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JONATHAN BARNES

PROOFS AND SYLLOGISMS IN GALEN

At the beginning of Book 8 of the *Doctrines of Plato and Hippocrates* Galen recalls that his first aim in the work had been to determine whether "there is one power or several which govern men", and his second to determine what these powers are and where in the body they are situated. The outcome of the long discussion was a vindication of the Platonic view of the soul against various Stoic eccentricities. The vindication was achieved "in accordance with the laws of proof" (*PHP* V 648.6 K); and it was, in principle, easily achieved. For the proofs which Galen rehearsed, and which Hippocrates and Plato had rehearsed before him, were simplicity itself; they had convinced all but the

¹ κατά τὸν ἀποδεικτικὸν νόμον: the phrase is repeated at PHP V 653.18; it is also found at Loc. Aff. VIII 201.6 and at Adv. Jul. XVIIIA 295-296, where Julian, "the sophist of the new dialectic" is mocked for his ignorance of ἀκολουθία — here the νόμος ἀποδείξεως is a rule of inference or principle of implication. I have not noticed the phrase outside Galen — but cf. Proclus, In Ti. I p.227 Diehl ("the law of the demonstrators requires us to inquire whether something is — where that is unknown — before we ask what it is"). At Diff. Puls. VIII 735.16 Galen talks of "dialectical laws" — which there prescribe the right way of constructing a definition. The same phrase is found again later: e.g. Proclus, In Alc. 334; In Parm. 651-652; Simplicius, In Cael. p.28.15-25 (on the right way of forming negations); Philoponus, Aet.M. 18, p.609 Rabe ("the dialectical laws say that the possible follows the possible" — i.e. if it is possible that P, and if P then Q, then it is possible that Q); and there is perhaps an ironical allusion in Sextus Empiricus, M.~8.108 (ή ... παρὰ τοῖς διαλεκτικοῖς νομοθεσία). The expression was known to Gellius, who latinizes it as lex disciplinae dialecticae (see 16.2.1-13). 'Logical law' in the sense of 'rule of dialectical debate' occurs at Asclepius, In Metaph. p.253.37-39 Hayduck. Note also, much earlier, Epictetus, Diss. 1.26.1 (νόμος ὑποθετικός).

philosophers; and if Galen's discussion had nevertheless run on and on, that was the fault of his asinine adversaries (*ibid*. 648-650).

The philosophers — Galen is thinking of the Stoics — produced anti-Platonic arguments by the bucketload. But even the most persuasive of them "were not constructed according to the demonstrative method" (*ibid*. 650). On the other hand,

all those who have not yet touched on philosophy — geometers, arithmeticians, calculators, astronomers, architects, and even musicians and clock-makers and rhetoricians and grammarians, and in general anyone who has been trained in a rational art [ἐν τέχνη λογικῆ] —

will recognize that the view of Hippocrates and Plato is correct (*ibid.* 652). And Posidonius, "the most scientific of the Stoics because he had trained in geometry", felt obliged to reject the Stoic doctrine and side with the truth (*ibid.*).

Not only was it not necessary for the account of the ruling part of the soul to fill five books — there was no need even for one entire book, at any rate for those who have learned what sort of thing a scientific proof is — something which, as I affirm, is appropriate to philosophers more than to geometers and arithmeticians and calculators and astronomers and architects, although the former have not practised it as the latter have. Thus Euclid, in a single theorem — the first in his book on *Phenomena* — showed in very few lines that the earth is in the middle of the universe and stands towards it as a point and a centre; and those who have learned the proof believe its conclusion as they believe that two twos are four. Yet some of the philosophers talk such nonsense about the magnitude and position of the earth that anyone would be ashamed of the whole profession. (*ibid*. 654)

The philosophers are "unwilling to use linear proofs [γραμμικαὶ ἀποδείξεις]", they "have not been trained to read linear proofs" (ibid. 656): hinc illae lacrimae.

A passage in *On my own Books* recounts how, as a young man, Galen recognized the importance of proof; how he therefore studied logic with the philosophers, both Peripatetics and Stoics; how he found them at loggerheads, disagreeing with one another and contradicting themselves — even though they too praised "linear proofs"; and how "I decided that I should distance myself from what they said and follow the character of linear proofs" (*Lib. Prop.* XIX 40-41).

Galen's main treatment of the subject, the fifteen youthful books *On Proof*, is lost; and although there are innumerable references to proof in the surviving works, there is no detailed discussion. But in outline his views are known. In particular, it is plain that proofs are syllogisms or formally valid deductions.

When Galen was a boy, two types of syllogism were taught in the schools: there was categorical syllogistic, historically associated with Aristotle and the Peripatetics; and there was hypothetical syllogistic, of which the Stoics were traditionally the patrons. The relation between the two syllogistics was disputed: in the *Introduction to Dialectic* Galen dismisses such disputes, maintaining that both types of syllogism are useful for proofs—categorical syllogisms especially for categorical propositions, hypothetical syllogisms especially for existential propositions (12.1-6; 14.1-2). The two syllogistics are thus complementary parts of logic.

But they do not exhaust the domain of logic:

There is also another, third species of syllogism useful for proofs — I call them relational, although the Peripatetics insist on $[\beta \iota \acute{\alpha} \zeta \circ \nu \tau \alpha \iota]^2$ counting them among the categoricals. There is no small use of them by the arithmeticians and calculators. (*Inst. Log.* 16.1)

² βιάζονται: grammar (middle or passive?) and sense ('force', 'be forced', 'insist'?) are alike uncertain; but the drift of Galen's remark is not obscure. — I excise 'σκεπτικοῖς καὶ': cf 16.5.

The arguments which Galen goes on to mention were not new to logicians. On the contrary, the Peripatetics — as Galen notices — had rudely forced them into categorical form; and the Stoics — though Galen does not say so — had called them "non-methodically concluding [ἀμεθόδως περαίνοντες]". Against the orthodox logicians of the day, Galen claimed that these arguments were members of a separate species of syllogism, and hence that there were three rather than two types of syllogistic.³

Galen was impressed by the proofs which arithmeticians and geometers and other serious scientists advanced, and he was ashamed of the ways in which philosophers tried to prove their doctrines. Galen recognized a third species of syllogism, apart from the two species which were taught in the philosophical schools, and he observed that members of the third species were particularly common in mathematical and scientific work. It is natural to put these facts together: scientific and philosophical practice showed up the inadequacy of orthodox school logic and spurred Galen to the discovery of the third species of syllogism. And it is natural to congratulate Galen: was not the logic of the schools indeed inadequate, and do not scientific proofs indeed use relational syllogisms?

In *PHP* 8, Galen offers six exemplary proofs: one each for the location of the rational and the affective parts of the soul, and four for the desiderative part (V 655-660). The last four are presented in a disconcertingly nonchalant style, but the first two are set out with some formality, thus:

(I) Where the origin of the nerves is, there is the ruling part. The origin of the nerves is in the brain. Therefore the ruling part is there.

³ Galen was always alive to the importance of distinguishing between what is relational and what is not — something which his opponents often failed to do (see e.g. *SMT* XI 569-570).

And

(II) Where the affections of the soul more evidently move the parts of the body, there is the affective part of the soul.

But the heart clearly undergoes a considerable change of motion in anger and fear.

Therefore the affective part of the soul is in it. (ibid. V 655)

Strictly speaking — or so a modern reader might be tempted to object — neither argument is formally valid: to turn the proofs into formally valid arguments, we should have to change 'here' to 'in the brain' in the conclusion of the first proof, and to make several adjustments to the second. But this objection confuses formality with pedantry: the form of an argument is fixed not by its superficial linguistic expression but by its underlying semantic structure. Replace 'there' by 'in the brain' in the conclusion of the first proof and you change nothing: the new words present exactly the same argument as the old.

However that may be, it may well seem that the two proofs share the same logical structure, so that it will be enough to consider the first of them — which has already been announced at PHP V 587-588 and stated at 649. Then what is the structure of proof (I)? what sort of syllogism underlies it? To be sure, there need be no uniquely correct answer to this question. For just as a proposition may have any number of different formal structures, so an argument — which is a set of propositions may have a multiplicity of valid forms. Galen perhaps recognized this: in a couple of passages, at Sem. IV 609 and SMT XI 499, he offers arguments which he says may be construed either categorically or hypothetically; and hence — or so it appears — he acknowledges that one and the same argument may exhibit two different and valid formal structures. In any event, Galen must have taken his paradigmatic proofs to be syllogisms, and hence he must have taken them to be categorical or hypothetical or relational — or any two, or all three.

In *PHP* Galen does not comment explicitly on the logic of the proofs. In general, although Galen is a stickler for demonstrative rigour, he rarely remarks upon the logical structure of the proofs which he commends. In particular, although he adverts to proofs a thousand times and more in his works, and although he must use the word 'syllogism' and its cognates several hundred times, the only texts outside *Inst.Log.* in which Galen characterizes an argument as categorical or hypothetical are the two from *Sem.* and *SMT*; and he never characterizes an argument as a relational syllogism.⁴

But perhaps the text of *PHP* points implicitly at the logical structure of the proofs? Three expressions which Galen uses have been or might be thought to advance the inquiry.

First, the adjective γραμμικός. Galen demands γραμμικαὶ ἀποδείξεις; and he surely took argument (I) to be one. Such proofs are especially associated with the geometers. Galen says that geometers make frequent use of relational syllogisms. Perhaps, then, γραμμικὴ ἀπόδειξις in Galen points to relational syllogisms.

Γραμμικός means 'to do with lines' or 'linear'. 'Η γραμμική θεωρία is the theory of lines (e.g. *UP* III 812.10; *Pecc.Dig.* V 86.16), γραμμικὰ θεωρήματα are theorems about lines (e.g. *UP* III 838.2; Nicomachus, *Intr.ar.* 2.21.1), and a πρώτη γραμμική στοιχείωσις is an elementary treatise on lines (Theon Smyrnaeus, p.16.19-20 Hiller). Some arithmeticians distinguished three types of numbers — the linear, the planar, the solid; but

⁴ It has been found remarkable that, outside *Inst.Log.*, Galen nowhere mentions his relational syllogisms (they are not to be sought at *PHP* V 796-797, *pace* de Lacy, pp.707-708, nor in the parallel passage at *Nat.Fac.* II 28); and it is tempting to invent chronological hypotheses to account for the fact. But Galen recognized relational syllogistic early in his life (*Inst.Log.* 17.1); and in any event, what needs explanation is not Galen's silence about relational syllogisms but his general reticence about logical form.

⁵ E.g. Nicomachus, *Intr.ar.* 2.6.1 (and often); [Iamblichus], *Theol.ar.* p.84.9-10 De Falco (= Speusippus, fr.2.4 Lang — but there is no reason to think that [Iamblichus] is quoting Speusippus); perhaps first — in adverbial form — at Anon. *In Tht.* 40, 19-23.

lines are also contrasted with numbers, so that Galen will refer to "numerical and linear theory which astronomy and architecture use as their base" (Pecc. Dig. V 80.4-6).

The adjective γραμμικός is often applied to arguments. In his discussion of Archimedes and the history of mechanics, Plutarch speaks of a λογική καὶ γραμμική ἀπόδειξις (Marc. 14 [305D]);⁶ and Sextus says that Aratus proved something γραμμικῶς (Math. 1.304). Sextus has in mind a geometrical proof — the proof of Euclid 4.15. Plutarch, too, is adverting to a geometrical argument. So perhaps a linear proof is a proof about lines, or a geometrical proof? But if Galen demands linear proofs for locating the parts of the soul, he is not looking for geometrical arguments.

When an Epicurean and a Stoic and a Peripatetic attempted to recommend their different views about the void, Galen found that "they possessed no demonstrative argument but only contingent and probable arguments — and sometimes not even those"; indeed "it was plainly evident to everyone present that none of them produced an argument which was compelling [άναγκαστικόν] or related to a linear proof [άποδείξεως γραμμικής έχόμενον] but only arguments made up of the sort of considerations which orators use" (Pecc. Dig. V 102-103).8 Galen is chiding the philosophers not for their want of geometrical arguments but for their want of arguments which are like geometrical arguments.

In what way like? One of the Hippocratic commentaries remarks, in an aside, that

linear proofs ... not only persuade those who are learning a discipline, but also have a reputation among laymen as being

⁶ The first surviving occurrence of γραμμικός in this, or any other, usage though Plutarch surely was not the first have used the word. The occurrences of the phrase 'linear proofs' in Cleomedes (1.8, p.84.14; 1.11, p.104,20 Ziegler) should not be ascribed to Posidonius.

⁷ Cf Math. 3.92; Nicomachus, Intr.ar. 2 ('linear and arithmetical proofs'); Ptolemy, Harm. 1.5 [p.12.8-9 Düring]; Alm. 1.1 [9.15-16, 142.6], and often. ⁸ Cf UP IV 20.9-10; Foet. Form. IV 695.10, 701-702; Aff. Dig. V 42.7.

very true — so they say that those who have shown something clearly and uncontroversially have used linear proofs [γραμμικαῖς οὖν ἀποδείξεσι κεχρῆσθαί φασι τοὺς ἐναργῶς τι καὶ ἀναμφιλέκτως δείξαντας]'. (HVA XV 439-440)

Laymen use the term 'linear proof' to commend an argument which they find compelling. There is a nice example of this use

in Origen (Cels. 8.11).

To call a proof 'linear' is not to describe its logical form, and hence not to say that it is a relational syllogism. A linear proof is an argument, of any form, which meets the standards of the geometers — an argument which exhibits the rigour, the brevity and the demonstrative power which, or so Galen professes to think, typically mark the work of the Greek geometers.

The second of the three expressions is the noun $\pi\rho\delta\sigma\lambda\eta\psi\iota\varsigma$; for Galen so characterizes one of the premisses of his proof. It is generally held that the term $\pi\rho\delta\sigma\lambda\eta\psi\iota\varsigma$ was introduced by the Stoic logicians, and that it is a term of art in hypothetical syllogistic, where it designates the second or non-complex premiss of a mixed hypothetical syllogism. So perhaps the presence of the word $\pi\rho\delta\sigma\lambda\eta\psi\iota\varsigma$ in the text signals that Galen takes argument (I) to have a hypothetical structure.

The term πρόσληψις was certainly used of Stoic syllogisms, and it was perhaps first used of hypothetical syllogisms by Stoic

logicians. Thus Diogenes Laertius reports that

an argument, as Crinis says, is something constituted from an assumption and an additional assumption and an inference [τὸ συνεστηκὸς ἐκ λήμματος καὶ προσλήψεως καὶ ἐπιφορᾶς]. (7.76)¹⁰

See PHP V 649.16 and PHP V 654.1; and note ή προσληφθεῖσα at 588.4
 — where Alexanderson's correction ή πρόσληψις commends itself.

¹⁰ See also 7.80, 82. — 7.76 does not imply that Crinis was the first Stoic to use πρόσληψις in this way. No doubt it was so used by Chrysippus, if not by Zeno.

The Peripatetics had another use for the term $\pi \rho \delta \sigma \lambda \eta \psi \iota \varsigma$; ¹¹ and that is no doubt why, to do the work of the Stoic $\pi \rho \delta \sigma \lambda \eta \psi \iota \varsigma$, they preferred $\mu \varepsilon \tau \dot{\alpha} \lambda \eta \psi \iota \varsigma$. ¹² But — as Philoponus noticed (*In APr.* 243.8) — it was $\pi \rho \delta \sigma \lambda \eta \psi \iota \varsigma$ which caught on; and Galen uses it dozens of times to designate the non-complex premiss of a hypothetical syllogism. ¹³

Nonetheless, the word $\pi \rho \delta \sigma \lambda \eta \psi \iota \zeta$ does not mean "non-complex premiss in a hypothetical syllogism": it means "additional assumption", or "supplementary premiss". You are trying to construct an argument for a given thesis. You take or are offered one pertinent proposition which may serve as an assumption or premiss; and then you hunt about for another — which will therefore be a further assumption or $\pi \rho \delta \sigma \lambda \eta \psi \iota \zeta$. Nothing in the sense of the word $\pi \rho \delta \sigma \lambda \eta \psi \iota \zeta$ ties it to any particular syllogistic structure. At least once Galen calls the complex premiss of a hypothetical syllogism a $\pi \rho \delta \sigma \lambda \eta \psi \iota \zeta$ (PHP V 429). Alexander once or twice speaks of the $\pi \rho \delta \sigma \lambda \eta \psi \iota \zeta$ in a categorical syllogism (e.g. In APr. 22.9-10; 285.21-26); and Galen once implies that one premiss of a categorical syllogism will be a $\pi \rho \delta \sigma \lambda \eta \psi \iota \zeta$ (Inst. Log. 7.4).

Thus the occurrence of $\pi\rho\delta\sigma\lambda\eta\psi\iota\zeta$ at *PHP* V 649 does not demonstrate that Galen took argument (I) to be a hypothetical syllogism. Nonetheless, the fact that, in the vast majority of its

See Alexander Aphrodisiensis, In APr. 324.16-18; cf 19.3-5 Wallies; Philo-

ponus, In APr. 242.22-243.8 Wallies.

See Alexander Aphrodisiensis, In APr. 378.12-18 Wallies; Galen, Inst. Log. 19.1 (syllogisms κατὰ πρόσληψιν).

Sextus Empiricus, PH 2.234; Math. 8.233; 8.333; 8.450. — It is inexact to speak of the complex and non-complex premisses of a mixed hypothetical syllogism inasmuch as both premisses may be complex. It is worse to speak of major and minor premisses; for those terms are defined for categorical syllogisms and make no sense when applied to hypotheticals. Something along the following lines is needed. An argument has a mixed hypothetical structure if and only if it has two premisses, one of the form 'f(A₁, A₂, ..., A_n)', where 'f' is an n-placed propositional connector and n > 1, and the other of the form 'f*(B₁, B₂, ..., B_m)', where 'f*' is an m-placed propositional connector and $1 \le m < n$ and each B_i is either an A_i or else the negation of an A_i. (If m = 1, then 'f*' is the empty operator 'It is the case that'.) The premiss containing the B_is is the $\pi \rho \delta \sigma \lambda \eta \psi \iota \varsigma$.

occurrences, $\pi\rho\delta\sigma\lambda\eta\psi\iota\zeta$ does indeed refer to the non-complex premiss of a hypothetical syllogism suggests that it might well have the same a reference at *PHP* V 649.¹⁴

The third of the three expressions is the adjective μυριώτατος. At PHP V 649 the first premiss of the proof is called ή τοῦ λόγου κυριωτάτη πρότασις. 15 The word is not explained; but it is tempting to take as a variant on ἡγεμονικός or 'guiding', which Galen uses at *Inst. Log.* 7.1-2 to designate the complex premiss of a mixed hypothetical syllogism. Inst. Log. 7.1-2 uses ήγεμονικός without stopping to explain it, and it is not found in any other ancient logical text. But it is clear that it is a relational term, that premisses are ήγεμονικαί of προσλήψεις; and it is clear, too, that a given proposition is ἡγεμονική of an additional assumption to the extent that it determines or limits the options for such items. Suppose that you are aiming at a hypothetical syllogism, and that you already possess a conditional premiss: then your additional premiss must be either the antecedent of the conditional or the negation of its consequent — the conditional in this way is ἡγεμονικός, it guides or controls its stable-mates.

This is the most authoritative premiss of the argument, being accepted by all doctors and philosophers; and its as it were additional assumption is, if

The origin of the nerves is in the brain and if false:

The origin of the nerves is in the heart.

Perhaps, then, the qualifying 'as it were' is there on account of the second and false supplementary premiss, which Galen does not want to call a genuine $\pi \rho \delta \sigma \lambda \eta \psi \iota \varsigma$.

¹⁴ At V 649 Galen does not style his second premiss a πρόσληψις simpliciter: he says οἶον πρόσληψις. The turn of phrase seems to imply that the second premiss is not a πρόσληψις of the ordinary kind — and hence, perhaps, that the proof is not a hypothetical syllogism of the ordinary kind. But this argument carries little weight; for at V 588 there is no protective οἶον. The context at V 649 is this:

¹⁵ Κυριώτατος also occurs at *PHP* V 25 and 260 (of λήμματα which the Stoics unaccountably omit from their arguments), and at 651 (of the first λῆμμα in Chrysippus' false proof). So far as I know, it is not used in any other text, Galenic or non-Galenic, of the premisses of hypothetical arguments.

Now at *Inst.Log.* 7.2 Galen contrasts "syllogisms from guiding assumptions" with "syllogisms from categorical propositions", so that he apparently takes guiding premisses to be restricted to hypothetical syllogistic. And at 7.4 he states that

hypothetical syllogisms have the additional assumption determined, categorical syllogisms do not.

This implies that categorical syllogisms do not have guiding premisses. If 'guiding' and 'most authoritative' mean the same, then we may infer that Galen's proofs in *PHP* were not construed by him as categorical syllogisms.

Yet 'most authoritative' and 'guiding' are not synonyms, nor even equivalent expressions. At *PHP* V 260 Galen applies πυριώτατος to a premiss which is not complex. At *PHP* V 649 the first premiss is πυριωτάτη of the argument and not of the πρόσληψις. Similarly, at *PHP* V 261, Galen refers to premisses which are πυρίαι of the conclusion. Hence 'authoritative' and 'guiding' are expressions with quite different senses.

Philoponus speaks once or twice of "the most authoritative" premisses of a syllogism. Here the superlative has no particular force: Philoponus uses it as a synonym of the positive form χύριος; and he uses it in a sense which Alexander had explained. The authoritative premisses of an argument are "the premisses which proximately [προσεχῶς] prove and syllogize the conclusion in question" (In APr. 281.32-282.1; cf 282.22-24). Alexander is commenting on APr. 42 b1, where Aristotle refers to the χυρίαι προτάσεις of an argument. Suppose that you infer C from A and B, having inferred A from D and E, and B from F and G. Then ABDEFG are all premisses for C;

¹⁶ See Philoponus, In APr. 260.35; 263.19; In Ph. 773.17-19.

but A and B are the authoritative premisses.¹⁷ Perhaps Galen took the term κύριος from the Peripatetics — and perhaps that suggests that he took argument (I) to be a categorical syllogism?

But in Galen πυριώτατος or πύριος does not mark a distinction between the premisses of a syllogism and the premisses of a prosyllogism. At PHP V 259, where Galen is discussing a Stoic argument which, from a structural point of view, is very similar to argument (I), the word πυριώτατος picks up a preceding ἀναγκαιότατος: the assumption in question is most authoritative inasmuch as it is an essential part of the argument. Presumably, then, a premiss is authoritative if it determines the conclusion—it is not an optional or redundant extra; and a premiss is most authoritative if it is indispensable to the argument—it cannot be replaced by anything else. ¹⁸

Galen's Greek does not disclose the logical structure of his proofs. Perhaps considerations of a more abstract sort will do the trick.

First, then, is argument (I) in fact a categorical syllogism? Galen's formulation does not use the tell-tale words 'all' and 'some', and the argument certainly does not wear a categorical heart on its sleeve. But perhaps a categorical heart beats beneath the cloth?

If so, then the three constituent propositions of the proof must be construed as categorical, and we must discover three terms for the syllogism. ¹⁹ There are several ways of extracting suitable terms. One of them is this: let A be 'brain', B 'origin

¹⁷ Note also e.g. Alexander Aphrodisiensis, *In APr.* 22.25-26; Proclus, *In Parm.* 696.18-21 Cousin (αὶ κυρίως προτάσεις).

¹⁸ Alexander once remarks that a universal premiss is κυριωτέρα than a particular inasmuch as "it is proper to syllogistic justification to prove something on the basis of a universal" (*APr.* 48.12-16). But this is not pertinent to Galen, whose most authoritative premisses may be either universal or particular (*PHP* V 260).

¹⁹ I assume, here and later, that the proofs do not rely on any unexpressed premisses: having criticized the Stoics for omitting premisses in their arguments (V 259-261), Galen can scarcely have offered an enthymeme as a paradigmatic proof.

of nerves', and C 'location of a ruling part'. Then construe the first premiss,

Where the origin of the nerves is, there is the ruling part, as predicating B of every C; the second premiss,

The origin of the nerves is in the brain, as predicating A of every B; and the conclusion,

The ruling part is there,

as predicating A of every C. Then build the following argument:

(IC) Every location of a ruling part is an origin of the nerves.Every origin of the nerves is a brain.Therefore, every location of a ruling part is a brain.

That is a perfect syllogism in Barbara.

Does Barbara therefore lie under Galen's proof? That is to say, is (IC) the same argument as (I)? Is the categorical structure overtly presented in (IC) also covertly present in (I)? Or rather, did Galen take (I) to be the same as (IC) or as some variant on (IC)? Aristotle had insisted that extracting the terms of a syllogism may be a subtle matter, the natural expression of an argument often hiding rather than exposing its logical structure. Galen was no doubt aware of this; and the argument in *On Semen* which he explicitly states to be categorical is far from patently so. Nonetheless, it is far from plain that (IC) — or any categorical variant on (IC) — is the same argument as (I). Moreover, if Galen did think of (I) as a categorical syllogism, he did nothing to advertise the fact.

²⁰ "You should not always try to set out terms in a single word — often there will be phrases for which there is no equivalent word. That is why it is difficult to analyse such syllogisms, and sometimes we fall into error ..." (Aristotle, *APr.* 48 a 29-32); cf Alexander Aprhodisiensis, *In APr.* 357.20-358.4.

What, next, of hypothetical syllogistic? Galen's proofs imitate, in their linguistic form, some pseudo-proofs which he ascribes to the Stoics. Thus Chrysippus argued as follows:

Where the affections of the soul are, there too is the ruling part.

The affections of the soul are in the heart.

Therefore the ruling part too is in it. (PHP V 651)

Now an argument offered by a Stoic as a formal proof seems likely to be taken for a hypothetical syllogism. So perhaps argument (I), whether or not it is categorical, was intended to be construed as a hypothetical syllogism? (And the word $\pi\rho\delta\sigma\lambda\eta$ - $\psi\iota\zeta$ designates, as it usually does, the second premiss of such a syllogism.)

It is unlikely that Chrysippus presented the argument in the form which Galen gives it at *PHP* V 651. Galen had already alluded to it at V 271, 294 and 361. He found it in Book I of Chrysippus' *On the Soul*, which he quotes:

Since anger arises there [i.e. in the heart], it is reasonable that the remaining desires too are there — and indeed the remaining affections and the reasonings and whatever is like them. (*PHP* V 294.9-11)

The argument which *PHP* ascribes to Chrysippus is something which Galen himself invented on the basis of this text. Still, it might be thought that if Galen invented an argument on Chrysippus' behalf, then he would give a Stoic structure.

If argument (I) is a hypothetical syllogism, then what sort of hypothetical is it? There is only one plausible answer: a hypothetical syllogism of the sort which the Stoics called 'first unproveds' (and which we call arguments in *modus ponens*). In that case, the first premiss of the proof,

Where the origin of the nerves is, there is the ruling part, must be construed as a conditional proposition. And it is, after all, equivalent to

If the origin of the nerves is in a given place, then the ruling part is there

— and that has a conditional look to it.

Modern logicians will say that looks deceive: the proposition is not conditional, it does not have the form 'If P, then Q'. (It is a quantified conditional, of the form ' $(\forall x)$ (if Fx, then Gx)'.) True; but why think that ancient $\sigma \nu \nu \eta \mu \mu \acute{\epsilon} \nu \alpha$ must have the form 'If P, then Q'? There are important issues here; yet they are beside the present point. For whether or not the proposition

If the origin of the nerves is in a given place, then the ruling part is there,

is a conditional, the argument

If the origin of the nerves is in a given place, then the ruling part is there.

The origin of the nerves is in the brain.

Therefore the ruling part is there.

is not a first unproved. A first unproved is an argument "which infers the consequent from a conditional and the antecedent" (Sext.Emp. PH 2.157); or, as Galen puts it,

in the case of a continuous hypothetical proposition — which the Chrysippeans call a connected principle — if we assume in addition the antecedent, we shall have the consequent as conclusion. (*Inst.Log.* 5.5)

The second premiss in the argument I have just constructed is not the antecedent of the first.

Argument (I) cannot be construed in this way as a first unproved. I find no other, more plausible, way of taking it as a first unproved — or as any other sort of hypothetical syllogism.

Is the argument a relational syllogism? What, after all, could be more appropriate than that Galen's paradigmatic proofs should show syllogisms which, in his view, were scientifically

more important than either the categoricals or the hypotheticals?²¹ But what is a relational syllogism? Galen offers no general characterization of the species; and the numerous examples which he produces in the third part of the *Introduction* do not invite an easy induction. Yet there appears to be a large hint towards a definition.

Having introduced a first group of relational syllogisms, Galen says that

common to all these syllogisms is the fact that they have the same construction on the basis of certain axioms. (*Inst.Log.* 16.5)

Similar phrases, which have been alleged to express Galen's 'metatheorem', recur in the following pages.²² Then is not a syllogism relational if and only if its construction, or its validity, or its probative power, depends on some axiom or axioms?

There are several difficulties here. First, the text in many of the relevant passages is peculiarly recalcitrant.²³ Secondly, the 'metatheorem' is expressed in several ways which are not evidently equivalent to one another. Thirdly, Galen never clearly explains exactly how axioms are supposed to underwrite relational syllogisms. Fourthly — and most pertinently —, he appears to hold that every syllogism, or at least every demonstrative syllogism, is underwritten by a universal axiom. At 16.10 he refers to "all the other demonstrative syllogisms". At 17.1 he says that

pretty well all syllogisms gain their construction from the warranty of the universal axioms which are superordinate to them.

²¹ The first illustrative argument in *Inst.Log.* is a relational syllogism — no casual choice.

²² See 16.10, 11, 12; 17.1, 2, 3, 7, 9; 18.1, 6, 8; and note 1.2-3.

²³ Nowhere more so that at 16.5: the MS reading makes neither sense nor syntax; I read σύστασιν ὡς for συστάσεως — alii alia.

At 17.2 he affirms that

it may be learned more clearly in the following way that all demonstrative syllogisms are such because of the warranty of universal axioms.

At 17.7 he remarks that

most of the items which men syllogize and prove are said in virtue of an axiom.

In some of these passages, 'all syllogisms' or 'all demonstrative syllogisms' may be limited, by the context, to all relational syllogisms. But 17.7, at least, cannot be read with such a limitation; and it follows that the fact of being founded on an axiom does not define the class of relational syllogisms.

Perhaps relational syllogisms are underwritten by relational axioms, and that is what distinguishes them from other syllogisms? But the problem is thereby displaced, not resolved: instead of asking "What makes a syllogism relational?" we shall ask "What makes an axiom relational?".

If you want to know what makes a syllogism relational, then it is best to ask first what makes a syllogism categorical or hypothetical. Categorical syllogisms, according to Aristotle, reduce to Barbara and Celarent, the first two syllogisms of the first figure (*APr.* 29 b 15-25). Barbara and Celarent are 'perfect' or 'complete'; that is to say, their validity is 'evident' (*ibid.* 24 b 22-24). Their validity is evident inasmuch as it flows directly from the definition of 'hold of every' and 'hold of no' — inasmuch as it follows from what was later called the principle *de omni et nullo* (*ibid.* 25 b 39-40; 26 a 27-28). Thus, following Aristotle at one remove, we might say this:

A syllogism is categorical if and only if its validity is guaranteed merely by the sense of the quantifiers ('of every', 'of no', 'of some', 'not of some').

Then for hypothetical syllogisms we might offer:

A syllogism is hypothetical if and only if its validity is guaranteed merely by the sense of the propositional connectors ('if', 'or', 'and', 'not').

And so in the case of relational syllogisms we should be looking for something of the following form:

A syllogism is relational if and only if its validity is guaranteed merely by the sense of Xs.

It seems reasonable to suppose that relational syllogisms must contain, in some essential fashion, relational terms — terms which fall in the Aristotelian class of τὰ πρός τι.²⁴ Hence:

A syllogism is relational if and only if its validity is guaranteed merely by the sense of relational terms.²⁵

It may then be imagined that the axioms which underwrite relational syllogisms are truths determined by the senses of the relational terms which the syllogisms contain.

At Inst. Log. 16.6 Galen remarks that with the aid of the axiom

Items equal to the same item are equal to one another

we can prove the first theorem of Euclid's Elements. The proof took this shape:

A is equal to C.

25 These definitions of the three types of syllogism (which deliberately leave open the possibility that an argument might belong to more than one type) require considerable refinement; nor do they purport to correspond to anything which an ancient logician may have thought or said. They are offered as a rough

indication of how one might try to carve syllogisms into species.

There are two opposite objections to this supposition. On the one hand, some of the examples of relational syllogisms in *Inst. Log.* 16-17 do not essentially contain relational predicates — yet perhaps Galen thinks that they do. On the other hand, at 16.12 Galen perhaps mean to allow that some relational syllogisms do not contain πρός τι predicates — but the text is crucially uncertain, and in fact the examples in question do contain two-placed predicates. In any event, if relational syllogisms are not to be defined in terms of relational predicates, I do not know how they are to be defined.

B is equal to C.

A is equal to B.

It is a relational syllogism inasmuch as its validity is guaranteed merely by the sense of the relational term 'equal to'; and the pertinent aspect of the sense of 'equal to' is given by the axiom which underwrites the proof.²⁶

Are the proofs at *PHP* V 649 relational syllogisms? Well, what are their relational terms? At this point argument (II) might be summoned into the box; for its first premiss contains the comparative adverb 'more evidently' [ἐπιφανέστερον], and comparatives generally express relations. But the second premiss contains no comparative — there the verb φαίνεται (which my translation turns into the adverb 'clearly') corresponds to and picks up ἐπιφανέστερον. Perhaps the text should be emended (say, by the addition of μᾶλλον), or perhaps some comparative notion should simply be 'understood'? Perhaps, on the other hand, the comparative form ἐπιφανέστερον here has no comparative force — Greek comparative adverbs are often 'positive' in sense.

However that may be, a modern logician will readily find a relational term in argument (I). For example, the second premiss —

The origin of the nerves is in the brain

— expresses a relation between the origin of the nerves and the brain, namely the relation of being located in. But perhaps Aristotle would have taken 'in the brain' to be not a relational term but a predicate in the category of 'where'; and perhaps Galen

In the text of Euclid, 1.1, the axiom is used as a premiss of the argument, not as an external underwriting; and it must be admitted that some passages in *Inst.Log.* suggest that Galen too construed the axioms as supplementary premisses. But this cannot — or at any rate should not — have been Galen's considered view.

would have agreed.²⁷ However, the second premiss also contains the term 'origin' or $\mathring{\alpha}\rho\chi\acute{\eta}$ which is indisputably relational and which Galen himself took to be relational (cf. *PHP* V 564).

The relationality emerges clearly if Galen's proof is rewritten as follows:

(IR) Where the nerves originate, there is the ruling part.
The nerves originate in the brain.
Therefore the ruling part is in the brain.

Is this argument relational? That is to say, does its validity depend merely on the sense of the relational term 'originate in'?

Suppose that we vary any or every term in the argument save only 'originate in': is the result always a valid argument? Or equivalently, if all the terms save 'originate in' are replaced by schematic letters, is every instance of the resulting schema a valid argument? Plainly, the answer is No. Replace 'there' in the first premiss by 'elsewhere' and the result is an invalid argument. Hence (IR) is not a relational syllogism. It does not follow that (I) is not a relational syllogism — still less that Galen did not take it to be one. But I can find nothing better than (IR), and (IR) will not wash.

Nor should we wish to find a relational syllogism in the text. For Galen's third species of syllogism is not a species at all. Consider again the Euclidean argument:

(A) A is equal to C. B is equal to C.

A is equal to C.

Galen took that to be on the same logical level as,

(B) If something moves, something is void.

²⁷ See Aristotle, *Cat.* 2 a 1-2 ("in the market-place", "in the Lyceum"); and note Galen's curious examples of predicates in the category of 'where': "second in position from the earth" (*Inst. Log.* 2.1); "in the middle of the universe" (*ibid.* 13.7).

Something moves.

Something is void.

That is to say, in effect he construed the two-placed predicate 'is equal to' as a logical constant, just as in (B) 'if' may be construed as a logical constant. And Galen's relational logic is that system of logic which treats all and only two-placed predicates as logical constants.

Then take the following argument:

(C) Something runs.

Something moves.

It is valid in virtue of the sense of the one-placed predicates which it contains. So it belongs to another, fourth species of syllogism, which we may call predicative syllogisms. Predicative syllogistic is that system of logic which construes all and only one-placed predicates as logical constants. It is on all fours with Galen's relational syllogistic.

There are predicative syllogisms, and there are relational syllogisms — they are all arguments of the sort which the mediaeval logicians called "material consequences" and the Stoics "unmethodically concluding arguments". There are unsurveyably many of them; and although some of them come in little clusters or families — so that you may be tempted to talk of a 'logic of identity' or a 'logic of causality' 28 —, they are not systematizable and they do not form a species in the way in which categorical arguments and hypothetical arguments are systematizable and form species. You might say, crudely, that there is no such thing as predicative logic. In exactly the same sense, there is no such thing as relational logic. 29

²⁸ But a modern logician who speaks of the 'logic of causality' will have in mind standard predicate logic afforced by some causal constants.

²⁹ "But modern textbooks usually contain a chapter called 'The Logic of Relations', so the subject must exist". Many textbooks contain such chapters; but the

Galen's paradigmatic proof is not a relational syllogism, it is not a hypothetical syllogism, it is scarcely a categorical syllogism. In other words, it is not caught in the net of Galen's logic — Galen is not equipped to elucidate its validity. But it is formally valid: how, then, is it best analysed?

Surely the connector 'Where ..., there — ' is the first item to think about. For any argument of the form

Where X, there Y.

X here.

Y here.

is valid. Next, it seems clear that from the first premiss,

Where the nerves originate, there is the ruling part, there follows, by 'universal instantiation':

If the nerves originate in the brain, then the ruling part is in the brain.

And from this conditional proposition and the second premiss of the argument,

The nerves originate in the brain,

there follows, by modus ponens:

The ruling part is in the brain.

Any modern reader with a smattering of formal logic will then be tempted to present the argument in a mixture of symbols and abbreviations, thus:

chapters have nothing to do with Galen's relational syllogistic: they do not describe a third sort of logic, after propositional logic and predicate logic. This is clear from the fact that they introduce no new rules of inference.

$$(\forall x)(Oax \supset Lbx)$$
Oac

 \overline{Lbc}

The validity of this argument can then be proved:

1 (1) $(\forall x)(Oax \supset Lbx)$	prem
2 (2) Oac	prem
1 (3) Oac ⊃ Lbc	1, UE
1, 2 (4) Lbc	2,3 MPP

The term 'originate in' or 'O' is not essential to the validity of the argument — it may be replaced by any other relational term and the result will be a valid argument. Moreover, the validity of the argument does not depend on the fact that it contains relational terms. For, as its proof indicates, it is an instance of the schema:

$$(\forall x)(Fx \supset Gx)$$
Fc
$$\overline{Gc}$$

In short, the argument may be analysed as a hybrid of the categorical and the hypothetical: its validity depends both on the sense of the quantifiers (in the inference from (1) to (3)) and also on the sense of the connectors (in the inference from (2) and (3) to (4)).

This analysis of argument (I) is still hopelessly inadequate;³⁰ and there are, in any event, several other ways of devising an

³⁰ It treats 'the nerves', 'the ruling part' and 'the brain' as singular terms, which they are not; and it does not bring out the fact — which is not explicit in Galen's own formulation — that the nerves, ruling part and brain all belong to the same animal. A paraphrase of the first premiss which begins to do justice to these facts is this:

In any animal, at whatever place all the nerves of that animal originate, at that place is located any ruling part of that animal.

analysis. But two things are, I think, clear: first, none of Galen's three syllogistics is capable of explaining the validity of the argument; and secondly, any adequate explanation will conjoin categorical and hypothetical features. Had Galen thought of uniting categorical and hypothetical syllogistic in some fashion, he would have been the third logician of history. Instead, he discovered a bogus third species of syllogism.

Any analysis of Galen's argument which takes this into account is formidably complex — and in fact exceeds the powers of standard predicate logic.

DISCUSSION

- J. Jouanna: J'ai été très intéressé par votre étude du vocabulaire relatif au syllogisme. Je voudrais vous demander, à propos de γραμμικαὶ ἀποδείξεις, s'il existe des emplois antérieurs à Galien.
- J. Barnes: One at least in Plutarch's Marcellus; and probably another assuming that Nicomachus wrote before Galen.
- M. Frede: I wonder if the term γραμμική ἀπόδειξις does not have roughly the force of 'proof more geometrico'. It might be worthwhile to look for more passages in which the expression is used. As for the use of γραμμικός, it might be relevant that διαγράμματα is used in the sense of 'geometrical proofs'. In any event, I think it is fairly clear that Galen did not think of the proofs as being linear in the sense in which Aristotelian proofs might be thought to be linear, proceeding from the prior to the posterior.
- J. Barnes: I agree that 'linear' has nothing to do with priority and posteriority, and I agree that more geometrico is a rough equivalent provided that the Latin tag is construed in a fairly generous fashion.
- J. Jouanna: Concernant κυριώτατος, ce superlatif est employé chez Galien parfois aussi avec πρῶτος, ce qui ne fait que renforcer ce que vous avez dit sur la synonymie de κυριώτατος avec ἡγεμονικός. Mais bien entendu, dans l'histoire de la langue grecque, κυριώτατος est plus ancien que ἡγεμονικός. Enfin, quel est le rapport exact entre κυριώτατος et ἀναγκαιότατος?
- J. Barnes: It is quite true that, in Galen, χυριώτατος often keeps company with πρῶτος; but in such cases Galen is alluding, consciously or not, to the 'to be or not to be' of ancient meta-

physics: οὐσία ... ἡ κυριώτατά τε καὶ πρώτως καὶ μάλιστα λεγομένη (Cat. 2 a 11-12); and I am not sure that this encourages a rapprochement with ἡγεμονικός. As for ἀναγκαιότατος, I am inclined to think — although the evidence is admittedly tenuous — that, at least in PHP, it is equivalent to or even synonymous with κυριώτατος.

M. Frede: I wonder whether the term μυριωτάτη πρότασις at PHP V 649.15 refers to the contribution which the premiss makes not to the validity of the argument but to its probative character.

J. Barnes: In other words, without that premiss the argument might still be a syllogism but it would not be a proof? That sounds very plausible. But I suspect that it is something determined by the context rather than by the sense of the word κυριώτατος: the word means something like 'indispensable', and the context will answer the question 'indispensable for what'?

T. Tieleman: You argue persuasively that Galen's syllogisms on the location of the parts of the soul display both categorical and hypothetical elements. As you say, Galen himself does not comment on the logic of the proofs. But at PHP V 219-220 he does explain the construction and the sources of the proof concerning the regent part. He does so in an Aristotelian manner insofar as he starts from a definition in order to arrive at a major premiss; and in the near context he refers to Aristotle and Theophrastus as the philosophers who have offered the best accounts of scientific proof (*ibid.* 213; cf. 222). This makes it the more remarkable that he does not use strictly categorical syllogisms. Might Galen's logical education help to explain his attitude? Are there any examples from the later tradition which show a similar hospitality to hypothetical forms of inference?

J. Barnes: The text at PHP V 219-220 is indeed pertinent. In my paper I should have said that, although hypothetical syllogistic is associated primarily with the Stoics, certain hypo-

thetical forms were discussed and accepted by the Peripatetic logicians (from Theophrastus onwards). So Galen might in principle have thought that his proofs were hypothetical in form and yet Peripatetic in spirit. But the fact remains that he says nothing about the logical structure of the proofs.

V. Barras: L'importance du syllogisme relationnel semble évidente dans le système de preuves que propose Galien. Tentant de relier — peut-être de façon artificielle — la logique de Galien à sa pratique d'anatomiste, je me demande si l'anatomie, qui peut être entendue comme l'art de mettre en lumière des relations cachées entre différents organes ou parties, n'a pas quelque affinité avec une logique 'relationnelle'. Votre exemple, "Where the nerves originate ...", abonde en termes relationnels qui sont à la fois ce que Galien anatomiste cherche à prouver.

- J. Barnes: Je vous remercie pour votre suggestion, qui me paraît bien fondée: assurément Galien anatomiste aura remarqué l'importance des relations pour son étude. Il vaut pourtant la peine d'ajouter que la présence d'un terme relationnel dans un argument n'est pas une condition suffisante pour que l'argument soit relationnel: les syllogismes catégoriques et hypothétiques peuvent contenir de tels termes, pourvu que leur relationnalité ne soit pas pertinente à la validité de l'argument.
- J. Jouanna: À propos du second texte que vous citez (PHP V 649) dans votre étude si suggestive sur les relations entre la logique et l'anatomie, je voudrais vous demander comment vous interpréter ταύτην τὴν πρότασιν. Pour ma part, je verrais une reprise de la πρόσληψις qui précède, à savoir la πρόσληψις ψευδής, "l'origine des nerfs est dans le cœur".
- J. Barnes: The passage to which you refer contains several related difficulties. The transmitted text is this:
 - ... ὅπου τῶν νεύρων ἡ ἀρχή ἐνταῦθα καὶ τὸ τῆς ψυχῆς ἡγεμονικόν.

αὕτη μὲν ἡ τοῦ λόγου κυριωτάτη πρότασις ὡμολογημένη πᾶσιν ἰατροῖς τε καὶ φιλοσόφοις: ἡ δ' οἶον πρόσληψις αὐτῆς ἀληθὴς μὲν ἡ ἀρχὴ τῶν νεύρων ἐν τῷ ἐγκεφάλῳ,

ψευδής δέ

ή ἀρχὴ τῶν νεύρων ἐν τῆ καρδία, γράφειν μὲν ταύτην τὴν πρότασιν ἢ καὶ λέγειν τοῖς ἀπείροις ἀνατομῆς δυναμένου τινός, οὐ μὴν δεῖξαί γε δυναμένου. πάντα γὰρ ἐν τοῖς ζώοις τὰ μόρια νεύρων μετέχει, κτλ. (V 649.13-650.3)

Your question concerns the reference of ταύτην την πρότασιν. In addition, one may wonder with what verbs the dative τοῖς ἀπείροις ἀνατομῆς should best be taken. And one may ask what is the sense of γράφειν. Each of these questions admits several answers which are, from a grammatical point of view, equally plausible. Different combinations of the different answers ascribe radically different doctrines to Galen. My own opinion, of which I am far from certain, is this. Galen wants, in this passage, to draw a contrast between the two premisses of the arguments by which he thinks the problem of the location of the parts of the soul is to be settled. One of those premisses will be a general truth — a truth of reason accepted by all doctors and philosophers. The other premiss, however, will be empirical, and in particular it will depend on anatomical knowledge — which is why the Stoic philosophers are incompetent in its regard. That being so, then, first, we should take ταύτην την πρότασιν to refer neither to the false πρόσληψις nor to the true. Rather, it picks up ή ... οἶον πρόσληψις αὐτῆς: Galen means that the second premiss of the argument, whatever it may be, is beyond the competence of those who are unskilled in anatomy. Secondly, τοῖς ἀπείροις ἀνατομῆς should be taken with all three infinitives — γράφειν, λέγειν, and δεῖξαι. Thirdly, γράφειν means 'draw', not 'write'. Hence the remark may be paraphrased as follows: "As for the second premiss of such arguments, if someone is ignorant of anatomy, you may draw it for them (they can understand a diagram showing the nerves all converging on the brain),

and you may even state it to them (they can, in principle, understand the medical terms you use); but you cannot show them its truth (they have not done the practical work which that demands)". Other, very different interpretations of the passage are indeed possible; but I think that the following sentence $\pi \acute{\alpha} \nu \tau \alpha \ \gamma \grave{\alpha} \rho \ldots$ tells strongly in favour of my reading.

and you may even care it to their (they can in principle under stand the modical teams who had been work cambon already that and the first truth (they have not come of a practical and which that their truth (they have noticed as deleteral incorparements of the passage demands). Other, were deleteral incorparements in the passage are indeed possible; but a daily that the following sentence are reduced possible; but a daily that they are required.