From bath-tub vortex to pump-intake vortex

Axial-flow-induced instability as their formation mechanism

by Y. N. Chen, Winterthur

A new theory will be introduced for the formation of the bath-tub vortex, basing on the transfer of the circulation with time from the outer wall towards central region. This transfer is caused by the axial flow through the central region so that the viscous fluid of the inner core will be continuously carried out by it with the drain, resulting in the narrowing of inner core and the widening of potential outer core, connected with the increase of the swirling speed. At the same time turbulence will be generated at the edge of the inner core owing to instability induced by the axial velocity in this three-dimensional vortex. This turbulence will then accelerate the energy transfer mentioned. As the turbulence generation is proportional to the velocity at the inner core edge and this velocity (both the swirl and axial components) increases with the narrowing of the inner core, a feedback mechanism arises between them and causes a very rapid advance of the depth of the air core once a certain stage of the inner core is reached.

The theory predicts, for the formation of the bath-tub vortex, a necessity of continuous feeding of swirling energy from the earth's rotation. Any residual vorticity without further energy supply will not be amplified. This theory corresponds very well to the observations made by Shapiro, Binnie, and Trefethen et al. The further theoretical prediction that the formation of the air core is similar to a vortex-breakdown phenomenon is also borne out by the experiments carried out by Binnie & Hoockings and Anwar, Weller & Amphlett.

The theory developed for the bath-tub vortex can be extended to the intake-vortex formed in the pump suction sump on account of the similar properties between both. The supply of the swirling energy for the intake-vortex originates from the swirl flow either incorporated in the incident flow to the suction pipe intake or induced by the preswirl of the pump impeller. If the suction pipe is positioned vertically the separation of flow in the wake will amplify the formation. The practical measures approved to be effective in the engineering world for the prevention of the intake-vortex formation can be very well covered by the present theory.

Für die Entstehung des Badewannenwirbels wird eine neue Theorie entwickelt, die auf einer mit der Zeit erfolgenden Übertragung der durch die Erdrotation erzeugten Zirkulation von der Aussenwand der Badewanne nach innen zum zentralen Gebiet des Wirbels basiert. Diese Übertragung wird durch die axiale Abströmung aus dem zentralen Gebiet des Wirbelkerns ermöglicht, da diese die zähe Flüssigkeit des inneren Wirbelkerns dauernd abführt. Das Resultat ist die Verkleinerung des inneren Wirbelkerns und die Erweiterung des äusseren Potentialfeldes, verbunden mit der Erhöhung der Rotationsgeschwindigkeit des Wirbels. Gleichzeitig wird Turbulenz am Rand des inneren Wirbelkerns infolge der durch die axiale Strömung induzierten Instabilität des drei-dimensional gewordenen Wirbels erzeugt. Diese Turbulenz dürfte dann die erwähnte Energieübertragung weiter beschleunigen. Da die Erzeugung der Turbulenz bekannterweise der Geschwindigkeit am Rand des inneren Wirbelkerns proportional ist, und da ausserdem diese Geschwindigkeit (einschliesslich der Umfangs- und Axialkomponenten) mit der Verkleinerung des inneren Wirbelkerns zunimmt, tritt ein Rückkopplungsmechanismus zwischen der Turbulenz-Erzeugung und der erwähnten Geschwindigkeit auf. Die Folge davon ist ein rasches Fortschreiten der Tiefe des mit Luft gefüllten Hohlkegels im Wirbelkern, sobald ein gewisses Entstehungs-Stadium desselben erreicht worden ist.

Aus der entwickelten Theorie ergibt sich ferner, dass für die Entstehung des Badewannenwirbels eine dauernde Speisung der Drallenergie durch die Rotation der Erde benötigt wird. Jede in der Strömung übriggebliebene Wirbelung ohne weitere Energiezufuhr kann nicht zur Bildung des Badewannenwirbels beitragen. Diese Theorie entspricht sehr gut den Beobachtungen von Shapiro, Binnie sowie Trefethen und seinen Mitautoren. Die weitere Schlussfolgerung aus der Theorie, dass das rasche Fortschreiten des mit Luft gefüllten Hohlwirbelkerns mit der Erscheinung des «Vortex-Breakdown» etwas gemeinsam zu haben scheint, findet ebenfalls Bestätigung in den experimentellen Resultaten von Binnie & Hoockings und Anwar, Weller & Amphlett.

Diese für den Badewannenwirbel entwickelte Theorie kann auf den in der Pumpen-Einlaufkammer entstehenden Zuströmungs-Hohlwirbel angewendet werden, da beide ganz ähnliche Eigenschaften aufweisen. Die Speisung der Drallenergie zum Pumpenzuströmungs-Hohlwirbel stammt von der Rotationsströmung, die entweder in der Zuströmung zum Saugrohr infolge der asymmetrischen Geschwindigkeitsverteilung in der Einlaufkammer entsteht oder durch den Vordrall des Pumpenlaufrades induziert wird. Wenn die Pumpe zudem ein vertikales Saugrohr aufweist, wird die Ablösung der Strömung hinter ihm die Bildung des Zuströmungs-Hohlwirbels noch weiter verstärken. Es wird schliesslich gezeigt, dass die praktischen Massnahmen, die sich in der technischen Fachwelt als wirksam gegen die Bildung des Zuströmungs-Hohlwirbels erwiesen haben, durch die vorliegende Theorie sehr gut belegt werden können.

Bath-tub vortex

History

The formation of a vortex in a tank (a hand basin, kitchen sink or bath-tub) when water drains out through a hole in the bottom of it has been experimentally vond theoretically investigated by a great number of authors. As the so-called bath-tub vortex has a close similarity with the intake vortex formed in the sump of a pump due to its suction action, its investigation emobies *not only an academic interest but also an engineering one*. The carefully controlled experiments of *Shapiro* and *Binnie* were conducted in the water of a tank which was allowed to settle for many hours before opening the drain in order to reduce the residual vorticity to a value less than that corresponding to the earth's angular velocity [1–4]. These authors obtain a counterclockwise rotation in the water which corresponds to the sense being expected in the *Northern Hemisphere* if *the earth's rotation* were the cause. This expectation is confirmed by the experiment conducted by *Trefethen, Bilger, Fink, Luxton & Tanner* in Sydney, Australia who obtain a clockwise rotation in this *Southern Hemisphere* [5]. *Shapiro* observes furthermore for the case of a residual clockwise circulation that during the first two-thirds of the draining period, the rotation was clockwise due to the initial swirl, but then changed over to a counterclockwise sense due to the earth rotation. He observes near the end of one experiment that the frequency of the revolution reaches about 0.3 Hz in a period of 15 min corresponding to an exponential increase of vorticity with time (e^{at}) with a factor of a = 0.01 s^{-1} (Marris, [2]). This frequency is 30,000 times greater than the effective angular speed of the earth's rotation at that latitude.

Binnie conducted a further experiment using for the water draining a vertical pipe extending some 13 inches over the base of the tank in order to avoid the viscous action in the boundary of the flow next to it. He obtains the same result as in a tank with a usual hole as the drain unit, namely that an initial residual clockwise swirl is overtaken by a counterclockwise swirl. This clearly documents that the viscous effect of the base boundary layer is not essential for the transmission of the earth's rotation via the tank to the contained vortex.

The *mathematical model* introduced by *Marris* using the *tank as a frame of reference* having angular velocity corresponding to the earth's rotation speed and the water rotating with it as a rigid body (i.e. with an initial swirl velocity proportional to $r\omega_E\sin\lambda$, where ω_E = earth angular velocity, λ = latitude of location of experiment) yields that the reversal of the initial residual clockwise swirl can only be predicted if a time-dependent vertical gradient of the peripheral velocity of the rotating water is present. As the *Binnie* experiment using a vertical draining pipe mentioned for preventing the viscous effect of the base boundary layer still promotes the swirl reversal, the time-dependent vertical gradient of the peripheral velocity cannot be a cause for the formation of the bath-tub vortex. The real cause must be some other factor which is not included in the theoretical considerations made hitherto. It is the purpose of the present paper to search for this still unknown factor.

Properties

The formation of the vortex in a sump can be induced by the preswirl of the impeller or by the uneven distribution of the incident flow. The phenomenon is the same as that in a bath-tub induced by the earth's rotation. As described by *Denny* about the vortex formation

stages with the increasing swirl angular velocity, the vortex appears at first as a small dimple on the free surface (Fig. 1a) which gradually deepens to form a cone-shaped hole (Fig. 1b, c, d), air bubbles break away from time and are swept into the bellmouth of the pump suction pipe; finally with strong swirl intensity the air core lengthens to reach the suction-inlet and allows continuous passage of air up the suction pipe to the pump (Fig. 1e) [6].
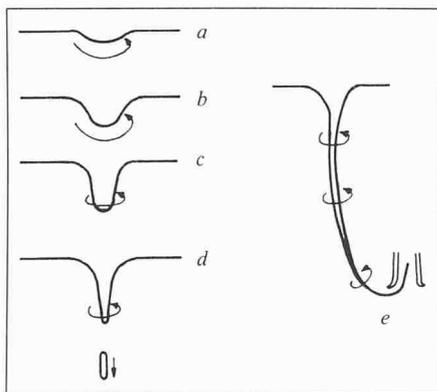

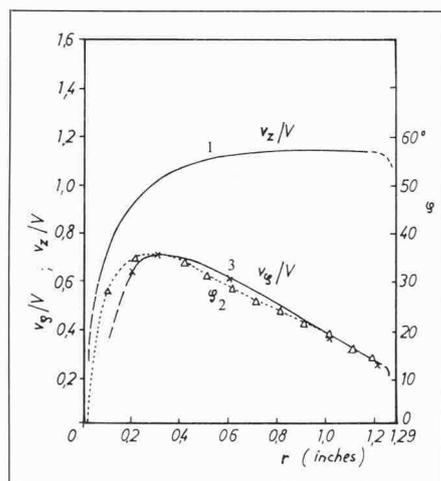Fig. 1. *Stages in development of an air-entraining vortex (Denny)*

Fig. 2. *Profiles of axial velocity, swirl velocity and swirl angle in the suction tube (Markland & Pope)*

The profile of the axial velocity and that of the angle of the flow direction in the suction pipe are reproduced in Fig. 2 as curves 1 and 2 according to the measurement carried out by *Markland & Pope* [7]. Then the distribution of the swirl velocity can be evaluated (curve 3 in Fig. 2). It shows a pattern of a *Rankine vortex* with a core rotating with a peripheral velocity $v_{\varphi, i}$ and an outer flow field roughly similar to a potential vortex. This vortex core must be a continuity of the air core shown in Fig. 1e.

The air core mentioned has diameters decreasing from the cone-shaped hole to the suction pipe. It serves as a rotating axis of the three-dimensional vortex extending from the surface of the sump to the pump inlet. As the pressure on the free surface of the air core is practically

equal to the atmospheric pressure if the effect of the resistance and velocity of the air flow through it is neglected, the swirl velocity of this free surface can be supposed to have very high values, surrounded by a potential vortex field, similar to the swirl flow as in the suction pipe. The flow field in the sump is thus a three-dimensional Rankine vortex with axial flow, but not a rigid-body rotating flow. As the bath-tub vortex is quite similar to the vortex formed in the sump induced by the intake swirl of a pump, it must also exhibit a pattern of the Rankine vortex. Then the mathematical model of a rigid-body rotation introduced by *Marris* deviates somewhat from the real flow condition. However, in his approach, the strong axial flow along the rotating axis of the vortex is well recognized and therefore modelled accordingly.

The experiment conducted by *Binnie & Hookings* reveals a series of interesting properties of the bath-tub vortex [8]. They detected a rotating axis of a dimple even though it is still very small, using the technique of the insertion of coloured water into the water tank. They found that "even very close to the dimple the water was rotating very nearly in horizontal planes, but a narrow spinning tube of liquid descended with considerable velocity from the base of the dimple into the draining tube. Its diameter increased with depth as it entrained the surrounding water" [8]. This observation is sketched in Fig. 3a, showing a strong axial flow along the spinning axis. This observation is extremely important for the understanding of the
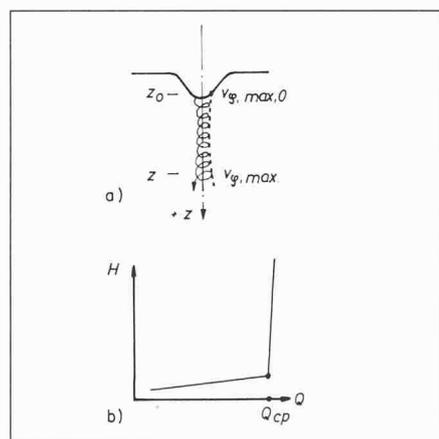

Fig. 3. *A flat vortex with a narrow spinning inner core*

property of both the bath-tub vortex and the intake vortex. This will be further discussed in the following section.

The measurement of *Binnie & Hookings* showed furthermore that the mass flow Q discharged from the central pipe of the water tank increased as the height H of the water level was increased, only below a critical value Q_{cr}. After this critical value was reached, an air core in the vortex from the water surface down

into the pipe was formed. As it entered the pipe its diameter increased and the axial velocity of the water flow accelerated furthermore considerably. The water flowed down helically, emerging from the outlet as a spinning annular jet. The mass flow remained practically at this critical value, even the height of the water level was further increased, see Fig. 3b. The limiting of the discharge to the critical value was thus caused by the sudden appearance of the powerful air core, which blocked the area of the pipe for the water drain. The sudden nature of the phenomenon without gradual transition must be connected with a kind of instability.

Theory

A two-dimensional vortex generally shows a rigid-body rotating pattern rather than a potential vortex, as shown in Fig. 4b for an edge vortex generated behind a sharp edge and in Fig. 4c for a *Karman vortex* shed by circular cylinder, as compiled in reference 9 using the experimental results obtained by *Howard & Mathews* and *Timme* [10 and 11]. The edge vortex has a rigid-body centre core surrounded by an outer core with a flat profile of the swirl velocity increasing outwards. The Karman vortex exhibits only a rigid-body core around which an outer potential vortex field is induced.

The three-dimensional vortex shed by a delta wing, however, shows quite a different pattern. The rigid-body centre core is very narrow, with a very high swirl velocity on its surface. This swirl velocity decreases then outwards rapidly in the outer core, see Fig. 4a. Its mean value obeys a law of about $r^{1/2} v_{\varphi} = const$ [10a].

The distribution of the axial velocity in Fig. 4a shows a strong rise inwards so that the very narrow rigid-body centre core is vented by a very strong axial flow. It is this venting effect which carries away the viscous fluid continuously and therefore makes the viscous, rigid-body inner core very narrow. The nearly inviscid outer core can then extend very closely to the axis. Because of its practically inviscid nature, the swirl velocity can therefore be transferred to a higher value inwards within this outer core. The very high swirl velocity on the inner surface of this core is thus a result of the very high axial velocity in the centre region associated with a very low pressure there. Here we find a close partnership between the strong axial flow in the centre region, a narrow rigid-body inner core with a great swirl velocity surrounded by a nearly inviscid outer core with a practically potential field.

There is another piece of evidence to judge the conclusion made previously. This can be evaluated from the measur-
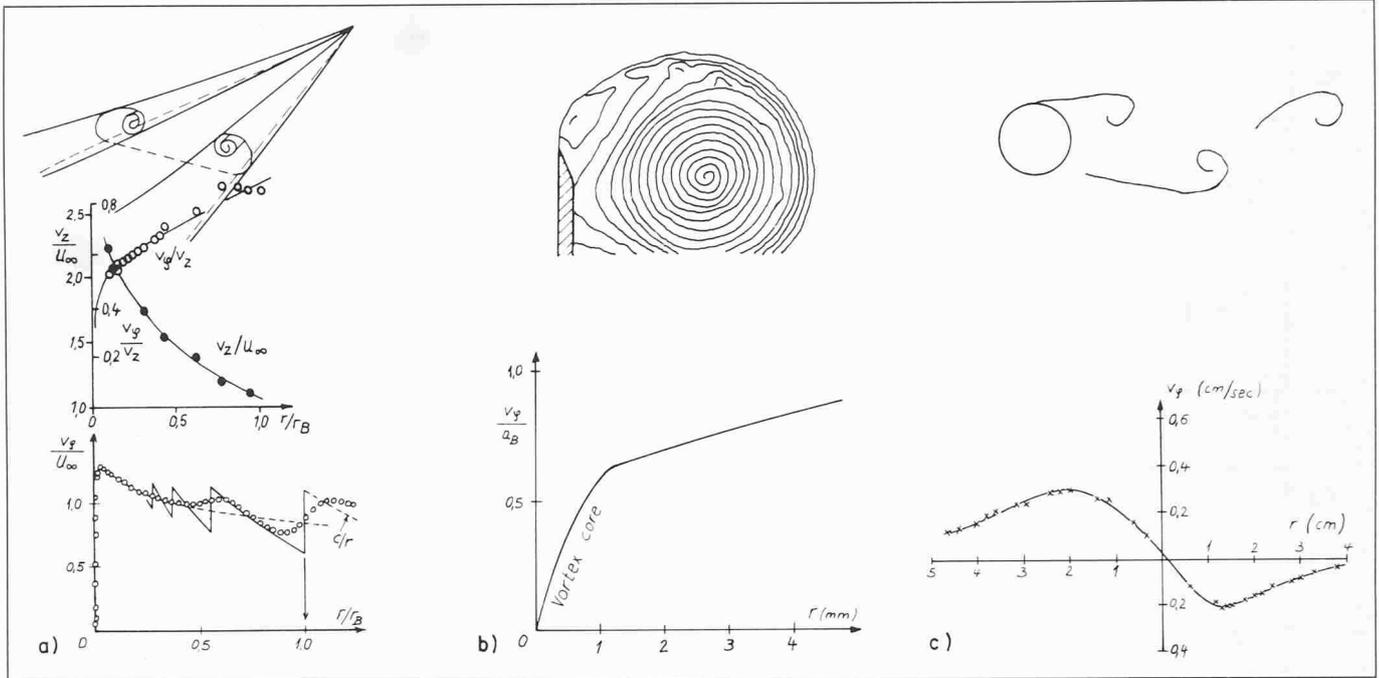
Fig. 4. Comparison between two-dimensional vortex (edge vortex in (b) and Karman vortex in (c) and three-dimensional vortex (conical vortex of a delta wing in (a) with respect to the swirl velocity distribution

ing result obtained by *Straub* as shown in Fig. 5 [12]. A swirling flow is generated in a tube with the exit through a nozzle (diameter ratio 0.52). The profile of the swirl velocity shows a rigid-body pattern just behind the tangential inlet, in which the axial velocity is still moderate. The swirl flow there is made of a viscous core only. However, the profile changes over to a pattern of a narrow rigid-body centre core surrounded by a practically potential vortex, as the swirl flow nears the outlet nozzle and therefore a strong axial velocity is built up in the centre region.
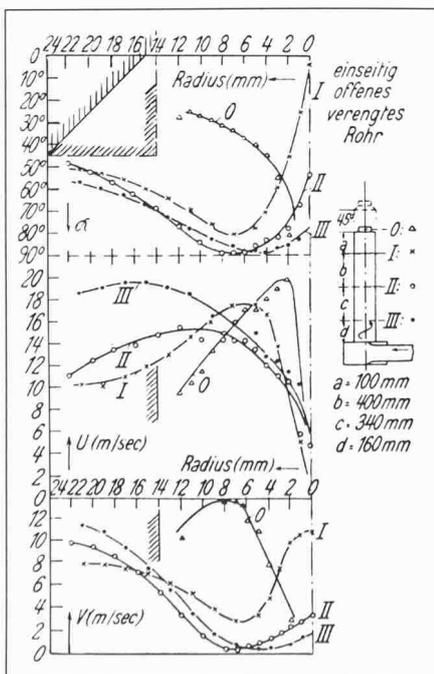
The relationship found in the two foregoing examples can be applied to the bath-tub vortex which incorporates nothing different than a swirling flow with a strong axial velocity in the centre region. The development of the swirling flow in the bath-tub from the initial rigid-body vortex according to the earth's rotation (for $t = 0$) to the Rankine vortex at different stages with increasing time t because of the axial outflow is sketched in Fig. 6 a. The initial angular frequency ω_o due to the earth rotation determines the swirl velocity $v_{\varphi, a}$ on the outer peripheral boundary of the bath-tub, namely

(1) $v_{\varphi}, a = r_a \omega_o$

This boundary condition is valid during the development of the Rankine vortex, see Fig. 6a. The energy of the rotating tank wall in the form of a circulation of $\Gamma = 2 \pi r_a v_{\varphi a} = $ const will be transferred inwards to the water in an inviscid process, and the rigid-body core gives way to the front of this travelling inviscid outer field. We thus obtain a *Rankine vortex* with an outer potential core, in which the following relation applies:

(2) $r_i v_{\varphi}, i = r_a v_{\varphi}, a = \Gamma / 2\pi = $ const

where r_i ist the radius of the rigid-body inner core. If we further overtake the result given by *Marris* about the exponential increase of the swirl frequency of the rigid-body inner core:

(3) $\omega = \omega_o e^{at}$

(4) $a = 0.01$

we obtain

$$\omega = v_{\varphi}, i / r_i = r_a v_{\varphi}, a / r_i^2 = \omega_o r_a^2 / r_i^2 \text{ or}$$

(5) $r_i^2 / r_a^2 = \omega_o / \omega = e^{-at}$ and

(6) $v_{\varphi, i}^2 / v_{\varphi, a}^2 = e^{at}$

The vorticity increases and the inner core radius decreases with time in accordance with the same exponential function. The relationship given in Eqs. (5) and (6) is plotted in Fig. 6b against the time t.

It is obvious from Fig. 6 that the generation of the bath-tub vortex needs a continuous feeding of energy from the earth's rotation in the form of a circulation of $\Gamma = 2\pi r_a v_{\varphi, a}$. Any residual vorticities without further energy supply sources can therefore not be maintained and amplified. They will die out in the course of time. This theory yields for the first time an adequate explanation for the corresponding experimental results obtained by *Shapiro* and *Binnie*.

According to Fig. 4a, the axial velocity distribution of a three-dimensional vortex can be supposed to obey the same law as the swirl velocity, namely

(7) $v_{z, i} = k v_{\varphi}, i = k v_{\varphi}, a e^{at} / 2$.

Binnie & Hooking found the following relationship

(8) $v_{z, av} / v_{\varphi}, i = \{(R^2 - r_i^2) / 2 r_i^2\}^{1/2}$

for a vortex at its final formation stage, at which a built-up air core of radius r_i penetrates through the draining tube (radius R) with an average axial velocity $v_{z, av}$ in water. The derivation of the above equation was based on the theory of travelling of an annular wave in the
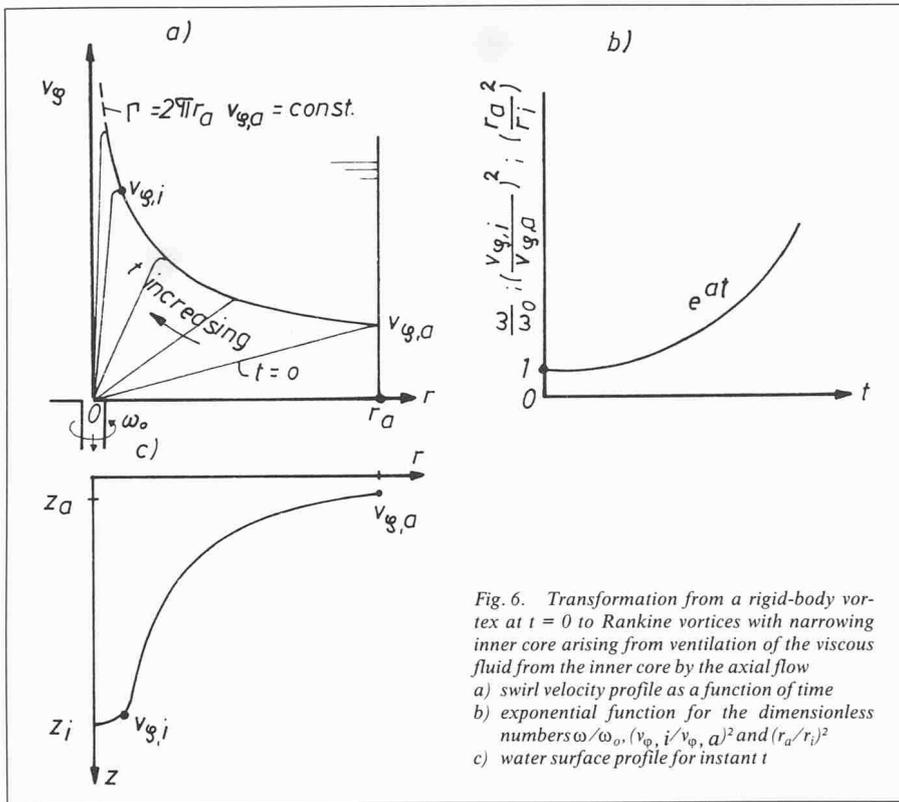

Fig. 5. Velocity distribution of a swirling flow in a tube with a nozzle at outlet (U = swirl velocity, V = axial velocity, φ = swirl angle)

Fig. 6. *Transformation from a rigid-body vortex at $t = 0$ to Rankine vortices with narrowing inner core arising from ventilation of the viscous fluid from the inner core by the axial flow*
a) *swirl velocity profile as a function of time*
b) *exponential function for the dimensionless numbers ω/ω_o, $(v_{\varphi,i}/v_{\varphi,a})^2$ and $(r_a/r_i)^2$*
c) *water surface profile for instant t*

axial direction on the rotating free surface. The authors found for this final formation stage of the air core a radius of $r_i / R = 0.5$, which yields $v_{z,av} / v_{\varphi,i} = 1.2$. This value then approximately corresponds to the factor k for a bath-tub vortex at its built-up stage:

(9) $\quad k = 1.2$

The energy equation yields the following equation about the conditions on the surface of the hollow core (Fig. 6c):

(10) $\quad -z_a + w^2_a / 2g + p_a / \gamma = -z_i + w^2_i / 2g + p_i / \gamma$

(11) $\quad$ or $w^2_i = w^2_a + 2g(z_i - z_a)$ where

(12) $\quad p_a = p_i = p_o$ atmospheric pressure

The velocity w_i at the inner core takes a spiral path with swirl and axial components:

(13) $\quad w_i = v_{\varphi,i} \hat{+} v_{z,i}$

For a rough calculation we neglect the axial component first, so that we may simplify Eq. (11) to the following form

(14) $\quad v^2_{\varphi,i} = v^2_{\varphi,a} + 2g(z_i - z_a)$ or

(15) $\quad (\Gamma / 2\pi r_i)^2 = (\Gamma / 2\pi r_a)^2 + 2g(z_i - z_a)$

and

(16) $\quad 1/r^2_i - 1/r^2_a = 2g(2\pi / \Gamma)^2 (z_i - z_a)$

The surface of the hollow core is a parabola, and the radius r_i of the inner core is proportional to the circulation $\Gamma = 2\pi r_a v_{\varphi,a}$, and inversely proportional to the depth

(17) $\quad S = z_i - z_a$

of the hollow core:

(18) $\quad r_i \sim \Gamma / S$

In reality, the generation of the air containing inner core is a process of instability as shown in part "Properties". Let us assume, for example, that a flat dimple is formed first on the surface of the water. A narrow inner core of spinning water will be generated descending with considerable axial velocity from the base of the dimple, as shown in Fig. 3a [8]. It is well known that a vortex with an axial flow is unstable. The stability of a vortex can be checked using the *Ludwieg map* as shown in fig. 7a according to the suggestion of *Brown* and *Chen* [17, 18]. The axes of the diagram are the dimensionless velocity gradients defined as follows

(19) $\quad \bar{c}_\varphi = (r / v_\varphi)(\partial v_\varphi / \partial r)$ for the swirl velocity and

(20) $\quad \bar{c}_z = (r / v_\varphi)(\partial v_z / \partial r)$ for the axial velocity

We obtain for the inner core with uniform rotation $\bar{c}_\varphi = + 1$, for the outer core with potential flow $\bar{c}_\varphi = - 1$ and for the boundary in between $\bar{c}_z = - k = - 1.2$. Referring to the *Ludwieg map,* this boundary is thus unstable. In fact, the swirl velocity distribution shown in Fig. 6 is greatly simplified.

The inner-core region tends toward the minimum kinetic energy state with a stress-free and uniform rotation. However, the conservation of angular momentum requires core edge velocities to

increase with an overshoot above the original potential flow values (Fig. 7b), as found by *Donaldson & Sullivan* and *Govindaraju & Saffman* for a built-up vortex [19, 20]. The instability of this core edge will thus be still more enhanced.

The intensity of turbulence formed at the inner core edge due to this instability mechanism is proportional to the velocity at this edge:

(21) $\quad (q^2)^{1/2} = \beta v_{\varphi,i} = \beta v_{\varphi,a} e^{at} / 2$

Thus the intensity of turbulence will reach considerable values at advanced stages of the bath-tub vortex having high angular velocity of the inner core. The spinning inner core will thus be very turbulent as it becomes small, so that the transfer of swirling energy from the outer core to the inner core will be increasingly accelerated. This process will promote still greater axial velocity through the inner core which in turn strengthens the instability of the vortex, and thus the turbulence. In this manner, the feedback loop is closed. The result is the rapidly deepening of the dimple until a hollow vortex with a deep air core down to the draining tube is formed.
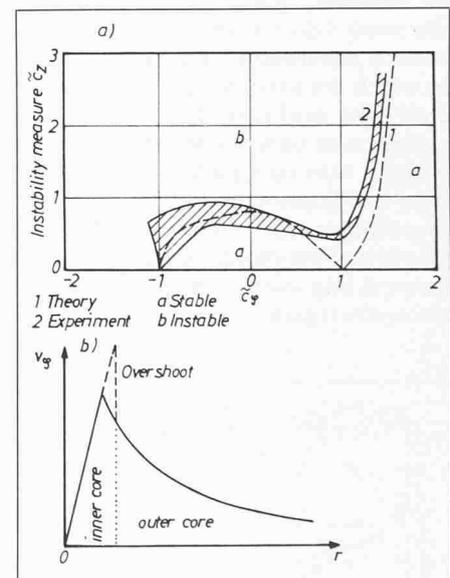

Fig. 7. *Instability of three-dimensional vortex due to axial flow*
a) *Instability map according to the theoretical and experimental investigations carried out by Ludwieg for rotating annular flow with axial velocity*
b) *Overshoot at the boundary between the inner and outer core near the final stage of the vortex formation (Brown, Donaldson & Sullivan, Govindaraju & Saffman)*

The rapid completion of the deep air core is thus a phenomenon of instability due to the feedback mentioned. This is quite similiar to the breakdown of a vortex to form an inner conical hollow surface when entering an expansion chamber, and when the swirl intensity of the vortex exceeds a critical value (Fig. 11c). An axial flow in the form of a toriodal vortex ring will be induced in-

side this hollow cone. The reversal axial flow zone occupied by the vortex ring can be strongly unstable, which then induces precessing vortex filaments leading to acoustic vibration in the surroundings, as encountered in combustion chambers [21]. The axial velocity arising in the bath-tub vortex finds somewhat of a counterpart in this vortex-breakdown process.

The generation of turbulence at the inner core edge finds further support in the experiment of *Binnie & Hookings* about the hollow bath-tub vortex in its initial formation stage without deep air core, as reproduced in Fig. 3a. The inner core composed of a narrow spinning tube of liquid exhibits an increasing radius as it descends downwards. If we form the *Bernoulli equation* including a term of the energy loss h_l due to turbulence:

$$(22) \quad v^2_{\varphi, i} = v^2_{\varphi, i, 0} + 2g\{(z - p/\gamma) \\ - (z_0 - p_0/\gamma) - h_l\}$$

and introduce

$$(23) \quad z - p/\gamma \cong z_0 - p_0/\gamma$$

we obtain

$$(24) \quad v^2_{\varphi, i} = v^2_{\varphi, i, 0} - 2gh_l$$

The swirl velocity of the surface of this spinning tube will decrease downwards due to energy loss, resulting in the increase of its radius owing to the conservation of the circulation. Thus, the generation of turbulence on the inner core edge is confirmed indirectly by the observation of *Binnie & Hookings*.

Vortices in pump intake sumps

Introduction

The formation of a vortex in the sump can lead to rotation of water in the suction pipe of the pump. In the extreme case, air can be drawn through the centre of the vortex into the intake, which will impair the performance of the pump, e.g. reduction in discharge, drop in efficiency, uneven loading of the impeller resulting in vibration and probably damage to the bearings.

If the pump has a horizontal intake, a vortex shown in Fig. 1 can be formed in the sump, see Fig. 8. As the properties of this vortex are the same as that of the bath-tub vortex, the theory developed before can then be applied for the present case.

For the maintenance of this intake vortex, a continuous feeding of energy in the form of a constant circulation is needed. This circulation can arise either from an uneven velocity distribution of the incident flow in the sump, resulting in a large-scale rotation in the flow (as

shown in Fig. 8), or from the preswirl of the impeller transmitted from the suction pipe to the sump. The vorticities in the water of the sump without any continuous energy feeding to keep their circulation cannot cause the generation of the intake vortex according to the theory.
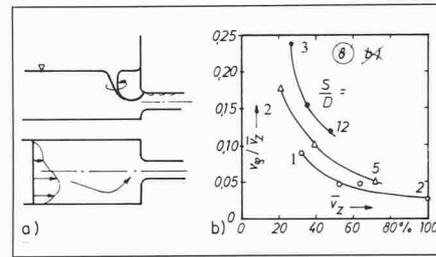

Fig. 8.   Formation of an intake vortex for a horizontal pump suction pipe (Prosser)

The measured result obtained by *Binnie & Hookings* as stated in part «Properties» about the sudden appearance of a powerful air core in a tank with a free drain was confirmed by *Anwar, Weller & Amphlett* in a recent work about the formation of the intake vortex in a model sump, using artificial swirl vanes at the suction pipe inlet of the pump as a generator of the preswirl to be sent into the sump [22]. These authors found that «with further increase in Q or decrease in the water level, the flow in the vortex reached a stage where a sudden expansion of the air core was observed: in this case a great deal of air was drawn down the inlet pipe.» The axial flow in the intake vortex will be, however, effected by the suction action of the pump, this in addition to the natural hydraulic head which is alone effective for the case of the bathtub vortex. Then, depending on the intensity of the pump impeller preswirl and/or the sump rotating flow on one hand, and the pump suction capability on the other hand, any ratio between the swirl and axial velocity in the suction pipe can be obtained.

The influence of the axial velocity on the initiation of the intake vortex of such an intensity that its air entraining inner core can just reach the suction pipe intake without any further expansion, can be evaluated from the experiment of the three authors quoted previously, in which the intensity of the swirl in the sump is varied by adjusting the attack angle of the swirl vane of the suction pipe. The evaluated result about the swirl velocity v_{φ} on the suction tube (normalized by $\bar{v}_z$) is shown in Fig. 8b as a function of the mean axial velocity $\bar{v}_z$ (in percentage of the total velocity w available from the hydraulic head corresponding to the submergence $S = z_{i,} - z_a$ Eq. (11)), with $S/D = 2$, 5 and 12 as parameter, see curves 1, 2 and 3.

Curves 1, 2 and 3 reveal, that with increasing axial velocity, the intensity of the preswirl needed for the initiation of

the intake vortex specified above decreases rapidly. This demonstrates very clearly that an axial velocity accelerates the formation of the intake vortex. These curves further show that for a higher water surface (a higher submergence S), a stronger preswirl in a proportion of approximately $S^{1/2}$ is needed for this initiation. This proportion corresponds very well to the theoretical consideration given in Eq. (11).

Anwar et al discovered a second weak vortex which appeared from the floor to the intake with the same rotating sense as the free-surface intake vortex. Both vortices then entered the intake with opposite direction of rotation. "The strength of the second vortex was reduced by roughening the floor using expanded metal mesh; this had no apparent effect on the air-entraining intake vortex." This second vortex will be referred to as submerged vortex in the literature. It obviously originates from the vorticity in the boundary layer on the sump floor as argued in the mathematical models of *Marris* and *Sibulkin* [2, 23]. This measurement yields a further evidence that the formation mechanism of the intake vortex will not be influenced by the boundary of the sump floor or just the same as that of the bath-tub vortex as shown by Binnie [3].

Vortex-separation of vertical suction pipe as cause of intake-vortex formation

The formation of the intake vortex for a suction pipe in the vertical position seems to be much more involved than for a suction pipe in the horizontal position. However, the experimental results obtained by *Denny* and *Markland & Pope,* as well as *Denny & Young* can
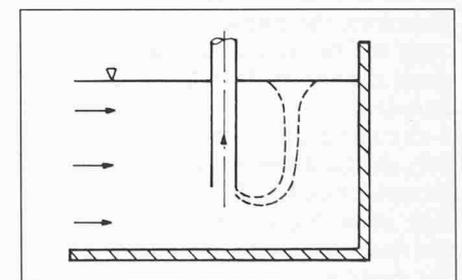

Fig. 9.   Formation of an intake vortex in the wake of a vertical pump suction pipe

serve as a basis for the establishment of a theory [6, 7, 13]. First of all it was observed by these authors that the intake vortex was always formed on the lee side of the suction pipe, see Fig. 9. *Markland* and *Pope* further found that "the normal condition observed was a pair of vortices intersecting the water surface at a radius of about 5 inches on the edge of the wake of the suction pipe (diameter 2.58 inches, with a bellmouth of diameter 4.6 inches at the tube inlet) in the general flow, and of opposite rotations.

After a struggle for mastery, one vortex would begin to entrain air, and this would continue for a period of approximately half a minute. The air core would then break, the vortex would usually decay to a certain extent and the other vortex would then grow and start to admit air. This cycle would repeat almost indefinitely. Occasionally, however, both vortices discharged air simultaneously for a period of several seconds. Only a small degree of asymmetry of general flow in the sump was capable of upsetting the formation of these two roughly equal and opposite vortices. ... (If however) a slow clockwise swirl was introduced into the sump, the swirl was sufficient to favour the vortex formation only on one side of the suction tube (i.e. the vortex with the clockwise rotation)." These authors carried out a further test with a streamline – shaped mantle sheet fitted around the suction pipe in order to prevent abrupt separation of flow in the wake (Fig. 10).
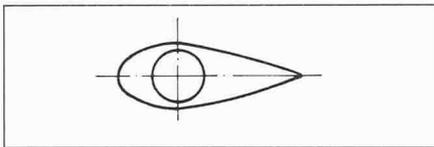

Fig. 10.   A vertical suction pipe with a streamlined overcoat

No air entrainment was then noted even with submergences (i.e. the depth of the suction pipe intake below water surface) of less than ½ inch and with maximum flow. Thus the formation of the intake vortex in the sump can apparently be connected with the separation of vortices of the suction pipe in its wake, because it always takes place on the lee side of the suction tube (Fig. 9). Therefore, the phenomena cited previously will be explained using the theory about the vortex shedding of a circular cylinder.

A circular cylinder in a cross flow can only shed a *Karman vortex* street if the *Reynolds* number $R = U_\infty d/\nu$ is higher than about 80, and if there is enough free space behind the cylinder to enable this shedding.

If these conditions cannot be fulfilled, only a twin vortex rotating as rigid-body in opposite sense will be formed behind the cylinder, attaching on its base. As is well known, this vortex pair is not stable at all. One vortex grows at the expense of the other one, and then diminishes again in order to enable the other one to grow. This alternate change between growth and diminution takes place within a certain period. The suction pipe of a pump will be usually installed in a channel very close to the rear wall. Then only a twin-vortex can be formed behind it, see Fig. 11a.

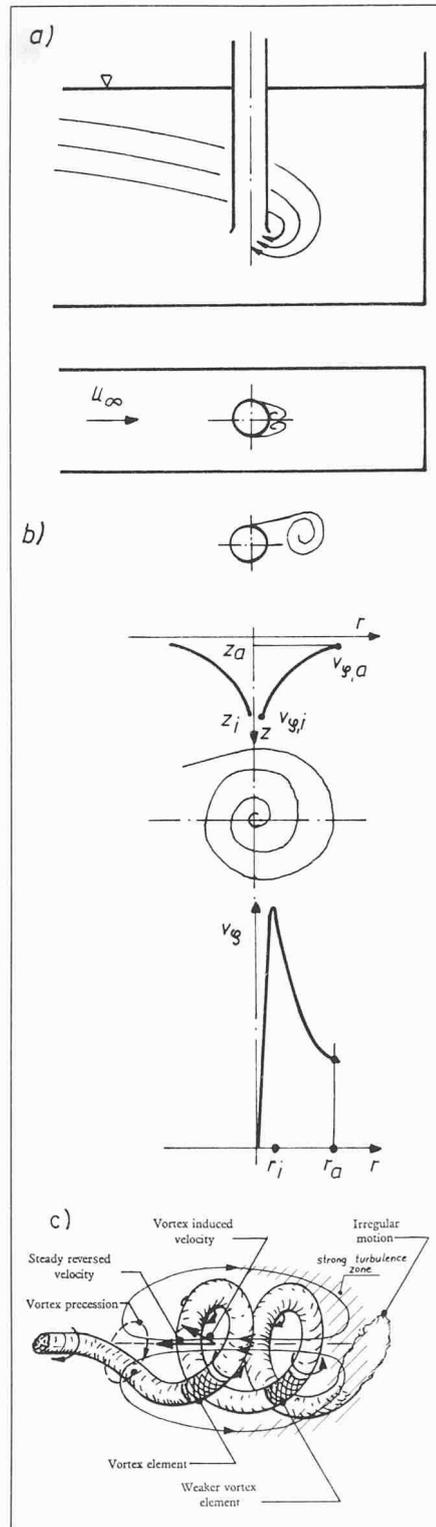As a certain amount of the flow will stream around the tube in an inclined

Fig. 11.   a) Formation of a twin vortex behind the suction pipe
b) Amplification of one of the vortex pair due to swirl in the flow
c) Vortex breakdown of a swirling flow with formation of axial flow in the form of a vortex ring within the hollow cone

direction to the bellmouth intake, there exists an axial flow through this vortex pair (Fig. 11a). This axial flow will make the twin vortex changing pattern from the rigid-body swirl to a *Rankine vortex* with a central core of increased swirl velocity, a mechanism, which has already been discussed before. The one of the vortex pair which is just in the growing period can then grow to a

strong Rankine vortex with a large air core, whilst the other vortex diminishes because of its decaying period. If a swirl is present in the flow, either caused by the uneven distribution of the flow or induced by the preswirl of the pump impeller, the one of the vortex pair having the same rotation sense as the swirl will then be amplified at the expense of the other one. Then, only one intake vortex will be formed in the wake of the suction tube (Fig. 11b). The dimension of the vortex is of the same order of magnitude as the diameter D of the suction pipe. The swirl velocity of the outer shell of the vortex is practically $v_{\varphi, a} = U_\infty/2$. Then a circulation of

$$(25) \quad \Gamma = \pi\, D\, U_\infty/2$$

is fed into the vortex by the separated shear layer. In the outer core of the vortex a potential flow field $\Gamma = $ const will be generated by the axial flow, so that a swirl velocity

$$(26) \quad v_{\varphi, i} = D\, U_\infty/4\, r_i$$

will be induced at the boundary of the rigid-body inner core, see Fig. 11b. This velocity can be very large if both the incident flow and the axial flow are strong. Then the inner vortex core will become hollow with air entrainment through it.

Measures of prevention of intake-vortex formation

It is thus clear that the effective measures for the prevention of the formation of the intake vortex in the sump must be first of all capable of keeping the formation of both the twin-vortex and the axial flow behind the suction pipe as weak as possible. This can be achieved by making the distance of the intake bellmouth to the side wall, the rear wall and the floor of the sump as small as possible.

A large sump length is also needed to make the flow have a uniform velocity distribution before it approaches the intake of the suction. Any asymmetry of the incident flow to the suction pipe is to be avoided. A number of examples is shown below.

The experimental result about the critical submergence, which is needed to prevent the air entrainment of the intake vortex to the suction pipe, as obtained by Iversen, is re-evaluated as shown in Fig. 12 with (S+C/2)/D as ordinate and the bottom clearance C/D between bellmouth and sump floor as parameter [14]. The critical submergence S decreases continuously with the decrease of the distance x of the bellmouth from the rear wall and with the decrease of the bottom clearance from C/D = 2.0 to 0.5. The same is true for the side wall clearance Z/D, see Fig. 13 according to the measurement carried
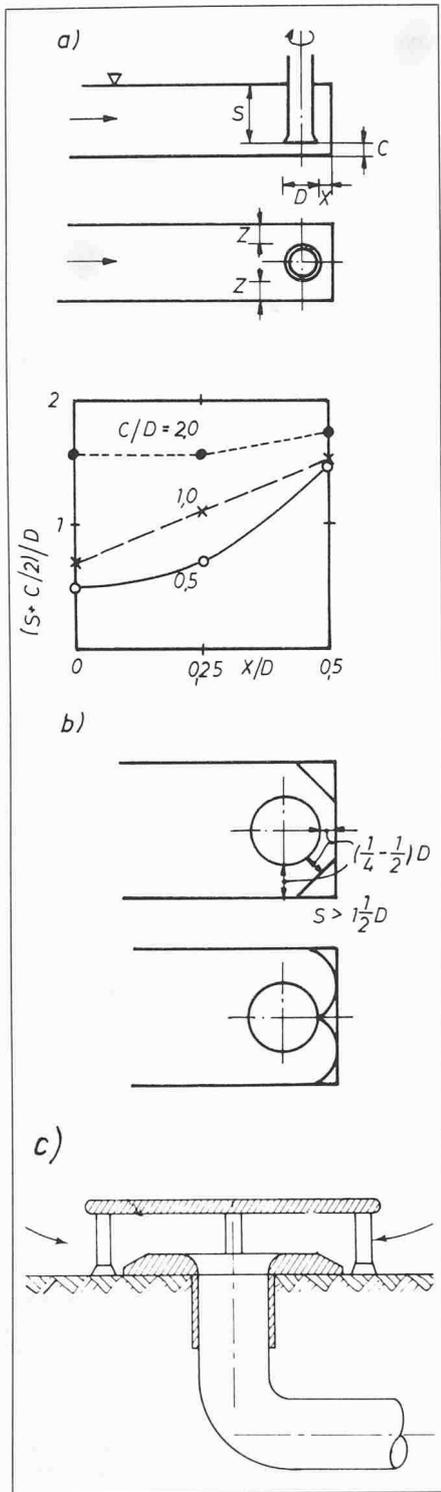
Fig. 12. a) *Plotting of the critical submergence* $(S+C/2)/D$ *against rear wall clearance* X/D *with floor clearance* C/D *as parameter*
b) *Recommendation for prevention of intake vortex (Prosser)*
c) *An extension plate over the vertical suction pipe inlet for reducing the tendency to form intake vortex*

$C/D = 1.$ $U_\infty = 13.5$ ft. per sec.

• Sumps 2 feet and 8 feet
○ Sump   1 foot
+ Sump   6 inches

Fig. 13. *Effect of wall clearance* $X/D = Z/D$ *on critical submergence (Denny)*

a 4-inch pipe in 8-foot sqare sump.   W = 16 inches
b $\frac{7}{8}$-inch pipe in 2-foot sump.   W = 4 inches
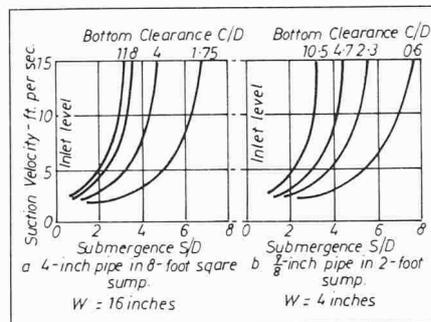
Fig. 14. *Effect of flow rate on critical submergence with bottom clearance as parammter (Denny)*

$C/D = 1.0.$ $D = 1\frac{1}{2}$ inches $U_\infty = 15$ ft. per sec.
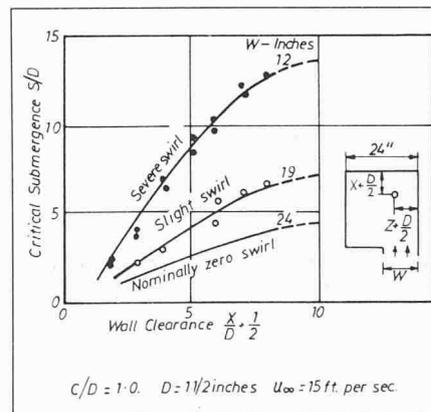
Fig. 15. *Critical submergence versus wall clearance* $X/D = Z/D$ *with swirl as parameter (Denny)*

out by *Denny* [6]. However, the higher flow will increase the critical submergence as shown in Fig. 14 according to the same author. If a swirl is present due to asymmetrically incident flow, the critical submergence will still be higher, see Fig. 15 given also by *Denny*.

A rear wall clearance of zero ($X/D = 0$) is of course not realistic, since the flow will only stream into the suction pipe from its frontal area. A certain value of
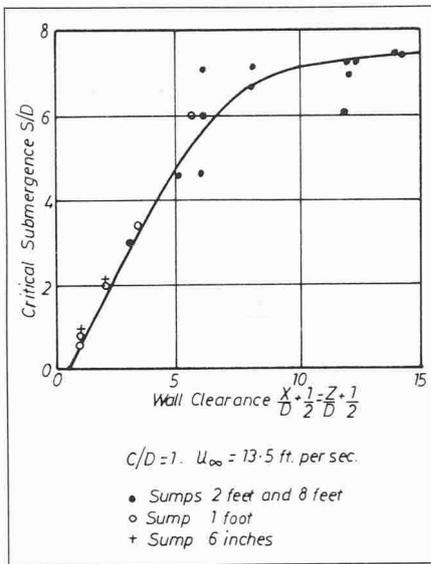
this clearance must be chosen in order to enable a somewhat uniform flow profile at the intake. This is obviously the case if $X/D = \frac{1}{4}$ is chosen in combination with $Z/D = 0.5$ (up to 1.0), $C/D = \frac{1}{2}$, and $S/D = 1\frac{1}{2}$, as recommended by *Prosser* [15]. Then, the stagnation pressure field next to the rear wall will be able to maintain a flow through the rear part of the suction pipe intake nearly as large as that through its fron-

tal part. Some design feature summarized in [15] is reproduced in Fig. 12b.

The effectiveness of the submerged roof of the suction sump (as described by *Siekmann & Holzhüter* in [16]) is purely due to the fact that the lower end of the suction pipe is exposed to the incident flow for as short a distance as possible in a sump without free surface (Fig. 16a). In this manner, the separation of the vortex behind the suction pipe can be prevented. The conical body installed on the floor opposite to the intake with a longitudinal vane in Fig. 16a will in addition act as a suppressor against the preswirl of the pump impeller to travel into the sump and thus to amplify the separated vortex behind the tube, should any be present. If, however, the submerged roof is too short, and the flow is separated in front of the suction pipe, as shown in Fig. 16b (according to *H. Addison* in [6]), the eddies generated by the flow separation can favour the formation of the intakevortex again, which would not be in the form of a free-surface vortex, but would rather appear as roughly periodically separated vortex travelling onto the pump impeller and then causing low frequency vibrations in the pump system. A similar phenomenon can arise at the intake of a suction pipe
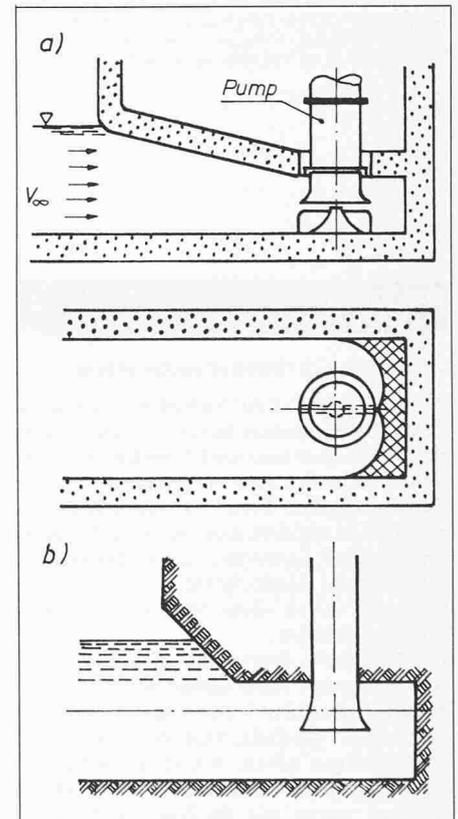

Fig. 16. *Eliminating the free surface of the sump using an overroof*
a) *long, inclined overroof for achieving a uniform, smooth flow in front of the suction pipe, whose intake end exposed to the flow is limited to a short length:*
b) *poorly designed overroof causing separation of flow in front of the suction pipe*

whose inlet is vertically connected with the bottom of the sump. An extension plate over the inlet can reduce the tendency to form intake vortex. However, improper designed guide vanes installed on it could lead to inadmissible separation of flow leading to vortex shedding periodically into the suction pipe, which then impairs the smooth run of the impeller. An example of an adequate design is given in Fig. 12c *(Prosser)*. The mean velocity at intake section should be less than 0.6 m/s.
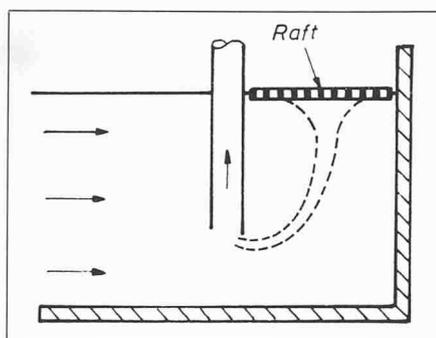
The floating raft showed by *Denny &*

Fig. 17. A floating raft for damping the intake vortex (Denny & Young)

Young has obviously its function in the damping of the intake vortex (Fig. 17, [13]).

***

Address of the author: *Dr. Y. N. Chen*, Research Laboratory for Vibration and Acoustics, Sulzer Brothers Limited, 8401 Winterthur.

References

[1] *Shapiro, A.H.*, Film "Vorticity". Educational Services Inc., Watertown, Mass., 1961. See also *Shapiro, A.H.*: Four minute film Loop No. FM-15, "The bath-tub vortex". Educational Service Inc., Watertown, Mass. 1963 (cited in [2])

[2] *Marris, A.W.*: "Theory of the bathtub vortex". Trans. ASME, Journal of Applied Mechanics 1967, March, pp. 11–15

[3] *Binnie, A.N.*: "Some experiments on the bath-tub vortex". Journal of Mechanical Engineering Science, vol. 6, 1964, pp. 256–257

[4] *Shapiro, A.H.*: "Bath-tub vortex". Nature, vol. 196, 1962, pp. 1080–1081

[5] *Trefethen, L.M., Bilger, R.W., Fink, P.T., Luxton, R.E.,* and *Tanner, R.I.*: "The bath-tub vortex in the Southern Hemisphere". Nature vol. 207, 1965, pp. 1084–1085

[6] *Denny, D.F.*: "An experimental study of air-entraining vortices in pump sumps". Proceedings, Institution of Mechanical Engineers vol. 170, 1956, pp. 106–125

[7] *Markland, E.* and *Pope, J.A.*: "Experiments on a small pump suction well, with particular reference to vortex formations". Proceedings, Institution of Mechanical Engineers, vol. 170, 1956, pp. 95–105

[8] *Binnie, A.M.* and *Hookings, G.A.*: "Laboratory experiments on whirlpools". Proc. Royal Society, series A, vol. 194, 1948, pp. 398–415

[9] *Chen, Y.N.*: "The vortex shedding". 2nd Colloquium on Industrial Aerodynamics, 1976, Fachhochschule Aachen. Proceedings pp. 107–133

[10] *Howard, L.N.* and *Mathews, D.L.*: "On the vortices produced in shock diffraction". Journal of Applied Physicc, vol. 27, No. 3, March 1956, pp. 223–231

[10a] *Earnshaw*, (cited by *Küchemann, D. & Weber, J.*: "Vortex motion". Zeitschrift für angewandte Mathematik und Mechanik, vol. 45, 1965, pp. 457–474)

[11] *Timme, A.*: "Über die Geschwindigkeitsverteilung in Wirbeln". Ingenieur-Archiv, vol. 25, 1957, pp. 205–225

[12] *Traub, E.*: "Über rotierende Strömungen in Rohren und ihre Anwendung zur Stabilisierung von elektrischen Flammenbogen". Annalen der Physik, 5. Folge, vol. 18, 1933, pp. 169–192

[13] *Denny, D.F.* and *Young, G.A.J.*: "The prevention of vortices and swirl at intakes". Proceedings IAHR Congress. Lisbon 1957, pp. C1/1 – 10 with figures 1–14

[14] *Iversen, H.W.*: "Studies of submergence requirements of high-specific-speed pumps". Trans. ASME vol. 75, 1953, pp. 635–641

[15] *Prosser, M.J.*: "The hydraulic design of pump sumps and intakes". British Hydromechanics Research Association, July 1977

[16] *Siekmann, H.* and *Holzhüter, E.*: "Pump intake design of high-specific-speed pumps". (Die Gestaltung der Einlaufkammer für spezifisch schnelläufige Pumpen), Pumpentagung Karlsruhe 73, 2.–4. Oct. 1973, Paper K3

[17] *Brown, C.E.*: "Aerodynamics of wake vortices". AIAA, Journal, vol. 11, 1973, pp. 531–536

[18] Chen, Y.N.: "Damping of Karman-vortex-induced vibration by axial flow". Sulzer Technical Review, 1978, Research Number, pp. 27–30

[19] *Donaldson, C. du P.* and *Sullivan, R.D.*: "Decay of an isolated vortex. Aircraft wake turbulence and its detection". Plenum Press, New York, 1971, pp. 389–411

[20] *Govindaraju, S.P.* and *Saffman, P.G.*: "Flow in a turbulent trailing vortex". The Physics of Fluids, vol. 14, 1971, pp. 2074–2080

[21] *Chen, Y.N.*: "Untersuchungen der Drallströmung im Feuerraum im Hinblick auf die Flammenstabilität (Studies into swirling flow in furnaces with a view to flame stability)". Gas Wärme international, vol. 28, 1979, pp. 35–50

[22] *Anwar, H.O., Weller, J.A.* and *Amphlett, M.B.*: "Similarity of free-vortex at horizontal intake". Journal of Hydraulic Research, vol. 16, 1978, No. 2, pp. 95–105

[23] *Sibulkin, M.*: "A note on the bathtub vortex". Journal of Fluid Mechanics, vol. 14, 1962, pp. 21–24

Umschau

Urmaterie auch chemisch nachgewiesen

Auf der Erde und mit Sicherheit auch in anderen Teilen unseres Sonnensystems gibt es Stoffe, die älter sind als 4,5 Milliarden Jahre – älter also als unser Planetensystem. Über diese «präsolare Materie» oder auch «Urmaterie» ist auf dem diesjährigen Jahreskongress (3. bis 7. September) der weltweiten Gesellschaft für Meteoritenforscher in Heidelberg zum ersten Male öffentlich ausgiebig diskutiert worden.

Die Urmaterie, deren Alter auf sechs bis sieben Milliarden Jahre datiert wurde, ist von Meteoritenforschern in Einschlüssen von Meteoriten entdeckt worden. Die ersten Überlegungen gehen, wie in Heidelberg zu erfahren war, auf das Jahr 1974 zurück. In Chicago waren die Professoren *L. Grossman, R. Clayton* und ihre Mitarbeiter auf die Idee gekommen, zu untersuchen, was die aus dem Kosmos auf die Erde niedergegangenen «Himmelskörper» in den Einschlüssen, deren Existenz der Wissenschaft schon lange bekannt war, eigentlich verbergen. Die moderne Forschung hat neue Methoden der Altersbestimmung von Materie entwickelt, die

auf dem radioaktiven Zerfall basiert. Sie feierte Triumphe bei der Altersbestimmung der von den Apollo-Missionen herbeigeholten Mondmaterie.

Als besonders ergiebig erwies sich, was die eingeschlossene Materie anbelangt, der etwa 2000 Kilogramm schwere Meteorit Allende, der grösste bisher auf die Erde niedergegangene Gesteinsmeteorit. Er war im Frühjahr 1969 auf mexikanisches Territorium etwa 300 Meilen westlich vom amerikanischen NASA-Zentrum in Houston gefallen. Clayton und seinem Team gelang es zum ersten Male, das Urmaterial in den Allende-Einschlüssen zu isolieren und den Beweis zu führen: Es ist älter als die Materie unseres Sonnensystems, muss also aus Bezirken der Milchstrasse stammen, die um Milliarden Jahre vor unserem Sonnensystem entstanden. «Also stimmt die Hypothese nicht, dass alles, woraus sich unser Sonnensystem zusammensetzt, gebildet wurde, als die Materie bei der Rotation von Gas- und Staubmassen (Nebel) sich zu flüssigen und festen Körpern kondensierte», folgerten Grossman und

Clayton.

In Heidelberg war darüber hinaus zu erfahren, dass inzwischen für die präsolare Herkunft des Einschlussmaterials auch der chemische Beweis geführt wurde. Er stammt von dem Ägypter Dr. *A. El Goresy*, der als Schüler des neunzigjährigen Heidelberger Mineralogen *Paul Ramdohr* am Heidelberger Max-Planck-Institut für Kernphysik arbeitet. Er stellte fest, dass die in Meteoriten eingeschlossene Materie «stark oxidiert» ist. Stammte sie aus dem Nebel unseres Planetensystems, so El Goresy, «könnte sie nicht oxidiert sein, denn unser solarer Nebel war durch die starke Häufigkeit des Wasserstoffs charakterisiert». Dies aber hätte «zum Verschwinden des Sauerstoffs durch Verbindung mit dem Wasserstoff geführt, so dass eine Oxidation der Einschlussmaterials nicht hätte stattfinden können». El Goresy konnte seine chemische Analyse mit einer Elektronenmikrosonde realisieren, die es ermöglicht, Materie in einer Grössenordnung von nur einem Millionsten Teil eines Kubikmillimeters chemisch zu analysieren.