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Internal Models for the Swiss Solvency Test

Introduction

With the introduction of the Swiss Solvency Test (SST), a wide range of models will be employed by insurers and reinsurers for regulatory purposes. Some companies will use the SST standard model [TD2006] which is supplied by the regulator but many will be using or developing internal models which either supplement or completely replace the standard model.

The Federal Office of Private Insurance (FOPI) decided early during the development of the SST that the use of internal models should be encouraged. It was the aim of FOPI that companies having similar risks should have similar capital requirements.

Expressed differently, FOPI wanted to achieve comparability of results rather than comparability of methods. If comparability of methods would have been of overriding importance, a standard formula should have been prescribed. A prime example of such an approach is Solvency 1 [see for instance AS2006]. The Solvency 1 calculation is simple and defined by fixed rules. This allows supervisors to check whether a company has followed the rules faithfully or not. However, the result of the calculation will in many cases have little bearing on the actual risk situation of the company. This is a general problem for rule-based solvency systems. Even if the model is calibrated to give the correct results for an average company, it will under- or overestimate the capital requirement for a great many companies. Therefore, comparability of result was of paramount importance to FOPI for the SST.

For many companies, a standard model would not be adequate to capture its risks adequately. These companies then have to use an internal model for the SST. Internal models are complex and regulatory requirements can not be captured by simple rules. Therefore, the internal models have to satisfy a number of principles so that the regulatory framework is flexible enough to encompass different internal models but specific enough to ensure comparability of results. FOPI actively promotes the use of internal model, since they are able to better reflect a company's risk and also because it is an integral part of risk management.

The SST Methodology

Risk-based solvency is interested in answering quantitative questions relating to the risk exposure of insurers. The question the SST is most interested in can be posed concisely as follows:

> Consider the economic balance sheet of a company, where all assets and liabilities are valued market consistently and define available capital as the market value of assets minus the market consistent value of liabilities. What is then the expected shortfall on the 1% tolerance level of the change of available capital due to financial market, credit and insurance risk within one year?

The above question – together with some definition on what is exactly meant by an economic balance sheet, by market consistent valuation and some further explanations – is sufficient to specify what is expected from a model for the SST. In the following we explain in more detail the SST methodology. An internal model which is consistent with the SST is a special class of an internal model. The valuation is based on a market consistent framework and the risk measure used is the expected shortfall or Tail Value at Risk over a time horizon of one year. For more on the methodology of the SST, see for instance [WP2004].

For a model to be consistent with the SST, it has to satisfy the following principles:

1. All assets and liabilities are valued market consistently

2. Risks considered are market, credit and insurance risks

3. Risk-bearing capital is defined as the difference of the market consistent value of assets less the market consistent value of liabilities, plus the market value margin

4. Target capital is defined as the sum of the Expected Shortfall of change of risk-bearing capital within one year at the 99% confidence level plus the market value margin

5. The market value margin is approximated by the cost of the present value of future required regulatory capital for the run-off of the portfolio of assets and liabilities

6. Under the SST, an insurer's capital adequacy is defined if its target capital is less than its risk bearing capital

7. The scope of the SST is legal entity and group / conglomerate level domiciled in Switzerland

8. Scenarios defined by the regulator as well as company specific scenarios have to be evaluated and, if relevant, aggregated within the target capital calculation

9. All relevant probabilistic states have to be modeled probabilistically

10. Partial and full internal models can and should be used. If the SST standard model is not applicable, then a partial or full internal model has to be used

11. The internal model has to be integrated into the core processes within the company

12. SST Report to supervisor such that a knowledgeable 3rd party can understand the results

13. Public disclosure of methodology of internal model such that a knowledgeable 3rd party can get a reasonably good impression on methodology and design decisions

14. Senior Management is responsible for the adherence to principles

Economic Valuation for the SST

The SST is based on a market consistent (economic) valuation. In a market consistent framework, consistency between the valuation of assets and liabilities can easily be achieved. Most statutory valuation systems lack this consistency and introduce artificial volatility into the risk quantification.

Market consistent valuation is based on an economic balance sheet. All financial instruments are included in an economic balance sheet (also guarantees and other off-balance sheet items) and have to be valued.

The market consistent value of a liability corresponds to the economic cost that the company is expected to incur during the whole run-off of the liabilities. This comprises the cost of meeting the claims, the cost for expenses and the cost for holding economic capital necessary to absorb deviations from the expected. To quantify the market consistent value of liabilities, two concepts need to be introduced: The replicating portfolio and the cost of capital margin (CoCM). A replicating portfolio for the liabilities is a portfolio of financial instruments which replicate the cash flows of the liabilities and which are traded in a deep and liquid

market. Since the instruments are traded, their prices can be observed. An insurer also needs a buffer for the risks that can not be hedged by the replicating portfolio, i.e. the basis risk. The cost to absorb the basis risks is taken into account by the CoCM.

For the SST, the market consistent value of liabilities is then defined as:

- observed transfer prices if the liabilities are traded;

- the market value of a replicating portfolio if the cash flows can be replicated with financial instruments which are traded in a deep and liquid market; and

- if the replication is not perfect under all possible states of the world, the remaining basis risk has to be captured with the cost of capital margin.

Usually, an insurer is a going concern and the CoCM is based on its own cost of capital and its own economic capital required to buffer the basis risk. However, in cases the company has financial problems and it becomes insolvent, FOPI requires that there is sufficient capital that the liabilities can either be transferred to another insurer or that the liabilities can be run-off. In case a company's financial situation becomes such that the liabilities have to be run off, the cost of capital margin is defined by the minimal required regulatory capital the company has to set up and the cost of capital corresponding to a company which has approximately a BBB rating[1].

There are therefore two separate cost of capital margins for the SST: a company specific one defined as the cost of future economic capital the company has to set up and a regulatory cost of capital margin, based on the cost of future regulatory capital.

If a company's financial situation deteriorates, the company specific CoCM (CoCM(company)) converges to the regulatory CoCM (CoCM(regulatory)). The CoCM(regulatory) can be higher or lower than the CoCM(company). While the cost of capital for a lower rated company increases, the future regulatory capital to buffer risks is lower than the future economic capital for a higher than BBB rated company.

For simplicity's sake, within the SST only the CoCM(regulatory) has to be determined since the focus of the SST is the situation of a company in financial distress where CoCM(company) equals CoCM(regulatory). However, it has to be noted that this is a simplification and not fundamental to the methodology. Conceptually it would have been more correct to base the economic balance

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[1]We are grateful to Pablo Koch-Medina and Hans-Peter Würmli for many illuminating discussions on the concept of economic valuation and cost of capital margins.

sheet for the SST on the CoCM(company) so that it optimally reflects the current financial position of the company. The CoCM(regulatory) then would have defined the lowest intervention level under which risk bearing capital is not allowed to drop.

In the following, we explain how the CoCM(regulatory) is determined.

The CoCM(regulatory) is defined as the cost of future regulatory capital a company has to set up when taking over the portfolio. Currently (in 2007) the cost of capital is set at 6%. This corresponds to the capital costs of a potential buying company rated BBB, which is approximately the minimal capital requirement under the SST. The future regulatory capitals $SCR(t)$ (Solvency Capital Requirement at time t to cover risks emanating during $[t, t+1)$) at times $t = 1, 2, \ldots$ are calculated assuming that the asset portfolio is changed to an optimal replicating portfolio, i.e. that the assets are chosen from a deep and liquid market such that the remaining basis risks is minimized. It is assumed that the portfolio transfer occurs at the end of year 0, so that the current SCR $(:= SCR(0))$ does not enter the calculation of the CoCM(regulatory).
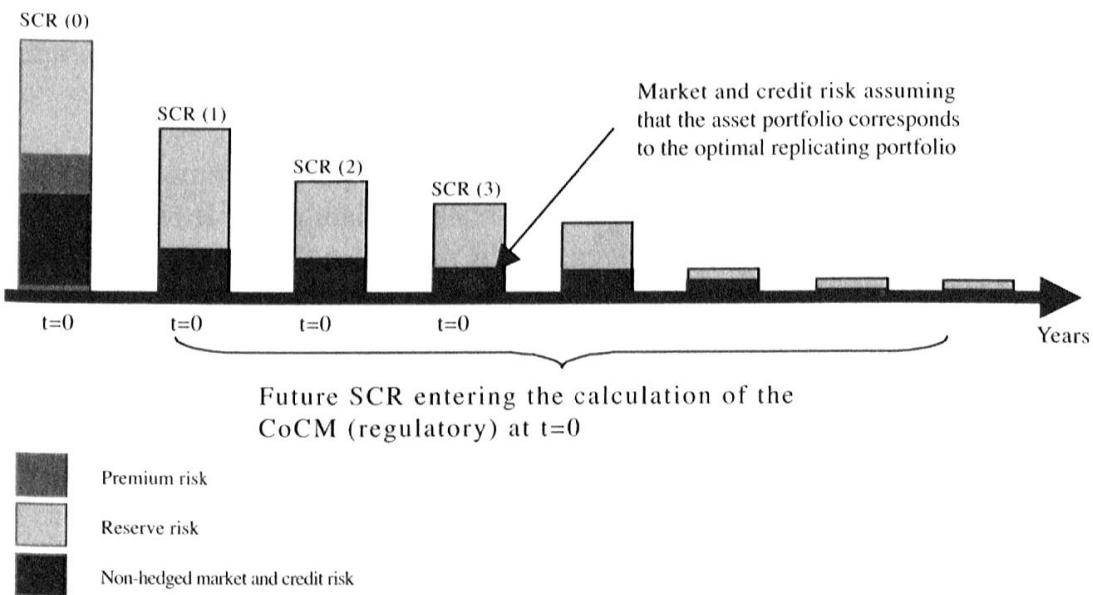
The choice of the replicating portfolio is not unique and to a certain degree up to the company to define. A company can chose to define a simple replicating portfolio which replicates the expected liability cash flows only. This replicating portfolio can then be composed of government bonds. At the other extreme, a company can put together a replicating portfolio which tries to replicate the liability cash flows under different economic scenarios. The more complex

replicating portfolio might then be composed not only of government bonds, but also of corporate bonds, swaptions and other derivative instruments. If a simple replicating portfolio is chosen, the remaining non-hedged financial market risk is higher than if a more complex replicating portfolio is chosen. This implies that the CoCM(regulatory) is then also higher. It is important that whatever the choice of the replicating portfolio the company has made, the financial instruments of which it is composed are chosen from a deep financial market.

To determine the CoCM(regulatory), the future SCR, given that the liabilities are run-off and that the assets are composed of a replicating portfolio have to be determined.

- In the first step for each year of the run-off period $t = 1, 2, \ldots$, the liability portfolio has to be determined.

- Then, given the liability portfolio at year k, the associated replicating portfolio for year k has to be put together.

- Given the assets and liabilities of year k, the corresponding SCR(k) has to be calculated.

- To calculate SCR(k) the company can either do an SST (i.e. a full calculation) or use a proxy for SCR(k).

During the field test 2004 and 2005, a simple proxy was used. It was assumed that future SCR develop proportionally as the best estimate of liabilities. However, in many cases this proxy might not be adequate. In particular, the smaller that liability portfolio gets, the more relevant relatively stochastic risk becomes. For certain lines of business (e.g. disability business), also other proxies might be better. It is the responsibility of the company to determine an optimal proxy for its business. Proxies used during the field test 2006 where for instance the sum at risk or the expected number of claims payments during a year.

The future SCR(k) have to take into account that there is often a certain amount of financial market risk which can not be hedged away using a replicating portfolio. This has to be taken into account. In the early years of the run-off, this non-hedgeable market risks is relatively higher than in later years. This is due to the fact, often during the beginning of the run-off, the duration of liabilities is longer than the duration of financial instruments which are traded in deep and liquid markets.

Market consistent valuation implies that the valuation and the risk quantification have to be realistic, without artificial implicit and explicit prudence. This does **not** imply that risks which are difficult to quantify or parameters which are difficult to estimate should **not** be assessed conservatively. This is consistent with a market

consistent methodology, since the market also takes into account illiquidity or lack of information by requiring a risk premium.

A market consistent framework implies that all options are to be assessed realistically. This is true for options owned by policy holders (e.g. an annuity option) but also for options owned by the insurer, for instance the option to distribute profits to the policy holders. Management's options have to be modeled realistically. Management needs to be able to formulate its management strategy in such a way that it can be captured within the model. This is a complex task since it might require the projection of management options not only for the next year put possibly for the whole duration of the liabilities.

For the SST, risk bearing capital is defined as the market value of assets less the discounted best estimate of liabilities. Part of risk bearing capital is however not available to the company. Risk bearing capital less the CoCM(regulatory) is available to cover the risks emanating during the 1 year time horizon, the CoCM(regulatory) covers the risks emanating from $t = 1$ until the whole run-off of the liabilities. Hence, if risk bearing capital would be less than the CoCM(regulatory) the company would be economically insolvent.

The figure below shows the economic balance sheet for the SST. Note that target capital is defined as the sum of the SCR and the CoCM(regulatory).
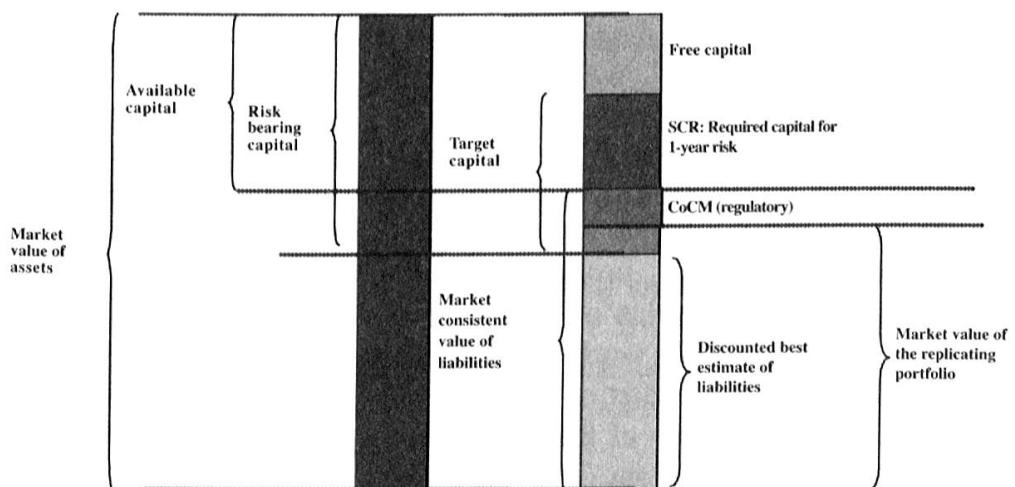
Risk in the SST

Risk within the SST is defined as the change of the economic balance sheet of the company between now and in 1 year's time. The current economic balance sheet is known and can be determined exactly. The economic balance sheet in

one year's time is unknown and will depend on events which will occur during the time horizons.
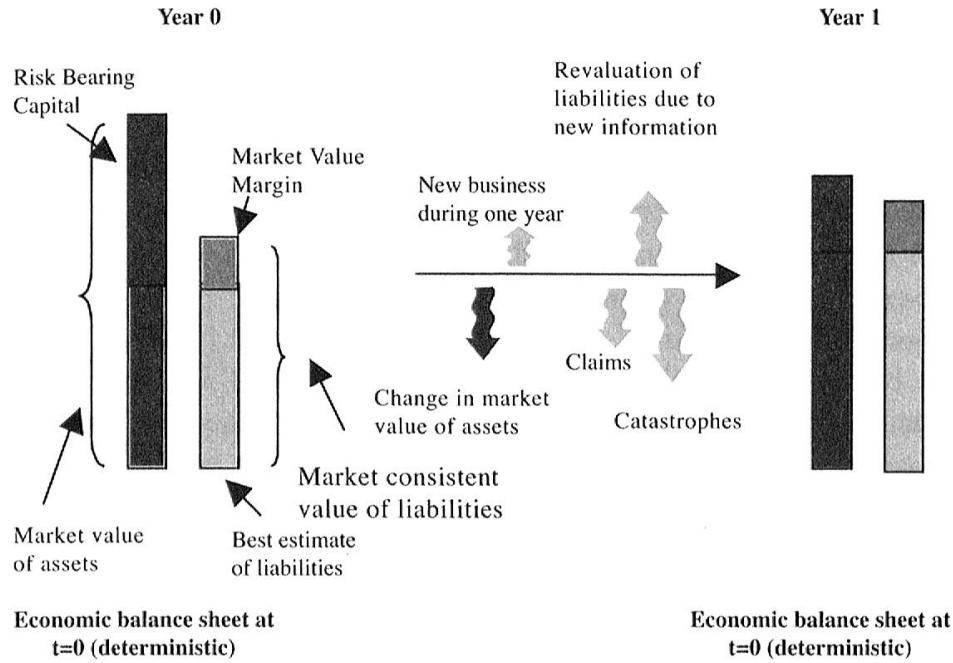
We can write the basic equations for the SST concisely in two lines. We first define a valuation function V which assigns to a financial instrument its currency value (e.g. to a bond its value in Swiss francs). We denote by F the space of all financial instruments (e.g. bonds, insurance policies, guarantees received, reinsurance recoverables, etc.). The value of a financial instrument depends on the state of the world. The state of the world is defined by – for instance – the interest rates, the expectation of the market on changes in FX rates, the mortality tables etc. The set of all possible states of the world at time t is denoted by S_t. The valuation is therefore a mapping from (F, S_t) into $\mathbb{R}$:

$$V : (F, S_t) \to \mathbb{R}$$
$$(f, s_t) \to V(f, s_t), \ f \in F, \ s_t \in S_t$$

Let $\{f_1, \ldots, f_n\}$ be the set of financial instruments of the company. Then the value of the company at time t is simply the sum of the values of the financial instruments the company holds at time t, given the state of the world $S_t = s_t$.

To make the notation more intuitive, we call the value of the company at time t the available capital $AC(t)$. The available capital at time $t = 0$ is deterministic. Since the state of the world at a future time $t > 0$ is unknown, available capital $AC(t)$ is a random variable. S_t depends on the current state of the world $S_0 = s_0$, since the evolution of the state of the world from time 0 to t depends on the initial position s_0. We can write

$$AC(t) = \sum_{i=1}^{n} V(f_i, S_t \mid S_0 = s_0) \,.$$

We denote by SCR(0) the required capital at $t = 0$ for taking on the risks which emanate during a given time horizon. For the SST, this time horizon is one year. We can now write the two lines describing the capital model for the SST succinctly as follows:

$$AC(t) = \sum_{i=1}^{n} V(f_i, S_t \mid S_0 = s_0) \,,$$
$$SCR(0) = -ES[AC(1) - AC(0)] \,.$$

where ES denotes the expected shortfall on a 1% tolerance level.

The equations above express that the valuation and risk quantification belong together. In contrast to Solvency 1, where the solvency margin is added somewhat ad-hoc on top of the liabilities, in the SST, the SCR is defined by the valuation and the risk measure. This consistency – together with other consistency requirements (e.g. between the valuation of assets and liabilities) – was of paramount importance during the development of the SST. Without consistency, a risk-based solvency framework easily becomes open to be arbitraged against.

The valuation of liabilities is at the core of a capital model. It is part of, and methodologically inseparable from, the calculation of required capital. It is an internal model with attendant requirements on processes, documentation, responsibilities, etc.

The above high level equations have to be translated into a model which then actually does the calculations. Depending on the level of simplification and methodology chosen, models can be very different but still be able to translate acceptably the equations above. For insurers and reinsurers, the challenge is to find a level of simplification such that the model is still able to capture the essential part of its risk exposure. For the supervisor, the challenge is then to decide if the model is acceptable. An acceptable model is a model which is reasonably consistent with the principles. Full consistency might be too restrictive a requirement for acceptability, but the deviations from the principles should not

be material. The difficulty for the supervisor lies in the fact that there are a vast (infinite) number of possible models and a much smaller but still infinite number of acceptable models.

The Structure of Internal Models

While internal models of different insurers superficially might look differently, the underlying structure is quite similar. The structure is easiest to explain for a model with a scenario approach. In such a model, scenarios are generated which allow the estimation of the value of assets and liabilities as well as the required solvency capital.

As above, the main input of the model is the portfolio of financial instruments $\{f_1, \ldots, f_m\} \subset F$ of the company. Depending on the type of company (e.g. life insurer, credit insurer, global reinsurer), the financial instruments can be quite different and the actual models have to reflect this fact. Hence, while the structure of an internal model for a small mutual and a global reinsurer is similar, the actual implementations of the models are of course of vastly different degrees of complexity.

Next, the states of the world on which the values of the financial instruments and therefore the value of the company depend have to be defined. For this, the states of the world have to be approximated by a vector of relevant risk factors. These risk factors have to be chosen carefully. Too many risk factors make the model unwieldy; too few would not capture the risk exposure of the company adequately. The SST standard model uses about 80 risk factors for market risk only. For some insurers with simple asset portfolio, this might be too much, for other insurers with complex assets, 80 risk factors might be too few to capture market risk adequately.

Then the stochastic evolution of the risk factors has to be modeled. This step is likely to be the most complex one. The model needs to capture the potential evolution of the risk factors over time, given the observed initial state $S_0 = s_0$. This implies that the model has to define the random behavior of the different risk factors as well as the dependency between the risk factors given an initial state for all future times $t > 0$. In addition, the model has to ensure that non-arbitrage conditions are satisfied during all times $t > 0$.

Given the dependency structure between the risk factors, the model then generates a number of (arbitrage-free) scenarios $(s^1(t) \mid s_0), \ldots, (s^N(t) \mid s_0)$ which capture possible states of the world at future times $t > 0$. The evolution of the possible states of the world $(s^j(t) \mid s_0)$, $j = 1, \ldots, N$, has to take into account optimally the available information of the financial and insurance market. The model has

to be calibrated against observed prices of traded financial instruments to ensure that the model is in-line with the market expectation.

The profit or loss $e(1)$ incurred by the company between time 0 and time 1 is the difference of the value of the company at time 1 and at time 0. Hence, given a scenario $s^j(1)$:

$$e_j(1) = \sum_{i=1}^{m} V(f_i, (s^j(1) \mid s_0)) - \sum_{i=1}^{m} V(f_i, s_0), \qquad j = 1, \ldots, n.$$

For the valuation of the company at time $t = 1$, the model has to generate for each simulated state of the world $(s^j(1) \mid s_0)$ further evolutions of the risk factors $(s^k(t) \mid s^j(1))$ for $k = 1, \ldots, N$, and all $j = 1, \ldots, N$ and $t > 1$. This is much more difficult than generating states of the world starting from the $t = 0$, since it is not possible anymore to calibrate the model to observed prices of financial instruments anymore.
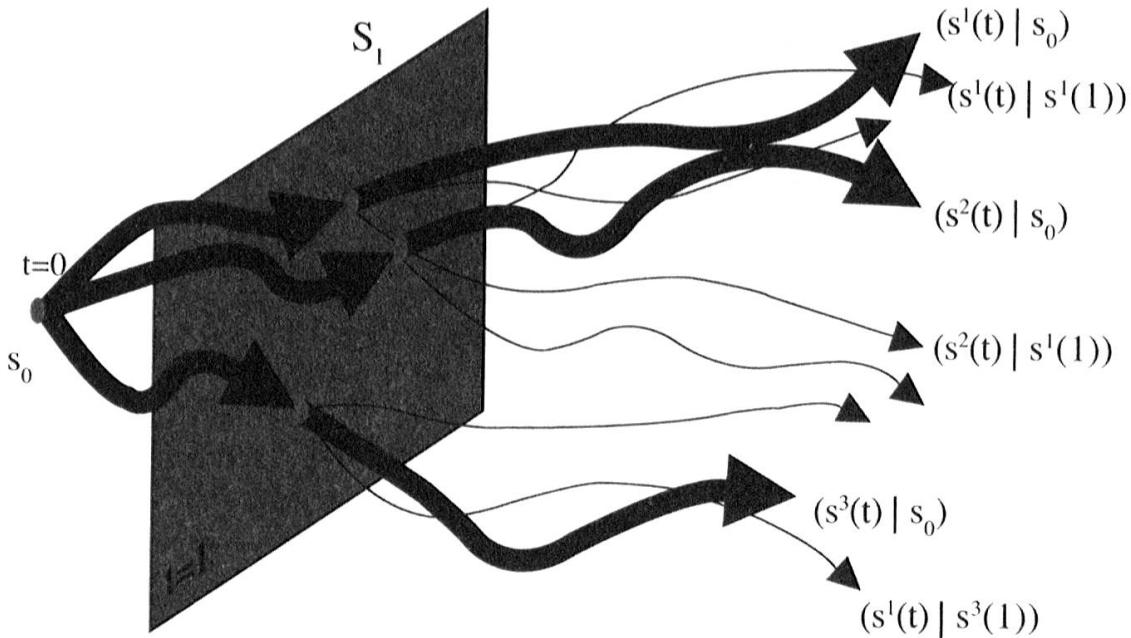
Finally, the necessary risk capital can easily be calculated using the simulated profit and losses $e_1(1), \ldots, e_N(1)$ for the different scenarios.

There are cases where this two-step procedure might not be sufficient. For instance, life companies selling policies with profit participation features such that policy holders profit yearly from the economic (or statutory) profit of the company during a business year, need – at least in theory – be able to capture the stochastic evolution of the risk factors for initial states $s^k(u)$ for $u = 2, \ldots, T$,

where T is the time until run-off of the assets and liabilities. The model has then to generate not only states of the world starting at s_0 and $s^j(1)$ but also starting at $s^k(u)$: $(s^l(t) \mid s^k(u))$, $u = 2, \ldots, T$, $t > u$.

The computational load for such a model becomes prohibitive. If for instance the duration of liabilities is 10 years and 100 scenarios have to be generated for the valuation at each time step, then the number of scenarios generated would be in the order of 10^{100}. The model then has to be simplified to allow reasonable run-speeds.
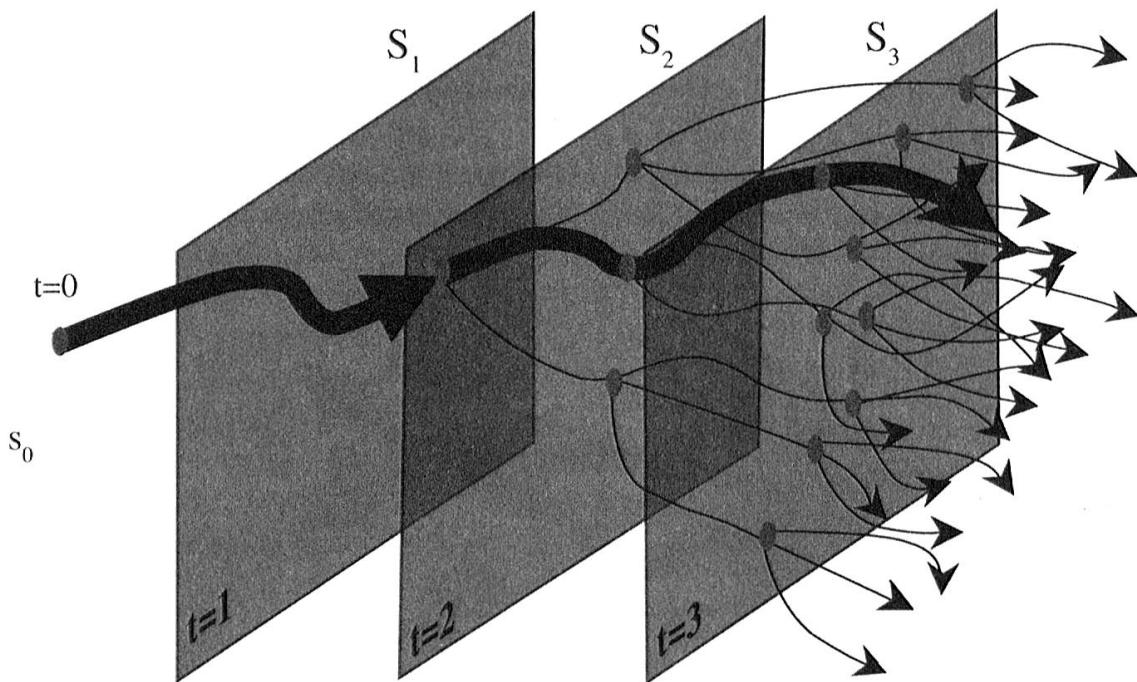
Group Models

The projection of the economic balance sheet of an insurance group can also be complex. As long as an insurance group is a going concern, it can often be considered with good reason as a consolidated entity. If a subsidiary – especially a major one – were to have a financial problem, it can rely on support from the group. However, if the insurance group is in financial distress, the situation can change profoundly. In case of financial problems, the group is likely to revert to a legalistic interpretation of its rights and obligations vis-à-vis its subsidiaries. The group might then exercise the limited liability put option it owns and let some subsidiary go into an insolvent run-off. Hence the model has to take into account this behavior for those scenarios where the group as a whole is in financial

distress. The economic balance sheet then is not a consolidated one anymore but segregates into a collection of separate economic balance sheets for the different legal entities of the group. This has consequences for the set-up of internal models needed to capture the risks of the group. The models need to be able to map the economic balance sheets of the different legal entities for different states of the world. This implies that it can not be based on a consolidated view of the group but needs to be able to take into account capital and risk transfer instruments between the legal entities of the group as well as the separate economic balance sheets of the legal entities. For more details, see [MG2006] and [MR2007].

Real Life Models

Most internal models of companies differ from the above outlined structure in some details. Approximations and short-cuts are used to improve calculation speed. Often the valuation at time $t = 1$ is simplified by just using an expected scenario given an initial state $(s^i(1) \mid s_0)$, reducing the number of scenarios which has to be generated from $O(N^2)$ to $O(N)$. It is however important that the modelers are always aware of the simplifications and check that the inaccuracies introduced are not material.

In the above outlined approach, the states of the world at time $t = 1$ are used to determine the required capital. Since for the SST, the SCR is defined as the expected shortfall of the change of risk bearing capital over a one-year time horizon on the 1% confidence level, a large number of scenarios have to be generated to estimate the SCR with reasonable stability. For the valuation of assets and liabilities at the hypothetical states $s^j(t)$, a smaller number of scenarios might be used. While the valuations of the company then become less exact, the over- and underestimations likely average away and the estimation of SCR is still adequate. However, the valuation of the company at time 0 should be done with great care since an under- or overestimation would introduce a systematic error in the calculations of the profit and losses $e^j(1)$.

Some models use two sets of scenarios: ones where states of the world are generated with physical, real-world probabilities and ones where the probabilities are risk-neutral. The physical scenarios are used to generate states of the world at $t = 1$. The risk-neutral scenarios are used for the valuation (at $t = 0$ and $t = 1$) and ensure that non-arbitrage holds.

Many internal models are an amalgamation of different models which are developed to deal with specific risks (e.g. credit risk or life insurance risk). Often, insurance, market and credit risks are determined separately, the expected shortfalls calculated for each risk and then the total capital requirement obtained

by using a correlation matrix for the separate shortfalls. The problem with such an approach is that the correlation depends – among other factors – on the portfolio of assets and liabilities. Even if the correlations have been estimated adequately, if the asset and liability portfolio changes, the correlation matrix might not be appropriate anymore.

A more general problem with many models is that parameters are often calibrated using the implicit assumptions that they are independent on the state of the world or on the financial state of the company. For instance, correlations between market risk factors in the event of a financial crisis are different than during normal periods. A company's loan might be rating dependent and has to be paid back in case of the insurer's financial distress. Often, dependencies between risk factors can change suddenly and non-linearly. While the financial market will not be influenced by a small random increase of mortality, a global substantial increase in the number of deaths (e.g. in case of a pandemic) will have an impact.

Outlook

The development of economic capital models for insurers is at the beginning. It is likely that the sophistication and complexity of the models in the future will vastly exceed the ones in use currently. The increase in processing speed and memory of future computers will not only make the run-speed faster but will allow qualitatively different models to be used. The development of multi-year models for financial market risk factors will allow more stable projections of possible future states of the world. While currently a number of economic scenario generators are commercially available, there will be improvements which will make them accessible for small- and mid-sized companies but will also make them sophisticated enough for use by complex, international insurance groups.

It is important that economic capital models which are being developed now for the SST are set up flexibly enough so that they can benefit from future improvements of calculation speed and methodologies. Since the development and implementation of an internal model is a multi-year project, the model framework should take into account not only the current but also future requirements and possibilities.

Internal models will have to be fully integrated within the processes of the company. For this, models will have to be flexible enough so that they are able to quantify the economic and statutory state of the company from different perspectives. While for the insurance supervisor, the policy holder perspective is of main interest, for senior management other points of view are also relevant, e.g. the shareholder perspective, bondholder perspective etc. An internal model

ideally is also able to calculate the future position of a company not only for an economic valuation framework, but also for other valuation systems (e.g. IAS, USGAAP, statutory valuation, etc.).

For an internal model to be useful to senior management, it has also to be able to quantify the effects of short- and long-term strategies. Senior management can then assess the effects of different policy holder participation schemes, investment strategies, risk mitigations like reinsurance, securitization etc. It will allow senior management to choose among a portfolio of strategies the optimal one.

Literature

[AS2005] Arne Sandstrom, Solvency: Models, Assessment and Regulation, Chapman & Hall, 2005

[MR2007] Thomas Luder, Modelling of Risks in Insurance Groups for the Swiss Solvency Test

[MG2006] Draft Modelling of Groups and Group Effects, Bundesamt für Privatversicherungen, Bern, 2006

[WP2004] White Paper of the Swiss Solvency Test, Bundesamt für Privatversicherungen, Bern, 2004

[TD2006] Technisches Dokument zum Swiss Solvency Test, Bundesamt für Privatversicherungen, Bern, 2006

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Abstract

For many companies, the standard model for the SST would not be sufficient to capture their risks adequately. These companies then have to use an internal model. Such models have to fulfill a number of principles to satisfy the regulatory requirements. The paper focuses on the SST methodology in general and on the structure of internal models in particular.

Zusammenfassung

Für viele Gesellschaften wird das Standardmodell des SST nicht ausreichen, um die Risiken adäquat darzustellen. Diese Gesellschaften müssen ein internes Modell entwickeln. Solche Modelle müssen eine ganze Anzahl von Prinzipien erfüllen um den regulatorischen Anforderungen zu genügen. Der Fokus des Artikels liegt auf der SST-Methodologie im Allgemeinen und der Struktur interner Modelle im Speziellen.

Résumé

Pour nombre de compagnies le modèle standard ne sera pas suffisant pour représenter les risques de manière adéquate. Ces compagnies doivent développer un modèle interne. De tels modèles doivent satisfaire un nombre de principes pour répondre aux exigences prudentielles. L'article présente la méthodologie du SST en général ainsi que la structure des modèles internes en particulier.