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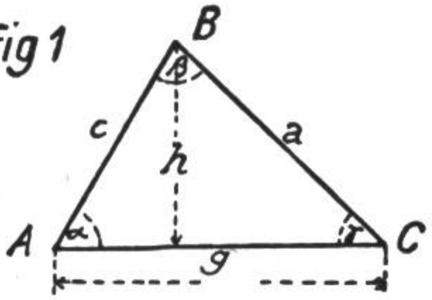
GEOMETRIE

Tafel I

Formeln zur Inhaltsberechnung von Flächen und Körpern.

Fig 1



Dreieck:

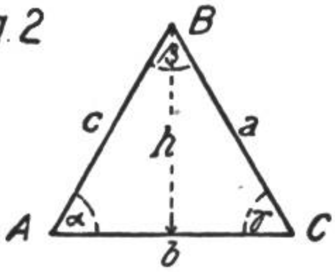
Grundlinie = g; Höhe = h; Fläche = F

$$F = \frac{g \times h}{2} = \frac{g}{2} \cdot h = \frac{h}{2} \cdot g;$$

$$g = \frac{2F}{h}; \quad h = \frac{2F}{g}$$

$$\sphericalangle \alpha + \sphericalangle \beta + \sphericalangle \gamma = 180^\circ = 2R$$

Fig. 2



Gleichseitiges Dreieck:

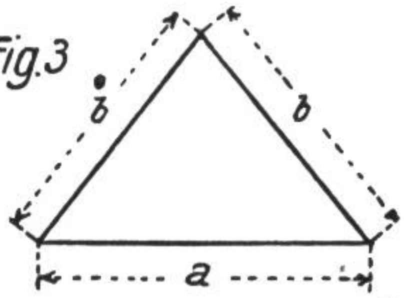
Seiten = $a = b = c$; $\sphericalangle \alpha = \beta = \gamma = 60^\circ$

$$F = \frac{a^2}{4}\sqrt{3} = 0.433\, a^2$$

(genauer $0,4330127\, a^2$)

$$h = \sqrt{a^2 - \frac{b^2}{4}} = \sqrt{a^2 - \frac{c^2}{4}};$$

Fig. 3

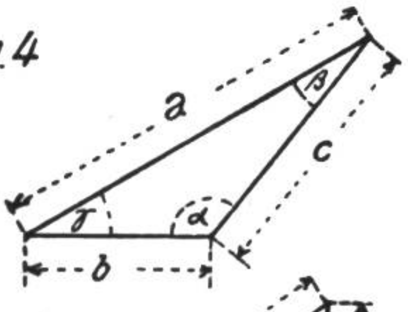


Gleichschenkliges Dreieck.

Grundlinie = a, gleiche Seiten = b

$$F = \frac{a}{4}\sqrt{(2b+a)\cdot(2b-a)} = \frac{a}{2}\sqrt{\left(b+\frac{a}{2}\right)\cdot\left(b-\frac{a}{2}\right)}.$$

Fig. 4

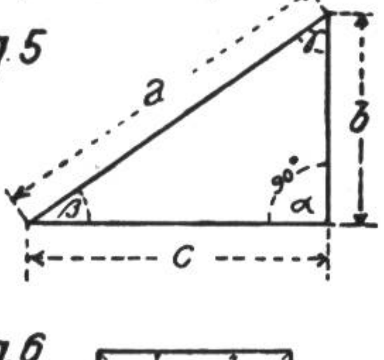


Ungleichseitiges Dreieck:

Seiten a, b und c, $s = \frac{a+b+c}{2}$

$$F = \sqrt{s(s-a)(s-b)(s-c)}.$$

Fig. 5



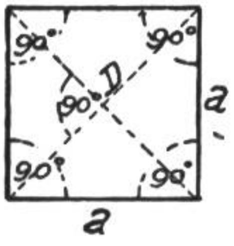
Rechtwinkliges Dreieck; $\sphericalangle \alpha = 90^\circ$

Hypothenuse = a, Katheten = b und c.

$$F = \frac{b \cdot c}{2}; \quad a^2 = b^2 + c^2; \quad a\sqrt{b^2 + c^2};$$

$$b = \sqrt{a^2 - c^2}; \quad c = \sqrt{a^2 - b^2}.$$

Fig. 6



Quadrat:

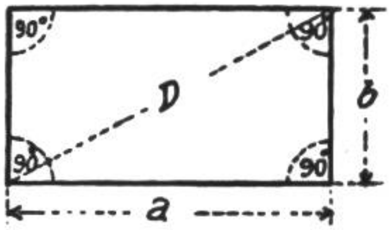
Seite = a. Diagonale = D.

$$F = a \times a = a^2 \qquad a = \sqrt{F}$$

$$D = \sqrt{a^2 + a^2} = \sqrt{2a^2} = a\sqrt{2} = a \cdot 1{,}4142$$

E. Pochan

Fig. 7



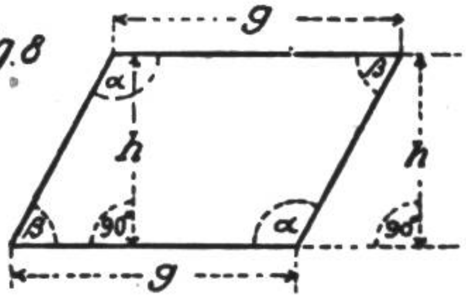
Rechteck:

Seiten = a und b, Diagonale = D

$$F = a \cdot b; \quad a = \frac{F}{b}; \quad b = \frac{F}{a};$$

$$D = \sqrt{a^2 + b^2}.$$

Fig. 8

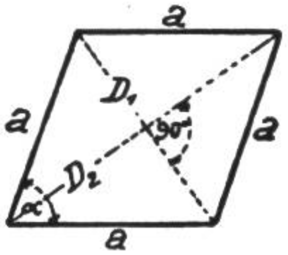


Parallelogramm:

Grundlinie = g; Höhe (rechtwinklig auf Grundlinie) = h

$$F = g \cdot h; \quad g = \frac{F}{h}; \quad h = \frac{F}{g};$$

Fig. 9

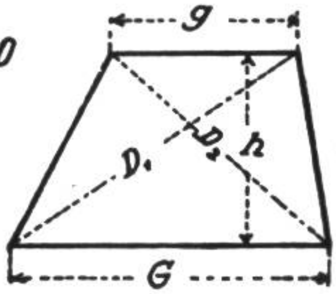


Rhombus:

Gleiche Seiten = a; Diagonalen D_1 u. D_2

$$F = a^2 \sin\alpha; \quad F = \frac{D_1 \cdot D_2}{2}.$$

Fig. 10



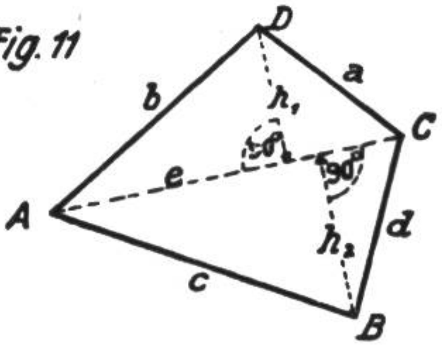
Trapez:

Parallelseiten = G und g; Höhe = h

Diagonalen = D_1 und D_2

$$F = \frac{G + g}{2} \cdot h;$$

Fig. 11

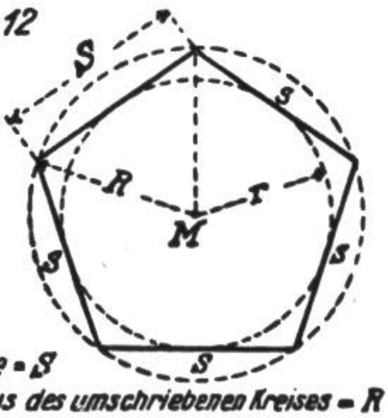


Trapezoid:

Diagonale $\overline{AC}$ und rechtwinklig darauf die Höhen h_1 und h_2

$$F = \frac{\overline{AC}}{2} \cdot h_1 + h_2 = \frac{e}{2} \cdot h_1 + h_2;$$

Fig. 12



Seite = S

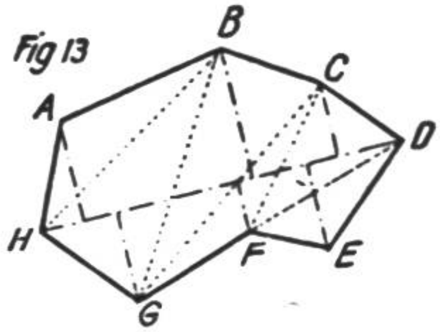
Radius des umschriebenen Kreises = R

Radius des eingeschriebenen Kreises = r

Reguläre Vielecke (Polygon):

Polygon	R	r	S	F
Dreieck	0.577 S	0.289 S	1.732 R od. 3.463 r	0.433 S^2 od. 1.299 R^2
Quadrat	0.707 S	0.500 S	1.414 R = 2.000 r	1.000 S^2 = 2.000 R^2
Fünfeck	0.851 S	0.695 S	1.176 R = 1.453 r	1.721 S^2 = 2.378 R^2
Sechseck	1.000 S	0.866 S	1.000 R = 1.155 r	2.598 S^2 = 2.598 R^2
Siebeneck	1.152 S	1.038 S	0.868 R = 0.963 r	3.364 S^2 = 2.736 R^2
Achteck	1.307 S	1.208 S	0.765 R = 0.828 r	4.828 S^2 = 2.828 R^2
Neuneck	1.462 S	1.374 S	0.684 R = 0.728 r	6.182 S^2 = 2.892 R^2
Zehneck	1.618 S	1.540 S	0.618 R = 0.649 r	7.694 S^2 = 2.939 R^2
Elfeck	1.775 S	1.704 S	0.563 R = 0.587 r	9.366 S^2 = 2.973 R^2
Zwölfeck	1.932 S	1.866 S	0.518 R = 0.536 r	11.190 S^2 = 3.000 R^2

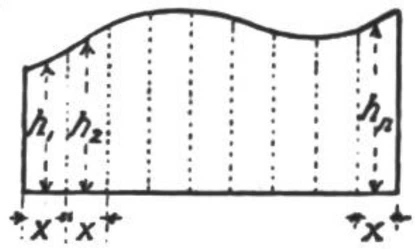
Fig 13



Unregelmassige Vielecke od. Flachen:

Die Fläche kann bestimmt werden durch Zerlegung des Vielecks in Dreiecke mittelst Diagonalen und Summierung der ermittelten Dreiecksflachen, oder durch Einteilung in Trapeze u. Dreiecke vermittelst der Koordinaten der Eckpunkte auf eine rechtwinklig gewählte Axe Fig 13.

Fig. 14

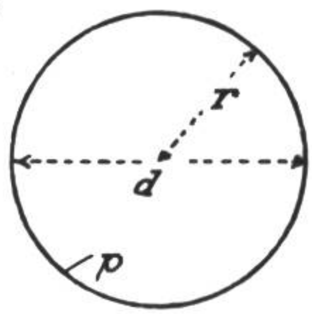


Durch Zerlegung in parallele Streifen gleicher Breite Fig. 14

$$F = x.(h_1 + h_2 + \ldots\ldots\ldots h_n)$$

Kreis:

Fig 15

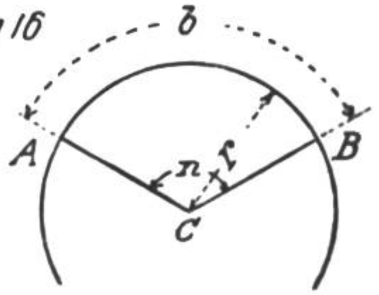


Durchmesser = d; Radius = r
Umfang = p; Inhalt = F

$$p = d.\pi = d.3{,}14159$$
$$= 2r\pi;$$
$$F = \frac{d^2\pi}{4} = 0{,}785\,d^2 = r^2\pi.$$
$$r = \sqrt{\frac{F}{\pi}} = 0{,}564\sqrt{F};$$
$$d = 2\sqrt{\frac{F}{\pi}} = 1{,}128\sqrt{F}.$$

Kreissektor: (ABC) Fig 16

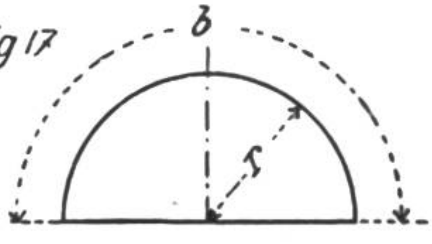
Fig 16



Radius = r; Bogen = b;
Zentriwinkel = n,

$$F = \frac{r^2\pi.n}{360} = \frac{b.r}{2}$$

Fig 17

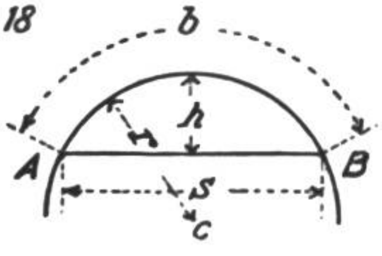


$$r = \sqrt{\frac{F}{\pi}\cdot\frac{360}{n}} = \frac{b}{\pi}\cdot\frac{180}{n};$$
$$n = 360\frac{F}{r^2\pi} = \frac{b}{r\pi}\cdot 180$$
$$b = 2r\pi.\frac{n}{360} = r\pi.\frac{n}{180}.$$

Halbkreis: Bogen = $b = \pi.r$, Fläche = $F = \frac{\pi.r^2}{2}$

Viertelkreis: Bogen = $b = \frac{\pi.r}{2}$; Fläche = $F = \frac{\pi.r^2}{4}$

Fig. 18



Kreisabschnitt: Fig. 18

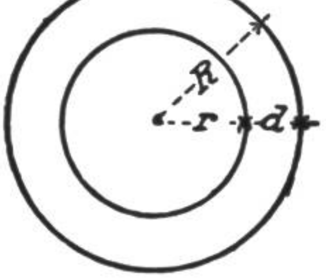
Sehne = s, Höhe = h, $F = \frac{2}{3}s.h$;

genau $F = \frac{r^2\pi n}{360} - \frac{1}{2}s\sqrt{r^2 - \frac{1}{4}s^2} = \frac{br - s.(r-h)}{2}$

$s = 2\sqrt{h(2r-h)}$; $r = \frac{s^2}{8h} - \frac{h}{2}$.

E. Pochon

Fig. 19



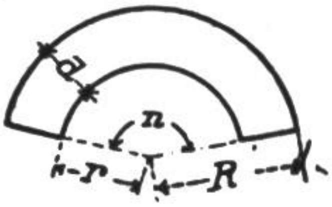
Kreisring:

Aeusserer Radius $= R$,
Innerer Radius $= r$,

$$F = R^2\pi - r^2\pi.$$
$$= \pi.(R+r).(R-r).$$

wenn d = radiale Breite des Kreisrings so ist $F = \pi.(2r+d).d$.

Fig. 20

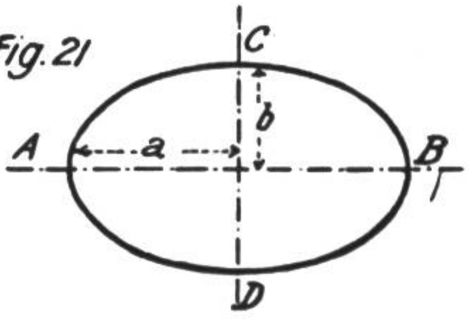


Kreisringstück: (Konzentrisch)

Aeusserer Radius $= R$,
Innerer Radius $= r$,
Zentriwinkel $= n$, radiale Breite $= d$

$$F = (R^2\pi - r^2\pi)\frac{n}{360} = (R^2 - r^2).\frac{\pi.n}{360}$$
$$= (R+r)d\,\frac{\pi.n}{360} = \quad = (R+r)d.n.0.0087$$

Fig. 21

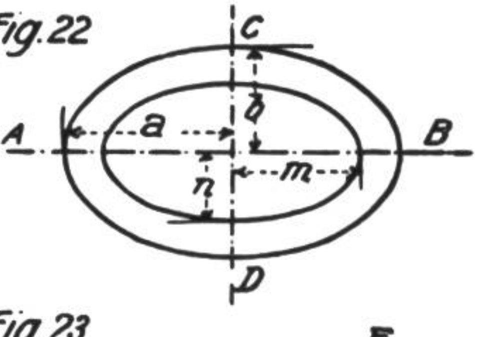


Ellipse:

Halbe Achsen der Ellipse $= a$ und b,

$$F = a.b.\pi;$$

Fig. 22

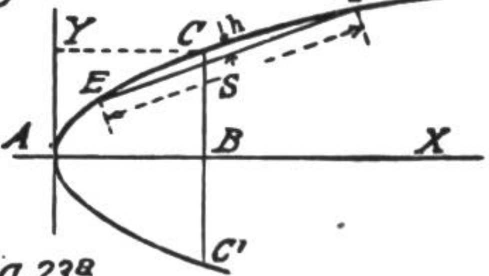


Elliptischer Ring:

Halbe Achsen der äussern Ellipse $= a, b$
Halbe Achsen der innern Ellipse $= m, n$

$$F = \pi.(a.b - m.n).$$

Fig. 23



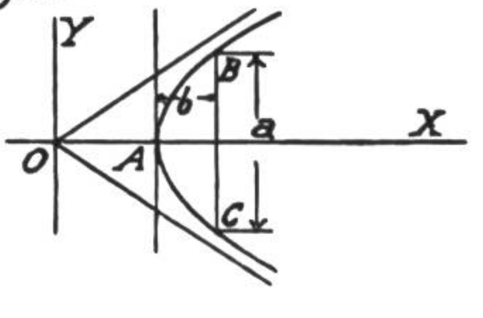
Parabelsegment ECF:

$$F = \frac{2}{3}\,s.h\,; \quad s = \overline{EF}\,;$$

Parabelfläche CAC':

$$F = \frac{2}{3}\,\overline{CC'}.\overline{AB}\,;$$

Fig. 23a

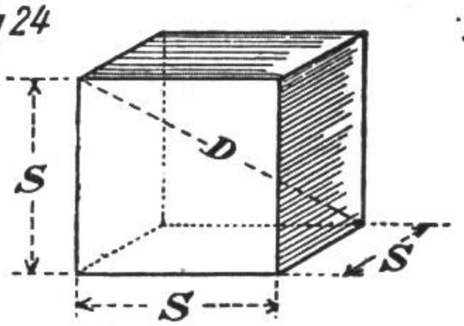


Hyperbelsegment ABC:

Sehne $= a$; Höhe $= b$

$$F\text{(annähernd)} = \frac{3}{5}\,b.a\,;$$

Fig. 24



Würfel:

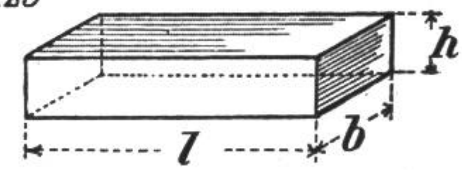
Seite $= S$, Inhalt $= K$; Oberfläche $= O$

$$K = S^3;\quad O = 6S^2;$$

$$S = \sqrt[3]{K}$$

Diagonale $= D = \sqrt{3S^2} = S\sqrt{3} =$ $S \cdot 1{,}732050$

Fig. 25



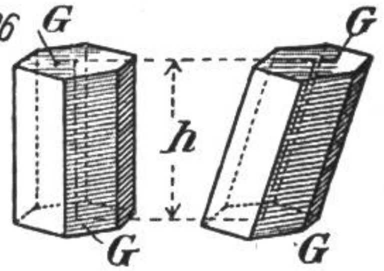
Parallelflach:

Länge $= l$, Breite $= b$, Höhe $= h$,

Inhalt $= K = l \cdot b \cdot h$

Oberfläche $= O = 2 \cdot (l b + l h + b h)$

Fig. 26



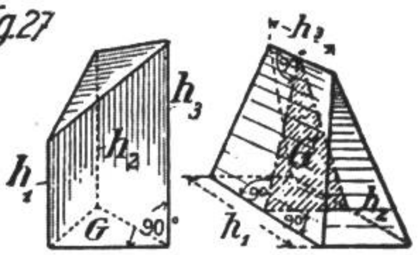
Prisma:

Grundfläche $= G$, Höhe $= h$,

Inhalt $= K = G \cdot h$.

Oberfläche $O =$ Umfang der Grundfläche $U \times h + 2G$.

Fig. 27



Schiefabgeschnittenes Prisma:

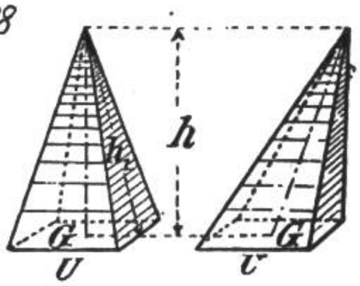
Flächeninhalt des senkrechten Querschnittes $= G$.

Länge der Kanten $= h, h_2 h_3 \ldots\ldots h_n$,

Inhalt $K = G \cdot \dfrac{h_1 + h_2 + h_3 + \ldots\ldots h_n}{n}$

Bei mehr als 3 Kanten ist das Prisma regelmässig oder hat dasselbe wenigstens Axensymetrie.

Fig. 28



Pyramide:

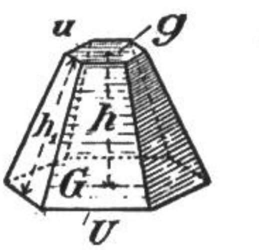
Grundfläche $= G$. Höhe $= h$,

$$K = \frac{G \cdot h}{3};\quad h = \frac{3K}{G};\quad G = \frac{3K}{h};$$

Mantel $M =$ Umfang der Grundfläche $U \times \frac{h_1}{2}$;

Oberfläche $O = M + G$

Fig. 29



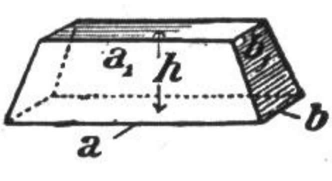
Abgestumpfte Pyramide:

Parallele Endflächen $= G, g$, ihr Abstand $= h$.

ihre Umfänge U, u. Mantel $M = \frac{U+u}{2} \cdot h_1$,

$$K = \frac{h}{3}\left(G + g + \sqrt{Gg}\right);\quad O = M + G + g.$$

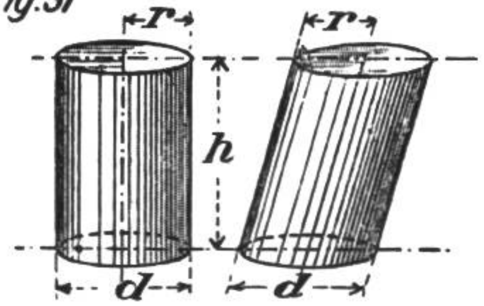
Fig. 30



Obelisk, Wall, (regelmässig aufgeschütteter Haufen)

$$K = \frac{1}{6} h \left[(2a + a_1)\, b + (2a_1 + a)\, b_1\right]$$

Fig. 31



Cylinder (Walze)

Radius = r, Durchmesser = d, Höhe = h

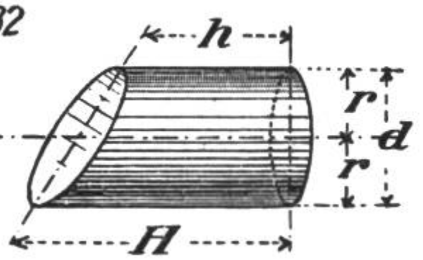
Inhalt $K = r^2\pi . h$ oder $\frac{d^2}{4} . \pi . h$

$r = \sqrt{\frac{K}{\pi h}}$; $h = \frac{K}{r^2\pi}$.

Mantel = $2r\pi . h$ oder $d\pi h$.

Oberfläche = $2r\pi . (r+h)$ oder $d\pi . (\frac{d}{2}+h)$.

Fig. 32



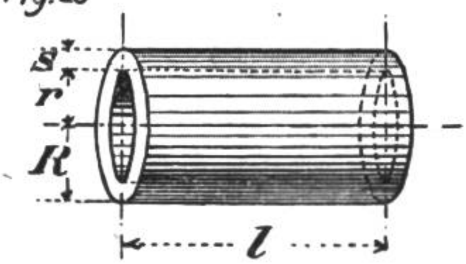
Schiefabgeschnittener Cylinder.

Grösste Höhe = H, kleinste Höhe = h,

Inhalt $K = r^2\pi . \frac{H+h}{2}$ oder $\frac{d^2}{4}\pi . \frac{H+d}{2}$

Mantel = $r\pi(H+h)$

Fig. 33



Hohlcylinder (Rohr):

Innerer Radius = r,

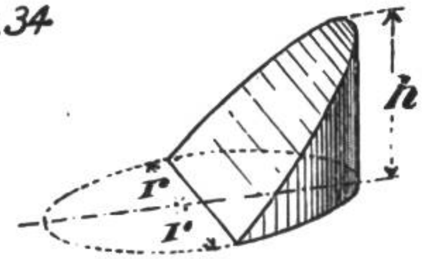
Aüsserer Radius = R, Länge = l,

Wandstärke = $s = R - r$,

Inhalt $K = \pi . l . (R^2 - r^2)$, oder

$K = \pi . l . s\,(2R - s)$ oder $\pi . l . s\,(2r + s)$.

Fig. 34



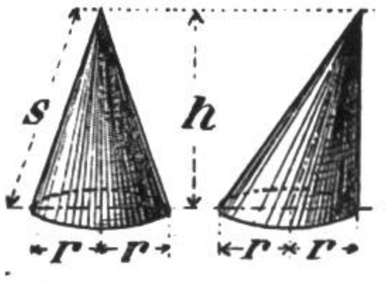
Cylinderhuf:

Radius der Grundfläche = r,

Höhe des Hufes = h, Mantel = $2rh$.

Inhalt: $K = \frac{2}{3} r^2 h$.

Fig. 35



Kegel:

Radius der Grundfläche = r.

Höhe = h. Seite = $s = \sqrt{r^2+h^2}$,

Mantel $M = \pi r \sqrt{r^2+h^2}$ oder $\pi . r . s$,

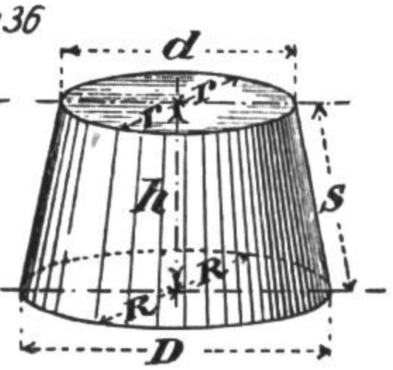
Oberfläche = $\pi . r^2 + r\pi . s$ oder $r\pi(r+s)$

oder = $\pi r . (r + \sqrt{r^2+h^2})$.

Inhalt $K = \frac{1}{3} r^2 \pi . h$;

$r = \sqrt{\frac{3K}{\pi . h}}$; $h = \frac{3K}{r^2\pi}$.

Fig. 36



Abgestumpfter Kegel:

Radien der parallelen Endflächen = R und r,

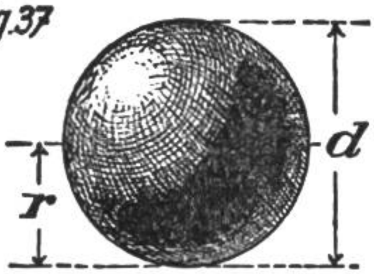
Durchmesser = D und d, Höhe = h, Seite = s,

Inhalt $K = \frac{1}{3}\pi h . (R^2 + Rr + r^2)$

$= \pi . h . (\frac{D^2 + Dd + d^2}{12})$

Mantel $M = \pi s(R+r)$.

Fig. 37



Kugel:

Radius = r; Durchmesser = d,

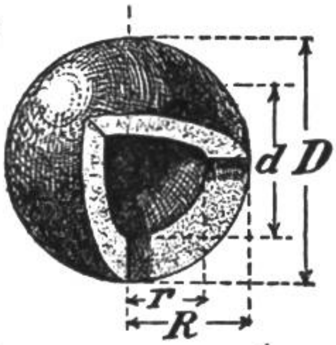
Oberfläche $O = 4r^2\pi = 12{,}566\, r^2$, oder $d^2\pi$.

Inhalt $K = \frac{4}{3} r^3 \pi = 4{,}189\, r^3$, $K = \frac{O . r}{3}$,

» $K = \frac{d^3\pi}{6} = 0{,}5236 . d^3$;

Radius $r = \frac{1}{2}\sqrt{\frac{O}{\pi}}$; $r = \sqrt[3]{\frac{3K}{4\pi}}$

Fig. 38



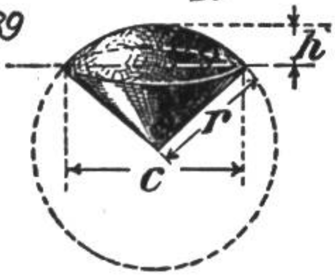
Hohlkugel:

Aeusserer Radius = R, innerer = r,

Aeusserer Durchmesser = D, innerer = d,

Inhalt $K = \frac{4\pi}{3}(R^3 - r^3) = \frac{\pi}{6}(D^3 - d^3)$.

Fig. 39



Kugelsektor:

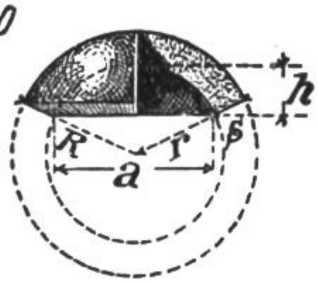
Radius der Kugel = r

Begrenzende Kalotte, Höhe = h, Durchm. = c,

Oberfläche $O = \frac{\pi r}{2}(4h + c)$

Inhalt $K = \frac{2}{3} r^2 \pi . h = 2{,}0944\, r^2 h$.

Fig. 40



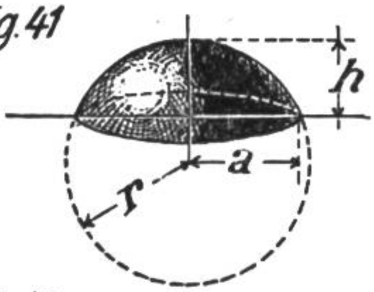
Hohlkugelsektor:

Aeusserer Radius = R innerer = r

Wanddicke = $R - r$ = s, $r = \frac{a^2 + 4h^2}{8h}$

Inhalt $K = 2{,}094 \frac{h}{r}(R^3 - r^3)$.

Fig. 41



Kugelsegment (Kugelkalotte):

Radius der Kugel = r,

Radius der Grundfläche = a,

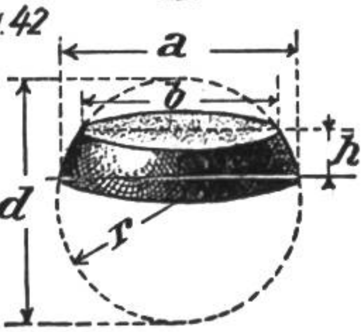
Höhe der Kalotte = h,

Oberfläche = $O = 2\pi r h = \pi . (a^2 + h^2)$

Inhalt $K = \frac{1}{6}\pi . h\,(3a^2 + h^2)$ oder

$= \frac{1}{3}\pi h^2 (3r - h)$.

Fig. 42



Kugelzone:

Höhe der Zone = h, Radius der Kugel = r

Durchmesser der Endflächen = a und b,

Mantel $M = 2r\pi h$, Oberfläche $O = M + \frac{a^2\pi}{4} + \frac{b^2\pi}{4}$.

Inhalt $K = \frac{1}{6}\pi h\,(3a^2 + 3b^2 + h^2)$.

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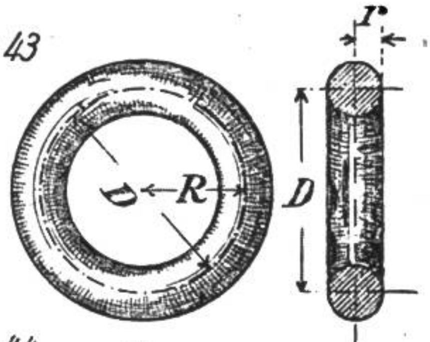
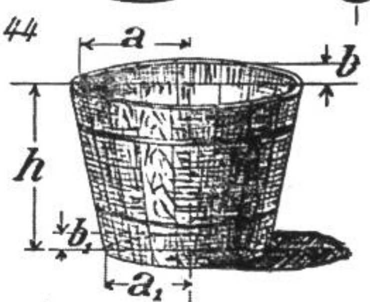
Fig 43



Cylindrischer Ring:

Radius des kreisförmigen Querschnittes = r,
Durchmesser des Ringes = D, Radius = R,
Inhalt $K = 2\pi^2 R r^2 = 2,467\, D d^2$.
Oberfläche $O = 4\pi^2 R r = 9,87\, D d$.

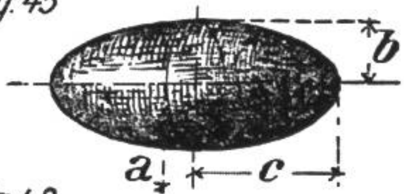
Fig. 44



Kübel.

Die unter sich parallelen Endflächen sind
Ellipsen mit den Halbachsen a b und $a_1 b_1$,
Höhe zwischen den Endflächen = h.
Inhalt $K = \frac{1}{6}\pi h\left[2(ab + a_1 b_1) + a b_1 + a_1 b\right]$

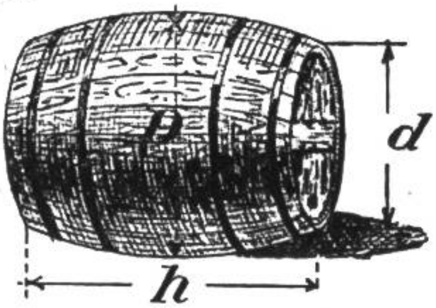
Fig. 45



Ellipsoid:

Bezeichnung der 3 Halbachsen = a, b, c.
Inhalt $K = \frac{4}{3}\pi\, a\, b\, c$.

Fig. 46

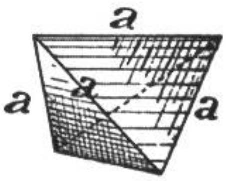


Fass.

Spunddurchmesser = D,
Bodendurchmesser = d,
Höhe (resp. Länge) = h.
Inhalt $K = 1,0453\, h\,(0,4 D^2 + 0,2 Dd + 0,15 d^2)$.

Reguläre Polyeder:

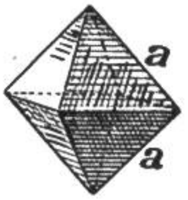
Fig. 47



Tetraeder: (4 gleichseitige Dreiecksflächen)

Länge der Kante = a, Oberfl. $O = a^2\sqrt{3} = 1,732\, a^2$
Inhalt $K = \frac{1}{12} a^3 \sqrt{2} = 0,11785\, a^3$

Fig. 48



Oktaeder: (8 gleichseitige Dreiecksflächen)

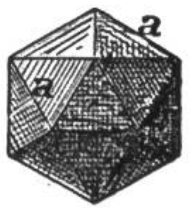
Kante = a, Oberfl. $O = 2a^2\sqrt{3} = 3,464116\, a^2$
Inhalt $K = \frac{a^3}{3}\sqrt{2} = 0,4714045\, a^3$.

Fig. 49

Dodekaeder: (12 regelmässige Fünfecke)

Kante = a, $O = 3a^2\sqrt{25 + 10\sqrt{5}} = 20,645729\, a^2$
Inhalt $K = \frac{a^3}{4}(15 + 7\sqrt{5}) = 7,663119\, a^3$.

Fig. 50



Ikosaeder: (20 gleichseitige Dreiecksflächen)

Kante = a, $O = 5a^2\sqrt{3} = 8,6602545\, a^2$.
Inhalt $K = \frac{5}{12} a^3 (3 + \sqrt{5}) = 2,181695\, a^3$.

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