
| 1000 | 610 | 23.6526 | 4.2517 | 1.6553 | 0.8842 |
| 1000 | 620 | 23.6800 | 4.3723 | 1.6826 | 0.9466 |
| 1000 | 630 | 23.6862 | 4.3998 | 1.6872 | 0.9641 |
| 1000 | 640 | 23.6896 | 4.4145 | 1.6939 | 0.9650 |
| 1000 | 650 | 23.6901 | 4.4169 | 1.6941 | 0.9670 |
| 800 | 25 | 22.6453 | –0.1884 | –0.0725 | –0.0433 |
| 800 | 100 | 22.7177 | 0.131 | 0.0495 | 0.0319 |
| 800 | 200 | 22.8187 | 0.5761 | 0.2162 | 0.1426 |
| 800 | 300 | 22.9267 | 1.0519 | 0.3942 | 0.2599 |
| 800 | 400 | 23.0548 | 1.6166 | 0.6067 | 0.3947 |
| 800 | 500 | 23.2261 | 2.3716 | 0.8953 | 0.5629 |
| 800 | 520 | 23.2671 | 2.5527 | 0.9651 | 0.6015 |
| 800 | 540 | 23.3231 | 2.7992 | 1.0654 | 0.6433 |
| 800 | 560 | 23.3814 | 3.0562 | 1.1657 | 0.6949 |
| 800 | 580 | 23.4519 | 3.3671 | 1.2944 | 0.7422 |
| 800 | 600 | 23.6815 | 4.379 | 1.6848 | 0.9488 |
| 800 | 610 | 23.6858 | 4.3978 | 1.6885 | 0.9596 |
| 800 | 620 | 23.6919 | 4.4248 | 1.6962 | 0.9704 |
| 800 | 630 | 23.6935 | 4.4317 | 1.6979 | 0.9737 |
| 800 | 640 | 23.6961 | 4.4432 | 1.7032 | 0.9743 |
| 800 | 650 | 23.6980 | 4.4518 | 1.7063 | 0.9765 |
| 600 | 25 | 22.6576 | –0.1338 | –0.0518 | –0.0303 |
| 600 | 100 | 22.7303 | 0.1865 | 0.0705 | 0.0454 |
| 600 | 200 | 22.8312 | 0.6311 | 0.2371 | 0.1556 |
| 600 | 300 | 22.9408 | 1.1144 | 0.4180 | 0.2744 |
| 600 | 400 | 23.0700 | 1.6838 | 0.6322 | 0.4102 |
| 600 | 500 | 23.2444 | 2.4525 | 0.9263 | 0.5805 |
| 600 | 520 | 23.2870 | 2.6403 | 0.9987 | 0.6205 |
| 600 | 540 | 23.3458 | 2.8994 | 1.1042 | 0.6641 |
| 600 | 560 | 23.4071 | 3.1696 | 1.2098 | 0.7179 |
| 600 | 580 | 23.4681 | 3.4385 | 1.3153 | 0.7702 |
| 600 | 600 | 23.6864 | 4.4006 | 1.6880 | 0.9633 |
| 600 | 610 | 23.6955 | 4.4406 | 1.7031 | 0.9720 |
| 600 | 620 | 23.6998 | 4.4595 | 1.7080 | 0.9806 |
| 600 | 630 | 23.7025 | 4.4714 | 1.7123 | 0.9835 |
| 600 | 640 | 23.7038 | 4.4770 | 1.7146 | 0.9844 |
| 600 | 650 | 23.7051 | 4.4829 | 1.716 | 0.9873 |

need for electrical or mechanical feed-throughs turned out to be a key factor for the overall success of this study.

Control of the experiments (temperature ramps and cycles, stepwise pressure bleed off) and all data acquisition was accomplished with DEC PDP-11 computers running the RT-11 operating system (Foreground-Background monitor). Voltages from thermocouples, LVDT amplifiers and pressure transducers were scanned with a low thermal-offset reed-relay scanner (HP 3495A) and routed to a digital voltmeter (HP 3456A). For instrument control, the HP-IB / IEEE-488 bus was used. Bias voltages driving temperature ramps were generated by a programmable digital to analog power supply (HP 59501A) and suitably arranged voltage-dividing resistors.

*Table 3* (continued)

| Pressure (bar) | Temp. (°C) | (V(P,T) (cm³) | $dV/V_0$ (%) | dlx/dlx0 (%) | dlz/dlz0 (%) |
|---|---|---|---|---|---|
| 400 | 25 | 22.6686 | −0.0855 | −0.0336 | −0.0183 |
| 400 | 100 | 22.7421 | 0.2383 | 0.0899 | 0.0583 |
| 400 | 200 | 22.8445 | 0.6896 | 0.2589 | 0.1703 |
| 400 | 300 | 22.9542 | 1.1732 | 0.4397 | 0.2893 |
| 400 | 400 | 23.0857 | 1.7529 | 0.6581 | 0.4267 |
| 400 | 500 | 23.2635 | 2.3367 | 0.9581 | 0.5998 |
| 400 | 520 | 23.3072 | 2.7291 | 1.0323 | 0.6405 |
| 400 | 540 | 23.3698 | 3.0050 | 1.1449 | 0.6863 |
| 400 | 560 | 23.4353 | 3.2939 | 1.2575 | 0.7442 |
| 400 | 580 | 23.5015 | 3.5857 | 1.3701 | 0.8045 |
| 400 | 600 | 23.7036 | 4.4764 | 1.7180 | 0.9770 |
| 400 | 610 | 23.7054 | 4.4841 | 1.7178 | 0.9849 |
| 400 | 620 | 23.7089 | 4.4996 | 1.7214 | 0.9927 |
| 400 | 630 | 23.7110 | 4.5089 | 1.7245 | 0.9955 |
| 400 | 640 | 23.7123 | 4.5146 | 1.7268 | 0.9965 |
| 400 | 650 | 23.7133 | 4.5192 | 1.7277 | 0.9991 |
| 200 | 25 | 22.6803 | −0.0338 | −0.0145 | −0.0048 |
| 200 | 100 | 22.7546 | 0.2934 | 0.1107 | 0.0717 |
| 200 | 200 | 22.8589 | 0.7531 | 0.2830 | 0.1852 |
| 200 | 300 | 22.9706 | 1.2455 | 0.4670 | 0.3064 |
| 200 | 400 | 23.1040 | 1.8336 | 0.6888 | 0.4451 |
| 200 | 500 | 23.2846 | 2.6298 | 0.9932 | 0.6211 |
| 200 | 520 | 23.3300 | 2.8298 | 1.0706 | 0.6629 |
| 200 | 540 | 23.3977 | 3.1279 | 1.1927 | 0.7112 |
| 200 | 560 | 23.4684 | 3.4398 | 1.3148 | 0.7725 |
| 200 | 580 | 23.6743 | 4.3472 | 1.7209 | 0.8464 |
| 200 | 600 | 23.7121 | 4.5138 | 1.7348 | 0.9798 |
| 200 | 610 | 23.7163 | 4.5323 | 1.7345 | 0.9983 |
| 200 | 620 | 23.7192 | 4.5451 | 1.7374 | 1.0049 |
| 200 | 630 | 23.7210 | 4.5531 | 1.7401 | 1.0073 |
| 200 | 640 | 23.7221 | 4.5580 | 1.7420 | 1.0082 |
| 200 | 650 | 23.7229 | 4.5614 | 1.7422 | 1.0111 |
| 1 | 25 | 22.6929 | 0.0215 | 0.0074 | 0.0067 |
| 1 | 100 | 22.7671 | 0.3488 | 0.1316 | 0.0852 |
| 1 | 200 | 22.8710 | 0.8065 | 0.3025 | 0.1994 |
| 1 | 300 | 22.9836 | 1.3028 | 0.4887 | 0.3199 |
| 1 | 400 | 23.1212 | 1.9094 | 0.7173 | 0.4630 |
| 1 | 500 | 23.3067 | 2.7270 | 1.0311 | 0.6409 |
| 1 | 520 | 23.3537 | 2.9340 | 1.1114 | 0.6836 |
| 1 | 540 | 23.4272 | 3.2581 | 1.2448 | 0.7346 |
| 1 | 560 | 23.5043 | 3.5981 | 1.3781 | 0.8007 |
| 1 | 580 | 23.7134 | 4.5197 | 1.7418 | 0.9716 |
| 1 | 600 | 23.7266 | 4.5779 | 1.7540 | 1.0036 |
| 1 | 610 | 23.7286 | 4.5867 | 1.7543 | 1.0115 |
| 1 | 620 | 23.7312 | 4.5981 | 1.1569 | 1.0174 |
| 1 | 630 | 23.7322 | 4.6024 | 1.7575 | 1.0203 |
| 1 | 640 | 23.7339 | 4.6101 | 1.7604 | 1.0220 |
| 1 | 650 | 23.7355 | 4.6168 | 1.7623 | 1.0247 |

## 5. Results of PVT measurements

Three samples of quartz were used for the investigation of its volumetric properties and the low/high transition at pressures up to 3.5 kbar:

(1) clear, colorless vein quartz (used for the development of the method)
(2) synthetic quartz cut parallel to the c(z)-axis
(3) smoky vein quartz cut parallel to the a(x)-axis

The samples were cut into cylindrical pieces of about 25 mm length and 5 mm OD, resulting in an overall weight of 1.5–2 g. A 0.8 mm hole was drilled half way along the cylinder axis to accommodate a 0.5 mm OD sheathed thermocouple. In accordance with earlier studies, the phase boundary between low and high-quartz proved to follow a straight line up to 3.5 kbar (Figs. 3, 4, 5 and Tables 1, 2, 3).

Our linear isothermal compressibility parallel a(x) and parallel c(z) deviates from those measured by BRIDGMAN (1948), see Fig. 3. Initially this difference was attributed to argon gas that had diffused into the crystal. To further investigate this effect we ran the same experiments with helium and nitrogen as the pressure medium. These runs yielded essentially the same differences and thereby practically ruled out the diffusion hypothesis. In order to suppress gas diffusion into the crystal lattice, future experiments could be carried out with gold plated/encapsulated samples because in another set of experiments gold turned out to be impermeable to argon and helium at these high pressures and temperatures.

## 6. Equations of State and fitting of PVT data

The α-quartz structure (with a space group of $P3_121$) consists of two sets of chains of $SiO_4$ tetrahedra forming spirals parallel to the c(z)-axis (HEMLEY et al., 1994, p. 43). The deformation of these tetrahedral chains is reflected by the dilatometric measurements. The linear thermal expansion and compressibility of quartz parallel to the crystallographic a(x)- and c(z)-axis were measured in the range up to 3.5 kbar and to 640 °C.

Thermal expansion agrees well with the data of SKINNER (1966), JAY (1933) and MAYER (1960) in showing also negative thermal expansion for high-quartz. Compressibility measurements at room temperature are consistent with the results of BRIDGMAN (1948 a and b; 1949) but show increased compressibility parallel to c(z) above about 2 kbar and for parallel to a(x) above about 3 kbar. Relative changes in length as a function of pressure and temperature are shown in Fig. 4. These data (see link to website in Table 3) were used to calculate an interpolated set of volumetric properties (Table 3) for a given P and T.

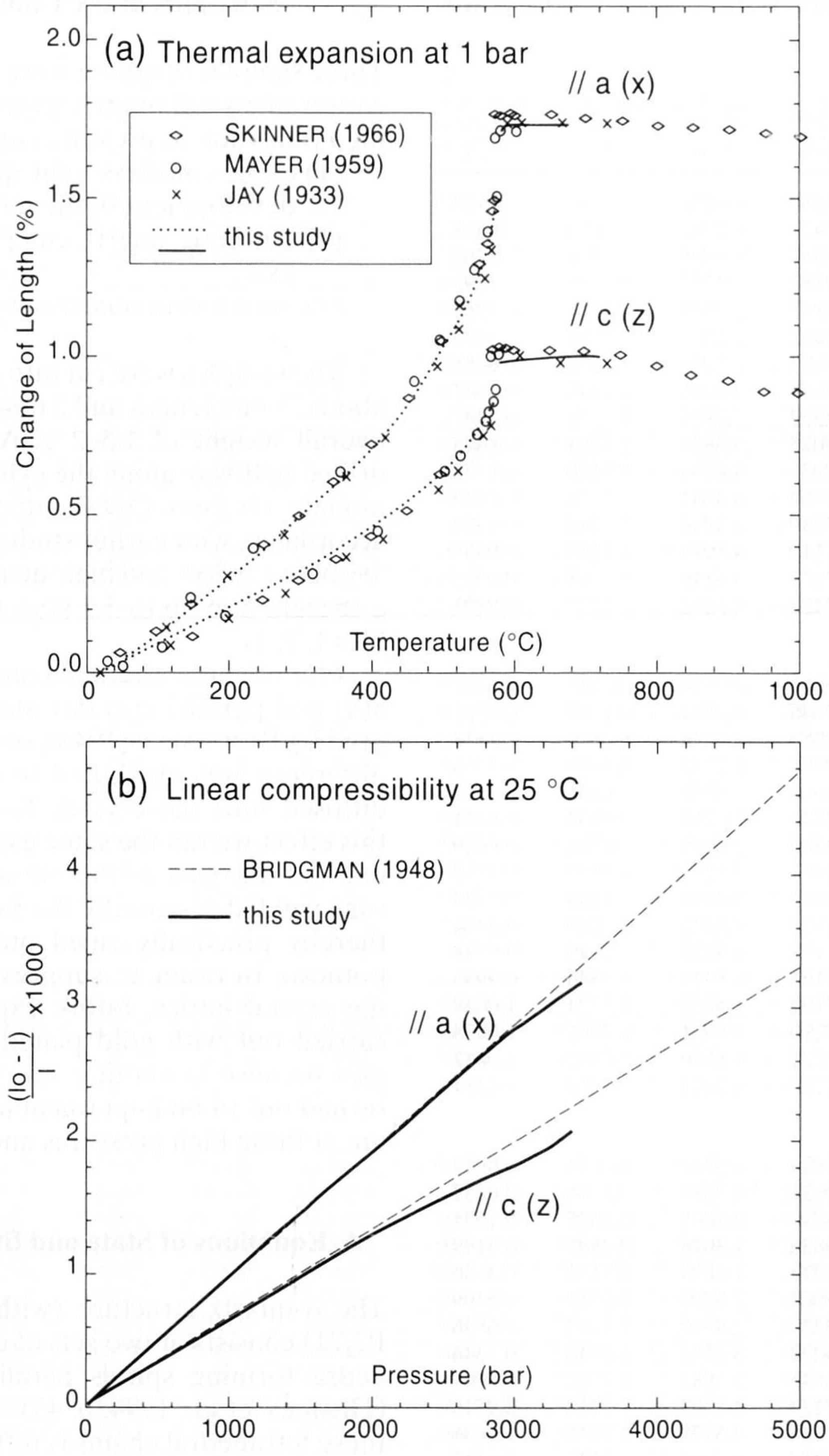


*Fig. 3* A comparison of our measurements with other studies. For measurement errors see Tables 1 and 2 and text. (a) Isobaric thermal expansion of quartz at 1 bar (JAY, 1933; SKINNER, 1966; MAYER, 1959; see also ACKERMANN and SORRELL, 1974). (b) Isothermal linear compressibility of quartz at 25 °C (BRIDGMAN, 1948). For a discussion of the deviation to the data of BRIDGMAN (1948) see text chapter five.

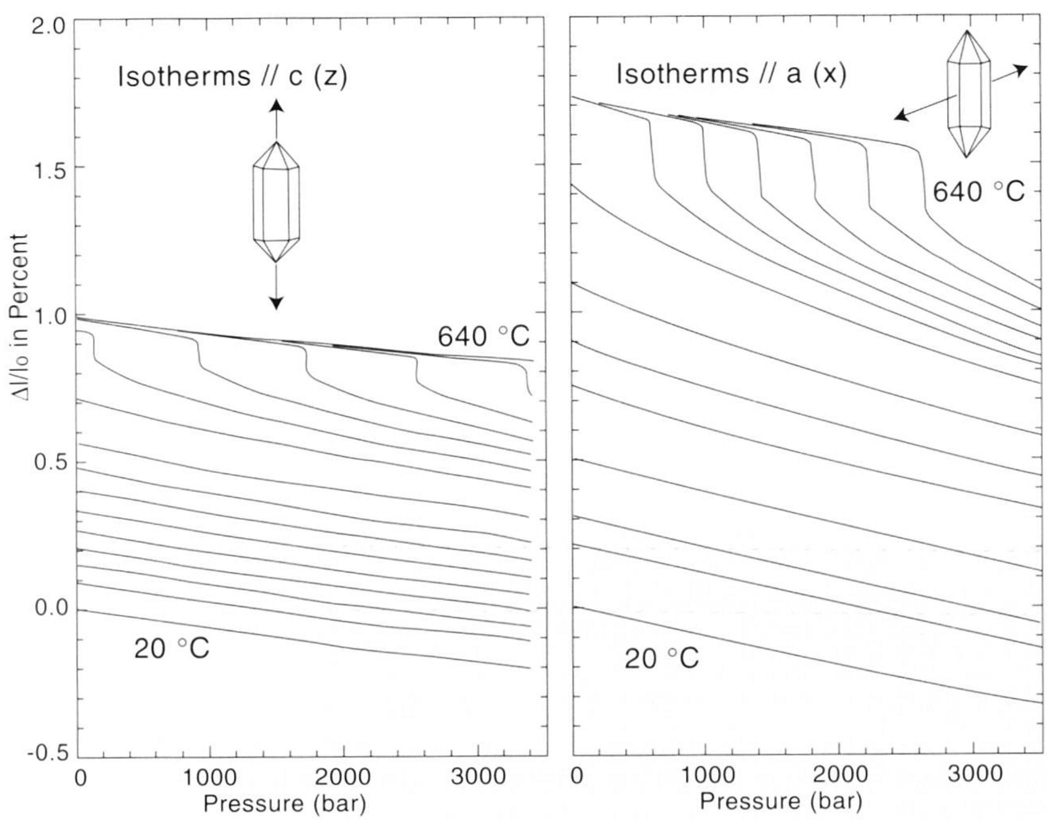


*Fig. 4* Isothermal linear compressibility of quartz parallel to c(z) and a(x) axes at nominal temperatures between 20 °C and 640 °C. For precise temperatures and contour intervals see the corresponding Table 2 (//c(z) axis) and Table 3 (//a(x) axis).

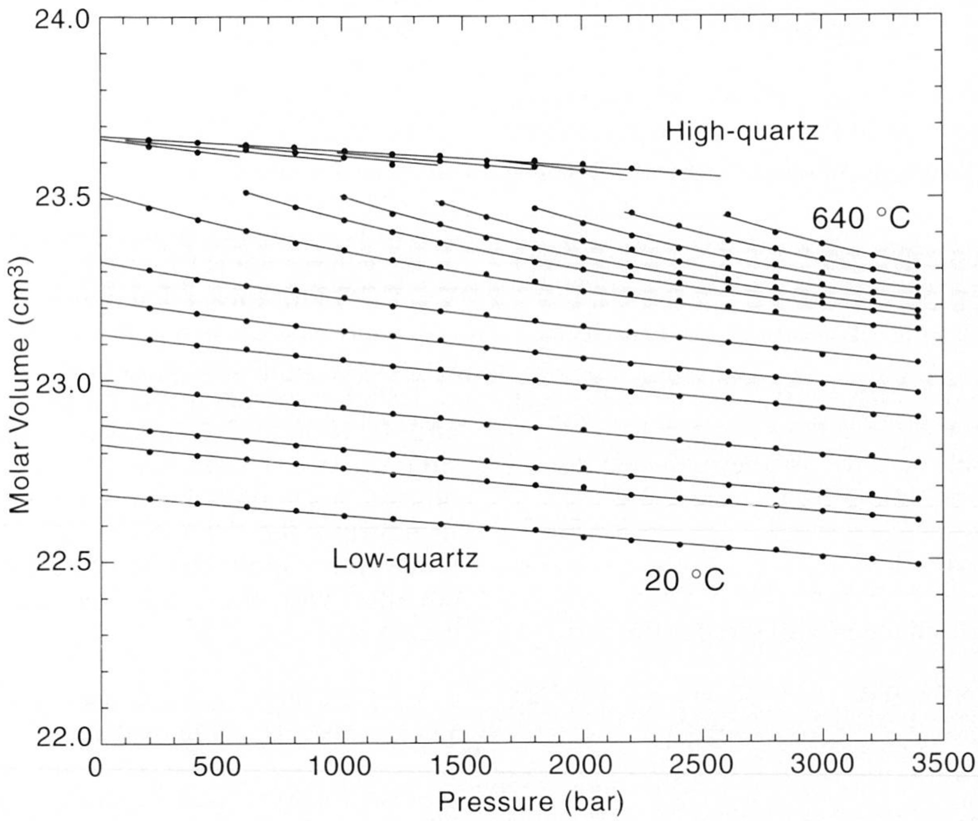


*Fig. 5* Interpolated isothermal volume vs. pressure relationship of quartz at nominal temperatures from 20 °C to 640 °C and pressures up to 3500 bar as derived from Table 4. Data measured in isothermal decreasing pressure scans starting at 3500 bar and proceeding downwards to ambient pressure in approximately 5 bar steps. The low to high quartz transition is anticipated by gradually accelerated increase in molar volume.

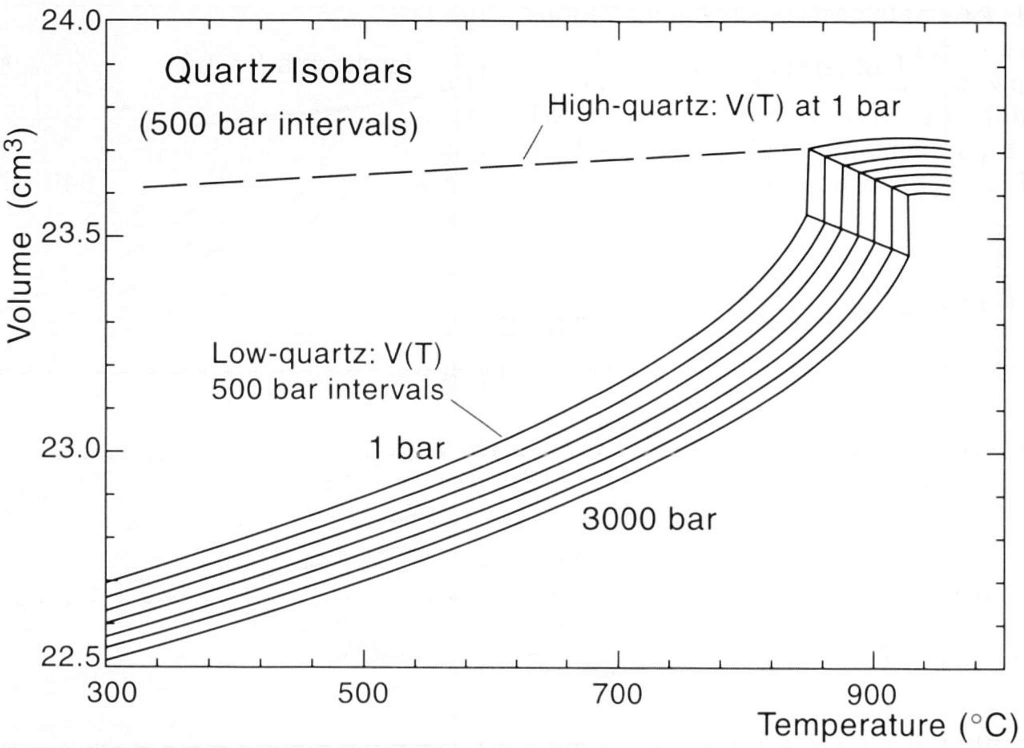


*Fig. 6* Volumetric data for quartz as a function of temperature and pressure from 1 bar to 3 kbar (at 500 bar intervals). The volumetric data for high quartz has been extrapolated to lower temperatures, following the method of MAYER (1960), to determine ordering parameters.

### 6.1. EQUATIONS OF STATE FOR LOW (α-) QUARTZ

Most Equations of State (EOS) for volumetric changes as functions of P and T are modified from the van der Waals modification of the ideal gas law

$$(P + a/V^2)(V-b) = RT \quad (1),$$

for example to a virial equation of the form

$$V = Vo + a/P + b/P^2 + c/P^3 \quad (2).$$

In the present case, isothermal changes of pressure with volume were formulated as

$$V = Vo + AP + BP^2 \quad (3),$$

and isobaric changes with temperature as

$$V = Vo + AT + BT^2 \quad (4).$$

The best type of fitting for low-quartz data was obtained with

$$V(P,T) = A/(P + B) + CP^2 + DP + E \quad (5)$$

with

$$A = a_1T + a_2$$
$$B = a_3T^2 + a_4T + a_5$$
$$C = a_6$$
$$D = a_7T^2 + a_8T + a_9$$
$$E = a_{10}T^2 + a_{11}T + a_{12}.$$

The molar volume of 22.688 $cm^3$ at 1 bar and 298.15 K was taken from ROBIE et al. (1979). The molar volume data at T and P were fit to a function V = V(P,T) for low and high-quartz separately, the type and degree being the same for both (the number of adjustable parameters was minimized).

280 data points in the range from 1 to 3 kbar and 300 K up to $T_c$ were taken as input for regression analysis. 3 additional points were calculated outside this limit by linear extrapolation to 200 K to improve the derivatives $(dV/dT)_P$ at the low temperature limit. The best fit constants (simplest equation with the lowest residuals) for low (α) quartz are:

| | | |
|---|---|---|
| $a_1$= 0.225007E+01 | $a_5$= −0.208346E+04 | $a_9$= −0.860444E−04 |
| $a_2$= −0.997038E+03 | $a_6$= 0.139958E−08 | $a_{10}$= 0.631069E−06 |
| $a_3$= −0.438645E−01 | $a_7$= −0.792110E−10 | $a_{11}$= 0.192899E−03 |
| $a_4$= 0.430831E+02 | $a_8$= 0.794849E−07 | $a_{12}$= 0.226296E+02 |

with a sum of squares of 99.98107 and a standard deviation of 0.004 $cm^3$, where the maximum deviation is 0.015 $cm^3$.

### 6.2. CRYSTALLOGRAPHIC EOS FOR LOW (α) QUARTZ

The EOS presented here is purely empirical, based on isobaric and isothermal, as well as poly-thermobaric, analysis of the experimental data. The mathematical relations were chosen in a purely pragmatic manner. The aim was to keep the fit function as simple as possible and represent the measured data with acceptable precision. Data for low and high-quartz were fitted independently to the same equation. Consequently it is not recommended that our function be extrapolated to obtain PVT values far outside the P-T-area covered by our experiments.

Recent developments of Equations of State for volumetric changes as functions of P and T are to be found in the fitting of high-pressure lattice parameter data (e.g., ANGEL, 2000). These equations developed for solids have followed the Birch-Murgnaghan equations in terms of expressing volume changes as functions of pressure and bulk moduli (reciprocal compressibility). We have used the data fitting program of Angel on his web-site (http://www.geol.vt.edu/rja/soft/) to fit our data in Table 3 to obtain a series of isothermal Birch-Murgnaghan equations. The raw data may thus be used by any reader to obtain their own preferred fit equations. The Birch-Murnaghan equation

$$P = 3K_0f_E(1 + 2f_E)^{5/2}\{1 + 3/2(K' - 4)f_E + 3/2 [K_0K'' + (K' - 4)(K' - 3) + 35/9]f_E^2\}$$

with $f_E = [(V_0/V)^{2/3} - 1)]/2$

was used to second order truncation (K'=4). Initial values of $V_0 = 22.6929 cm^3$ and $K_0 = 45 GPa$ were chosen. The resulting fitted values for $V_0$, $K_0$ and K" are listed in Table 4 (note: K' is fixed to 4). The quality of the isothermal fit decreases rapidly with increasing temperature, reflecting the softening of low-quartz approaching the α-β phase transition.

*Table 4* Fit parameters to a series of isothermal Birch-Murgnaghan equations obtained using the fitting routines of ANGEL (2000).

| Temp [°C] | $V_0$ [$cm^3$] | $K_0$ [GPa] | K' [ ] | K" [$GPa^{-1}$] |
|---|---|---|---|---|
| 25 | 22.69270 | 37.92129 | 4.0 | –0.01026 |
| 100 | 22.76634 | 37.45919 | 4.0 | –0.01038 |
| 200 | 22.87007 | 35.87710 | 4.0 | –0.01084 |
| 300 | 22.98223 | 33.69838 | 4.0 | –0.01154 |
| 400 | 23.11651 | 30.64829 | 4.0 | –0.01269 |
| 500 | 23.29894 | 26.44702 | 4.0 | –0.01470 |
| 520 | 23.34470 | 25.09329 | 4.0 | –0.01550 |

### 6.3. EQUATIONS OF STATE FOR HIGH (β) QUARTZ

For high (β) quartz the same function (equation 5) was used for fitting and the following constants were obtained:

| | | |
|---|---|---|
| $b_1$= –0.440968E+00 | $b_5$= –0.319299E+00 | $b_9$= –0.247462E–02 |
| $b_2$= 0.195727E+00 | $b_6$= –0.383799E–09 | $b_{10}$= –0.400260E–05 |
| $b_3$= 0.678930E–01 | $b_7$= –0.271121E–08 | $b_{11}$= 0.742196E–02 |
| $b_4$= 0.171344E+00 | $b_8$= 0.513888E–05 | $b_{12}$= 0.202996E+02 |

with a sum of squares of 97.38915 and a standard deviation of 0.005 $cm^3$, where the maximum deviation is 0.024 $cm^3$.

### 6.4. NEGATIVE THERMAL EXPANSION OF HIGH (β) QUARTZ

Our thermal expansion data show that high-quartz exhibits negative thermal expansion (i.e. high (β)-quartz shrinks when it is heated, see the recent discussion by WELCHE et al., 1998). Also our data show that the axial ratio c/a decreases from about 1.1001 at room temperature to a nearly constant value of about 1.092 above the transition temperature (RAZ, 1983, Fig. 46, p. 73). Similar observations were implied in earlier high temperature x-ray studies (ACKERMANN and SORRELL, 1974; JAY, 1933; BERGER et al., 1966) but are not visible in all fits of volumetric data (e.g., DOROGOKUPETS, 1995, Fig. 8, p. 8496). TUCKER et al., (2000a) have recently noted that the volume of high-quartz is lower than would be calculated from the actual Si–O bond lengths. This and other examples of lattice shrinking are interpreted by DOVE et al. (2000, p. 26) in terms of Rigid Unit Modes of linked polyhedra in framework structures.

More recent data, reviewed by CARPENTER (2000, p. 46, Fig. 8), indicate that it is the c(z)-direction which shows the principal negative thermal expansion, whereas the a(x)-direction shows little change or is slightly positive from their data. Our data show that the a(x)-direction shows weak negative thermal expansion.

### 6.5. RELATIVE MERITS OF THE DILATOMETER VS. DIAMOND ANVIL CELLS

In comparison to x-ray diamond anvil cell (DAC) studies, our dilatometric approach to PVT measurement suffers from a number of limitations but also offers features not known with DAC's. The major drawback certainly lies in the technical upper pressure limit of approx. 15 kbar to our technique. Proceeding beyond crustal conditions remains outside the realm of cold-seal bomb and internally heated pressure vessel dilatometry. On the other hand, resolution and accuracy of the dilatometry method are better. Thanks to the commercially available piezo-electric transducers and charge amplifiers, pressures in experimental apparatus may be resolved close to 1 ppm. The dilatometric technique fitted together with piezo-pressure sensors provides an extremely powerful method for the detailed study of phase transitions and their kinetics. A new future application, proposed here, will permit combination of our PVT data to DAC's driven by piezo-electric force actuators rather than by fine pitched mechanical screws.

## 7. Other physical properties reflecting these volumetric changes

MAYER (1960) measured the isobaric thermal expansion of quartz in both a(x) and c(z) directions at room pressure. He defined a parameter, like the ordering parameter in Landau theory, based upon the difference between the actual low-quartz volume at temperature and the down temperature extrapolated high-quartz volume, which has zero value above the transition temperature and tends towards infinity towards low temperature (RAZ, 1983, Fig. 45 p. 72). We have incorporated this method into the construction of Fig. 6. These data are consistent with results from Raman spectroscopic data of soft modes (HOECHLI and SCOTT, 1971; BACHHEIMER and DOLINO, 1977; BARRON et al., 1982), and from elastic constant measurements for compliance $C_{14}$ (HOECHLI, 1970; AXE and SHIRANE, 1970, and the recent summaries in the volume edited by REDFERN and CARPENTER, 2000).

## 8. Open questions about Equations of State

The ease of fitting a mathematical function to a given data set depends upon the extent of its change. In our case this is reflected by the values of the partial first and higher derivatives (divergences in Fig. 3) of the volume function (compressibility $(\delta V/\delta P)_T$ and thermal expansion $(\delta V/\delta T)_P$). Various requirements may be of importance when choosing a mathematical relation to represent this function:

(a) best possible approximation to the measured data set,
(b) reflection of a known or assumed physical relation among the variables,
(c) and/or as a consequence of (b) the possibility of sound extrapolation beyond the measurement range of the data set.

Our polynomial functions were selected principally with requirement (a) in mind. We found that the fit parameters to a series of isothermal Birch-Murgnaghan equations (Table 4) were quite imprecise, even though they relate to requirements (b) and (c). Likewise, classical Equations of State are not adequate close to phase transitions (see DOROGOKUPETS, 1995). Within the P-T-range covered by our measurements, noticeable divergence from linearity occurs in the vicinity of the low to high-quartz phase transition. This feature anticipates the advent of the phase transition (Fig. 6). Unfortunately, a quantitative determination of onset points on the low side of the transition is not feasible with our technique.

While there is an attraction to the elegance of a known or assumed physical relation among the variables (requirement b above), we can rarely fulfill the expected simplicity. Today with the development of computer based mathematical tools we can present measured raw data in tables and interpolate them as needed. This can also be done for simple determination of derivatives and inversion of the function with respect to all variables, etc. Development of Equations of State (EOS) for extrapolation of data beyond the range of measurements will continue as remarked for requirement (c) above. Our PVT data may be used for testing EOS derived from ab initio calculations.

## 9. Applications and outlook

Our data have several applications involving the physical behaviour of quartz and other common minerals. Most studies of fluid inclusions in minerals assume rigid cavities which means that the volume and shape of the inclusions should not change during cooling and decompression. This simplification may now be corrected with our data for quartz.

A future application of our data concerns the method for calibrating and measurement in Diamond Anvil Cells. Our PVT data for quartz would permit piezo-electric force actuators to be used rather than the crude method of fine pitched mechanical screws as used at present.

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