Zeitschrift: Mitteilungen / Schweizerische Aktuarvereinigung = Bulletin / Association Suisse des Actuaires = Bulletin / Swiss Association of Actuaries

Herausgeber: Schweizerische Aktuarvereinigung

Band: - (2005)

Heft: 1

Artikel: Closing and projecting life tables using log-linear models

Autor: Denuit, Michel / Goderniaux, Anne-Cécile

DOI: https://doi.org/10.5169/seals-967316

Michel Denuit and Anne-Cécile Goderniaux, Louvain-la-Neuve, Virton

Closing and Projecting Life Tables using Log-Linear Models

1 Introduction and Motivation

1.1 Mortality trends

In most industrialized countries, the human mortality globally declined during the 20th century; see, e.g., McDonald et al. (1998). Over the next few decades, the EU countries will undergo a large demographic transformation induced by the retirement of the baby-boomers and the dramatic increase in their longevity.

The past 100 years have seen many improvements in life expectancy, but the pattern of the improvement is changing markedly. In the first half of the 20th century, infectious diseases were almost eradicated and successfull vaccines were developed and made available. This led to massive improvements in mortality among the young. However, cancer and heart disease kept mortality rates stable for older people. Since then, substantial increases in longevity have been achieved at later ages.

1.2 Consequences for the insurance market

When mortality at old ages was relatively stable, insurers could predict how long a group of insured people were likely to live, allowing underwriters to price annuities relatively accurately. Now that the uncertainty about longevity has increased, insurers are no longer sure how much to charge for annuities.

The mortality improvements described above thus pose a challenge for the planning of public retirement systems as well as for the private life annuities business. When long-term living benefits are concerned, the calculation of expected present values (for pricing or reserving) requires an appropriate mortality projection in order to avoid underestimation of future costs. More generally, such trends affect any insurance cover providing some kind of "living benefits", such as long term care benefits or lifetime sickness benefits.

In recent years many insurers lost money on annuities and may continue to do so because they underestimated mortality improvements. The problems experienced by Equitable Life provide a stark example of the perils that may arise when annuity rates fall below guaranteed levels. Actuaries reacted by constructing projected life tables. Such tables incorporate a forecast of future mortality trends and are doubly indexed: by age x and calendar time t.

Extensive simulation studies have been conducted by BROUHNS ET AL. (2002b). On the basis of the ruin probabilities, it has been clearly shown in this paper that pricing with an obsolete static life table leads in most cases to insolvency for the insurance company. These results enlighten the importance of mortality projections. Nevertheless, it is possible to counteract the longevity risk by sufficiently high financial returns on the reserves (see BROUHNS ET AL. (2002b) for details).

Of course, the projected life table may itself underestimate future mortality improvements. Life annuity contracts typically run for several decades so that a life table which may seem to be on the safe side at the beginning of the contract might well turn out not to be so. Moreover, contrarily to financial assets (that can be very volatile), changes in forces of mortality slowly occur and pose a long term, but permanent, problem. Reinsurance treaties covering longevity risk are usually expensive and many life insurance companies are reluctant to buy long-term reinsurance coverage (because of substantial credit risk). In that respect, securitization offers an interesting alternative to reinsurance. DENUIT ET AL. (2004) designed "survival bonds" based on a public mortality index. The classical Lee-Carter model for mortality forecasting is then used to price a risky coupon bond based on this index. The linear version of Lee-Carter proposed in this paper could also be used to price these bonds, avoiding the selection of the appropriate ARIMA model.

1.3 Mortality projection models

Projected life tables can be constructed according to various approaches. Elementary approaches simply extrapolate observed trends in the sequence of annual death probabilities at each age. Other approaches rely on analytical mortality laws, as Makeham, Weibull or Heligman-Pollard. Mortality trends are represented assuming that the parameters of the mortality law are functions of the calendar year. See FELIPE ET AL. (2002) for a recent application to Spanish data.

LEE & CARTER (1992) expressed the secular change in mortality as a function of a single time index. This index is then modelled and forecast as a stochastic time series using standard Box-Jenkins methods. From this forecast of the general level of mortality, the actual age-specific rates are derived using the estimated age effects. For a review of recent applications of the Lee-Carter methodology, we refer the interested readers to LEE (2000). Let us mention that BROUHNS ET AL. (2002a,b) embedded the Lee-Carter model in a Poisson regression framework. We refer the interested reader to TABEAU ET AL. (2001) for various perspectives on forecasting mortality, and to PITACCO (2004) for more details about early attempts to project mortality for annuitants.

1.4 Scope of the paper

In the Lee-Carter approach (as well as in its Poisson counterpart), time is modelled as a factor and Box-Jenkins methodology is used to model the resulting time series. In this paper, we suggest to linearize the time index involved in the Lee-Carter model. Basically, we model time as a known covariate (as, e.g., in SITHOLE ET AL. (2000)): time now enters the model in a linear way on the log-scale. This avoids the two-step estimation procedure caused by the modelling of the fitted time index as a time series.
Before projecting the mortality to the future, some processing of the data is needed for actuarial applications. This is due to the fact that often in practice, very few observations are available for high ages: it is common to observe death counts only up to an age around 99. Moreover, there are substantial erratic variations at these old ages. For actuarial computations, we need to smooth the mortality pattern and to extrapolate it above age 100. Therefore, yearly life tables have to be closed before projection. Several methods have been proposed by demographers and actuaries, including e.g. LINDBERGSON (2001), RENSHAW & HABERMAN (2003), COALE & GUO (1989) as well as COALE & KISKER (1990). However, no approach provides systematically satisfactory results. A simple and powerful closure procedure based on a constrained log-quadratic regression is proposed in this paper.
A drawback of the projection methods that can be found in the literature is that the projected mortality rates tend to zero as calendar time tends to infinity. This is obviously unrealistic, since we expect that the mortality will approach some asymptotic level as time goes on. This paper shows how limit life tables (e.g., the life tables proposed by demographers) can be incorporated in the mortality projection model to control the long-term behavior of the mortality rates.

1.5 Agenda

The paper is organized as follows. Section 2 introduces some notation and assumptions. The data used to illustrate this paper are presented there. Then

Section 3 describes the classical Lee-Carter approach for mortality projections. It is shown there how the time index involved in this model can be linearized. Mortality at very old ages is discussed in details. Section 4 examines long term consequences of the projection model. In particular, optimal life tables are included in order to control the asymptotic level of mortality rates. The final Section 5 concludes.

2 Notation, assumption and data

2.1 Notation

We analyze the changes in mortality as a function of both age x and time t. Henceforth,

- $T_x(t)$ is the remaining lifetime of an individual aged x on January the first of year t; this individual will die at age $x + T_x(t)$ in year $t + T_x(t)$.
- $q_x(t)$ is the probability that an x-aged individual in calendar year t dies before reaching age $x + 1$, i.e. $q_x(t) = \Pr[T_x(t) \leq 1]$.
- $p_x(t) = 1 - q_x(t)$ is the probability that an x-aged individual in calendar year t reaches age $x + 1$, i.e. $p_x(t) = \Pr[T_x(t) > 1]$.
- $\mu_x(t)$ is the mortality force at age x during calendar year t, i.e. the risk that an individual aged x alive at time t dies instantaneously.

A projected life table is represented as a two-dimensional array containing the $q_x(t)$'s. Rows are usually indexed by age x and columns correspond to calendar years t in the spirit of Lexis diagrams. For pricing and reserving, actuaries will use diagonals of these tables, following cohorts of policyholders across calendar time (exactly as in a Lexis diagram). For instance, the net single premium of an annuity sold to an x-aged individual in year t is given by

$$a_x(t) = \sum_{k \geq 0} \left\{ \prod_{j=0}^{k} p_{x+j}(t+j) \right\} v^{k+1}$$

where v is the yearly discount factor. To compute $a_x(t)$, all the elements of the diagonal starting at cell (x, t) corresponding to age x and year t are used to produce the one-year survival probabilities $p_x(t) = 1 - q_x(t)$, $p_{x+1}(t+1) = 1 - q_{x+1}(t+1)$, $p_{x+2}(t+2) = 1 - q_{x+2}(t+2)$, etc. To facilitate the computations of premiums and reserves, it is therefore interesting to have simple analytical expressions for the $q_x(t)$'s, like those proposed in this paper.

2.2 *Assumption*

In this paper, we assume that the age-specific mortality rates are constant within bands of age and time, but allowed to vary from one band to the next. Specifically, given any integer age x and calendar year t, it is supposed that

$$\mu_{x+\xi}(t+\tau) = \mu_x(t) \quad \text{for } 0 \le \xi, \tau < 1 . \tag{2.1}$$

Under (2.1), we have for integer age x and calendar year t that

$$p_x(t) = \exp(-\mu_x(t)) . \tag{2.2}$$

The nature of (2.1) is best illustrated with the aid of a Lexis diagram, i.e. of a coordinate system that has calendar time as abscissa and age as coordinate. Both time scales are divided into yearly bands, which partition the Lexis plane into rectangular segments. Model (2.1) assumes that the mortality rate is constant within each rectangle, but allows it to vary between rectangles.

2.3 *Data*

The data used to illustrate this paper relate to the Belgian population, males and females separately. From 1948 to 1993, death probabilities have been computed by the BfP (for "Bureau fédéral du Plan") on the basis of the number of deaths by gender, age and year of birth, as well as the corresponding initial exposure-to-risk (on January the 1st of each year). From 1994 to 2001, death probabilities were computed by INS (for "Institut National de Statistique") and published on a yearly basis.

Only Belgian males are considered in this work. The $\widehat{q_x(t)}$'s are available for $t = 1948, \ldots, 2001$ and

$$x = \begin{cases} 0, \ldots, 100 & \text{for } t = 1948, \ldots, 1993 \\ 0, \ldots, 101 & \text{for } t = 1994, \ldots, 1998 \\ 0, \ldots, 105 & \text{for } t = 1999, \ldots, 2001 . \end{cases}$$

No preliminary smoothing procedure is applied. The data are displayed in Figure 2.1. The conventional way to examine mortality is to plot the logarithm of the mortality rates against age. As calendar time also enters the problem, we plot here the mortality surface $(x, t) \mapsto \widehat{q_x(t)}$. The mortality surface on the log-scale

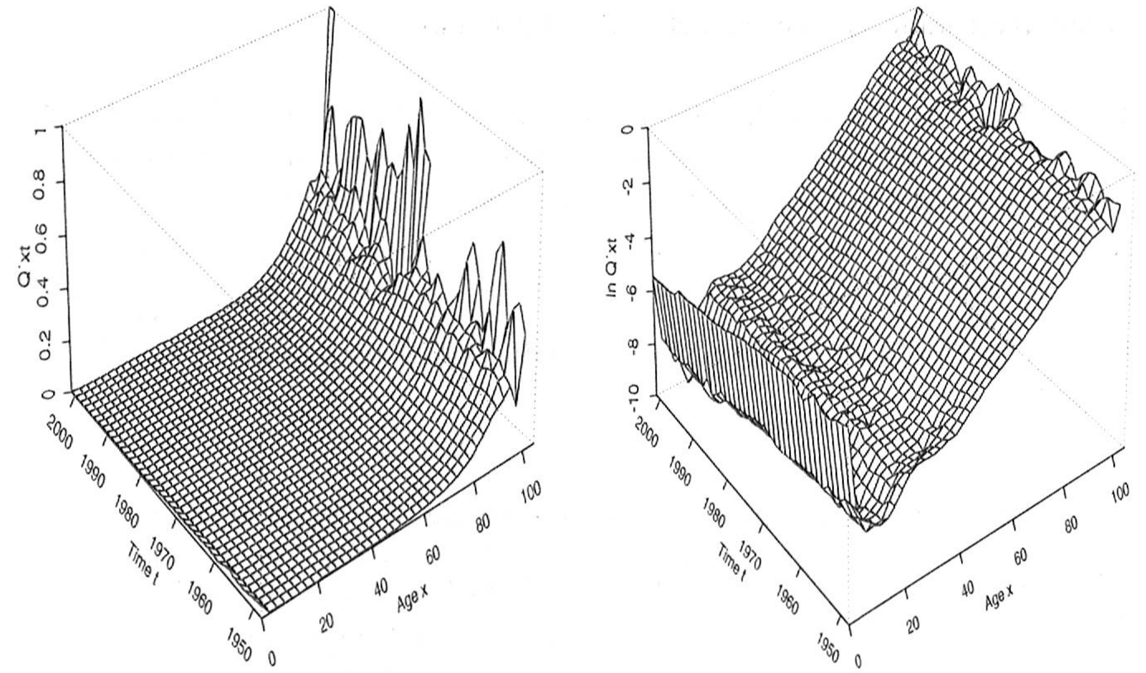

Figure 2.1: Rough death probabilities $\widehat{q_x(t)}$ on the original (left) and log-scale (right).

shows the classical pattern of mortality: severe mortality at young ages, then accident hump and finally the considerable variations at advanced ages discussed in the introduction.

Formula (2.2) allows us to compute the forces of mortality as

$$\widehat{\mu_x(t)} = -\ln\left(1 - \widehat{q_x(t)}\right) \quad \text{provided } \widehat{q_x(t)} < 1\,.$$

The $\widehat{\mu_x(t)}$'s will be the target for modelling.

3 Log-linear model for mortality projection

3.1 Lee-Carter classical methodology

Let us recall the basic features of the classical Lee-Carter approach. The latter is in essence a relational model

$$\ln \widehat{\mu_x}(t) = \alpha_x + \beta_x\kappa_t + \epsilon_x(t) \quad \text{with } \epsilon_x(t) \text{ iid } \mathcal{N}or(0,\sigma^2)\,, \tag{3.1}$$

where the parameters are subject to the constraints

$$\sum_t \kappa_t = 0 \quad \text{and} \quad \sum_x \beta_x = 1 \tag{3.2}$$

ensuring model identification.
The parameters involved in the model (3.1) are estimated using a matrix of age-specific observed forces of mortality with the help of singular value decomposition (SVD) techniques. Specifically, the $\widehat{\alpha}_x$'s, $\widehat{\beta}_x$'s and $\widehat{\kappa}_t$'s are such that they minimize

$$\sum_{x,t} \left(\ln \widehat{\mu_x}(t) - \alpha_x - \beta_x \kappa_t \right)^2 . \tag{3.3}$$

The minimization of (3.3) consists in taking for $\widehat{\alpha_x}$ the row average of the $\ln \widehat{\mu_x}(t)$'s, that is,

$$\widehat{\alpha_x} = \frac{1}{54} \sum_{t=1948}^{2001} \ln \widehat{\mu_x(t)}, \quad x = 0, \ldots, 99 ,$$

and to get the $\widehat{\beta_x}$'s and $\widehat{\kappa_t}$'s from the first term of a SVD of the matrix

$$\mathbf{M} = \begin{pmatrix} \ln \widehat{\mu_0(1948)} - \widehat{\alpha_0} & \cdots & \ln \widehat{\mu_0(2001)} - \widehat{\alpha_0} \\ \vdots & \ddots & \vdots \\ \ln \widehat{\mu_{99}(1948)} - \widehat{\alpha_{99}} & \cdots & \ln \widehat{\mu_{99}(2001)} - \widehat{\alpha_{99}} \end{pmatrix} .$$

This yields a single time-varying index of mortality κ_t. The estimations are displayed in Figures 3.1 and 3.2.
When the model (3.1) is fit by minimizing (3.3), interpretation of the parameters is quite simple:

- the fitted value of α_x exactly equals the average of $\ln \widehat{\mu_x}(t)$ over time t so that $\exp \alpha_x$ is the general shape of the mortality schedule;

- the actual forces of mortality change according to an overall mortality index κ_t modulated by an age response β_x. The shape of the β_x profile tells which rates decline rapidly and which slowly over time in response of change in κ_t.

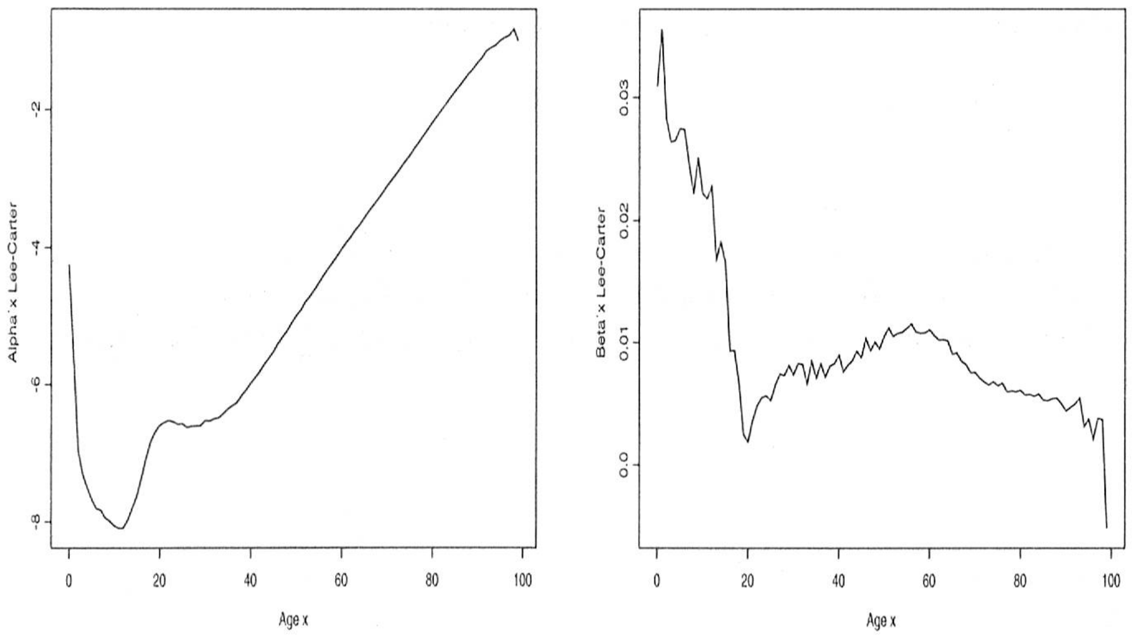

Figure 3.1: Graph of the $\widehat{\alpha_x}'s$ (left) and $\widehat{\beta_x}'s$ (right).

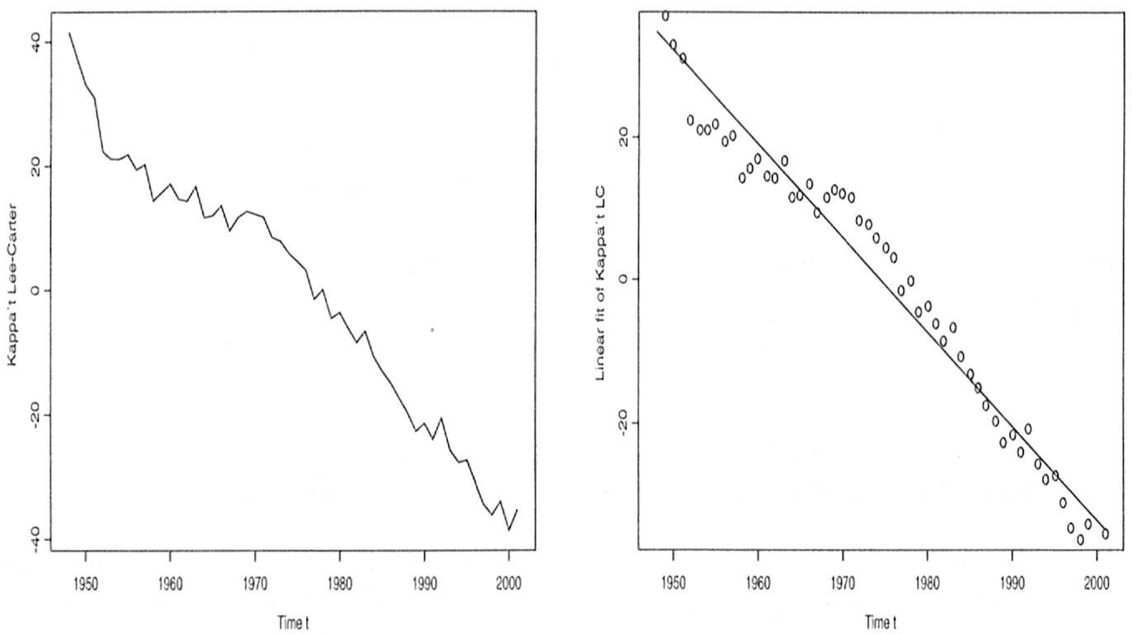

Figure 3.2: Graph of the $\widehat{\kappa_t}'s$ (left) and their linear fit (right).

3.2 *Linearization of the Lee-Carter model*

In the Lee-Carter approach, the time factor κ_t is intrinsically viewed as a stochastic process and Box-Jenkins techniques are then used to estimate and forecast κ_t within an ARIMA time series model. Figure 3.2 shows an approximately linear pattern of the $\widehat{\kappa}_t$'s after 1970. Based on this empirical evidence, we are now going to linearize the time index κ_t.
More specifically, we first determine the calendar year $t^* \leq 1991$ such that the series $\{\widehat{\kappa}_t,\ t = t^*, \ldots, 2001\}$ is best approximated by a straight line. To this end, we aim to maximize the adjustment coefficient R^2 (which is the classical goodness-of-fit criterion in linear regression). Figure 3.3 depicts the value of R^2 according to the number of observations included in the fit. The optimal starting year $t^* = 1970$ is confirmed. The right panel of Figure 3.3 depicts the linear fit to the $\widehat{\kappa}_t$'s for $t \geq 1970$.

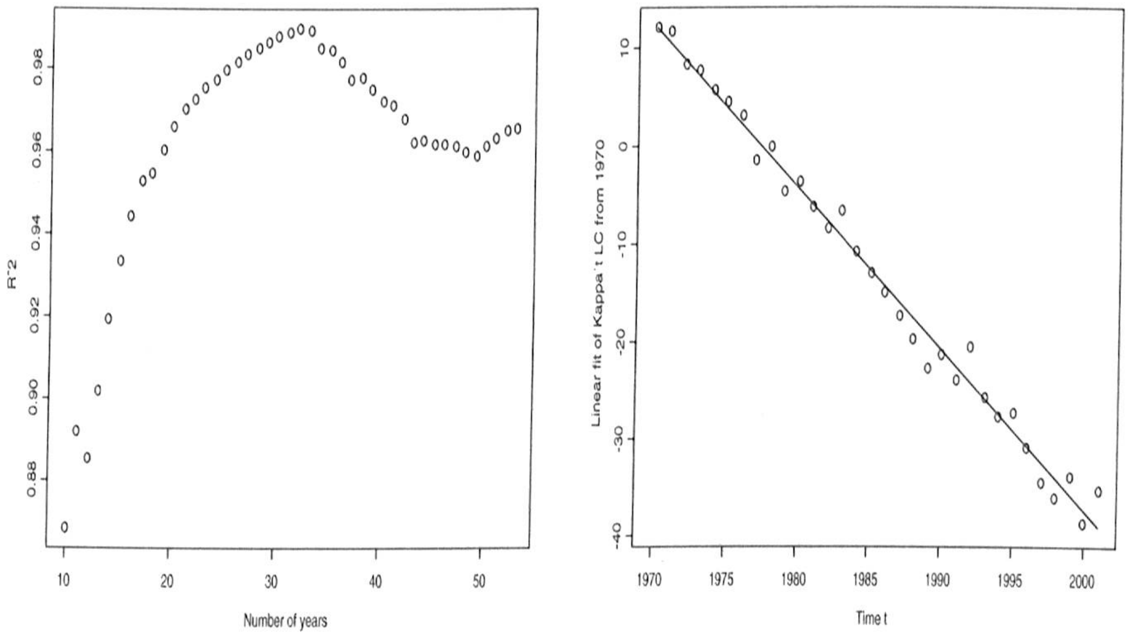

Figure 3.3: Values of R^2 (left) and optimal linear fit of the $\widehat{\kappa}_t{}'s$ (right).

We now consider the model

$$\ln \widehat{\mu_x(t)} = \alpha_x + \beta_x t_c + \epsilon_{xt} \quad \text{with } \epsilon_{xt} \text{ iid } \mathcal{N}or(0, \sigma^2) \tag{3.4}$$

for $t \geq 1970$ where calendar time has been centered

$$t_c = t - \bar{t} \quad \text{with } \bar{t} = 1985.5\,.$$

The OLS estimations $\widehat{\alpha_x}'s$ and $\widehat{\beta_x}'s$ of the parameters involved in (3.4) are displayed in Figure 3.4. The inspection of Figure 3.4 reveal suspect results for ages $x \geq 95$. We will address this problem in the next section.

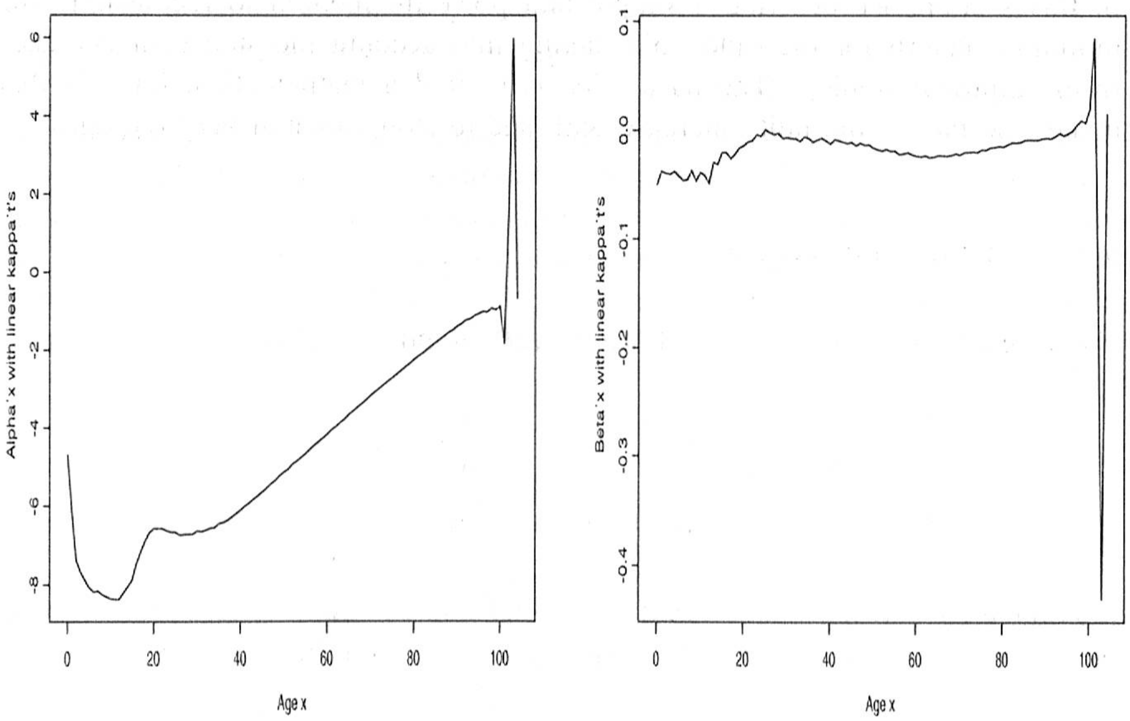

Figure 3.4: Graph of the $\widehat{\alpha_x}'s$ (left) and $\widehat{\beta_x}'s$ (right) involved in the model (3.4).

3.3 Completion of the data

3.3.1 Motivation

Data at old ages produce suspect results (because of small risk exposures): the pattern at old and very old ages is heavily affected by random fluctuations. Recently, some in-depth demographic studies provided a sound knowledge about the slope of the mortality curve at very old ages. It has been documented that the force of mortality is slowly increasing at these ages, approaching a rather flat shape. The deceleration of the rate of mortality increase can be explained by the selective survival of healthier individuals to older ages (see e.g. HORIUCHI & WILMOTH (1998) for more details).

Demographers and actuaries suggested various techniques to complete forces of mortality at old ages. Let us mention the influential works by LINDBERGSON (2001) and COALE ET AL. (1989,1990). We refer the interested reader to BUETTNER (2002) for an interesting discussion.
In this section, we describe a simple and powerful method to reconstitute the mortality pattern for the oldest-old, taking into account the statistical evidence from empirical studies. The basic idea is to find a mathematical formula that makes the rate of mortality increase with age to slow down at very old ages.

3.3.2 Completion procedure

The original data set $\widehat{q_x(t)}$, $t = 1970, \ldots, 2001$ and

$$x = \begin{cases} 0, \ldots, 100 & \text{for } t = 1970, \ldots, 1993 \\ 0, \ldots, 101 & \text{for } t = 1994, \ldots, 1998 \\ 0, \ldots, 105 & \text{for } t = 1999, \ldots, 2001 \,, \end{cases}$$

is completed into $\widehat{q_x(t)}$, $t = 1970, \ldots, 2001$ and $x = 0, \ldots, 125$. The starting point is standard: a constrained log-quadratic regression model of the form

$$\ln \widehat{q_x(t)} = a_t + b_t x + c_t x^2 + \epsilon_{xt} \quad \text{with } \epsilon_{xt} \text{ iid } \mathcal{N}or(0, \sigma^2)$$

is fitted separately to each calendar year $t = 1970, \ldots, 2001$ and to ages 75 and over. Then, we impose:

(i) a closure constraint $q_{130}(t) = 1$ for all t: even if the human life span shows no sign of approaching a fixed limit imposed by biology or other factors (see, e.g., WILMOTH (2000)), it seems reasonable to retain as a working assumption that the limit age 130 will not be exceeded.

(ii) an inflexion constraint $q'_{130}(t) = 0$ for all t that makes the rate of mortality increase with age to slow down at very old ages, as expected.

These two constraints yield the following relation between the a_t's, b_t's and c_t's for each calendar time t:

$$a_t + b_t x + c_t x^2 = c_t (130 - x)^2 .$$

The parameter c_t is then estimated on the basis of the observations $\{\widehat{q_x(t)}$, $x = 75, 76, \ldots\}$ relating to year t. It is worth mentioning that the two requirements underlying the modelling of the $q_x(t)$ for high x are in line with the empirical demographic evidence.

The completed data set is then obtained as follows. We keep the original $\widehat{q_x(t)}$ for $x = 0, \ldots, 85$ and we replace the death probabilities for older ages with the fitted values coming from the constrained quadratic regression. The results for calendar years 1975 and 2000 can be seen in Figure 3.5. This furnishes a rectangular array of data: $\widehat{q_x(t)}$ for $t = 1970, \ldots, 2001$ and $x = 0, \ldots, 130$. The completed data set is displayed in Figure 3.6.

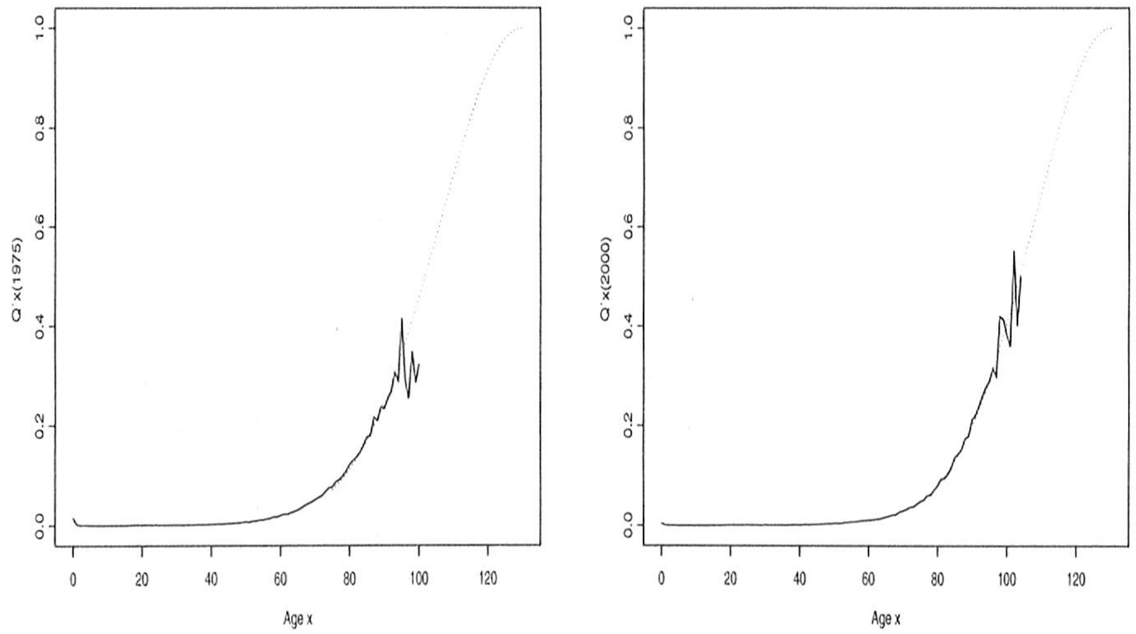

Figure 3.5: Completed $\widehat{q_x(1975)}$'s and $\widehat{q_x(2000)}$'s obtained from the constrained log-quadratic regression model.

Remark Some smoothing may be needed around age 85: in such a case, a simple geometric averaging

$$\widehat{q_x(t)}^{\text{smooth}} = \left(\widehat{q_{x-2}(t)}\widehat{q_{x-1}(t)}\widehat{q_x(t)}\widehat{q_{x+1}(t)}\widehat{q_{x+2}(t)}\right)^{1/5}$$

for $x = 81, \ldots, 90$ should be enough. Here, no smoothing is applied around age 85.

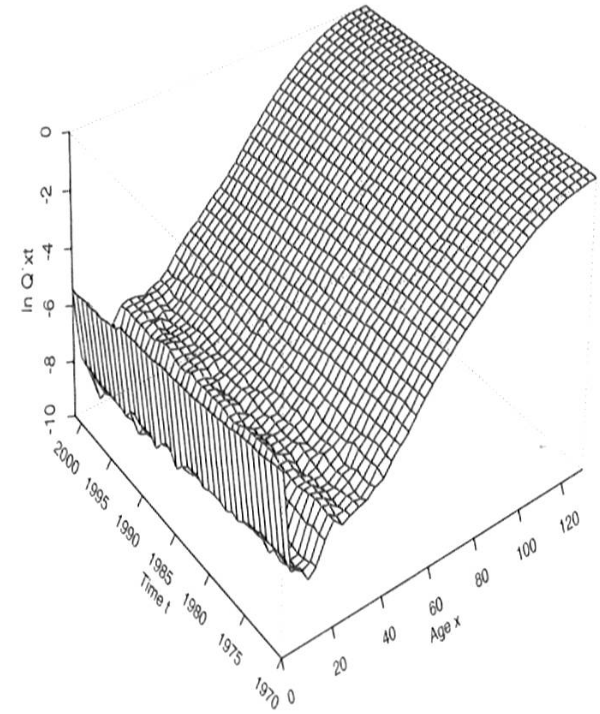

Figure 3.6: Completed $\widehat{q_x(t)}$'s (log-scale).

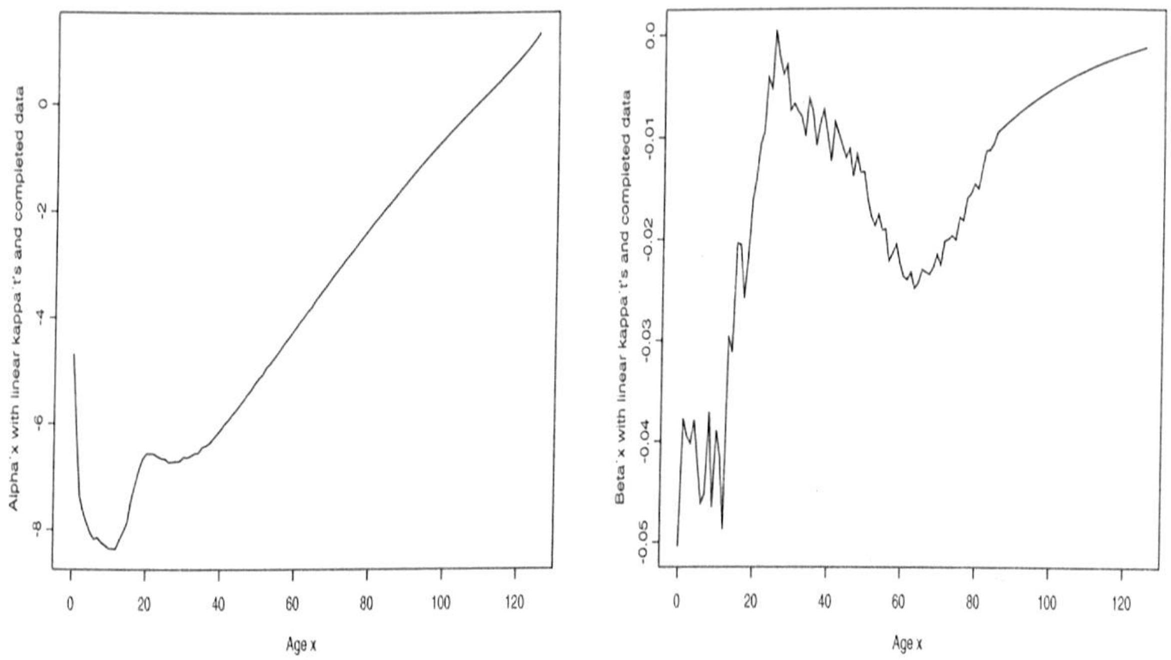

Figure 3.7: Graph of the $\widehat{\alpha_x}'s$ (left) and $\widehat{\beta_x}'s$ (right) involved in the model (3.4) obtained from completed data.

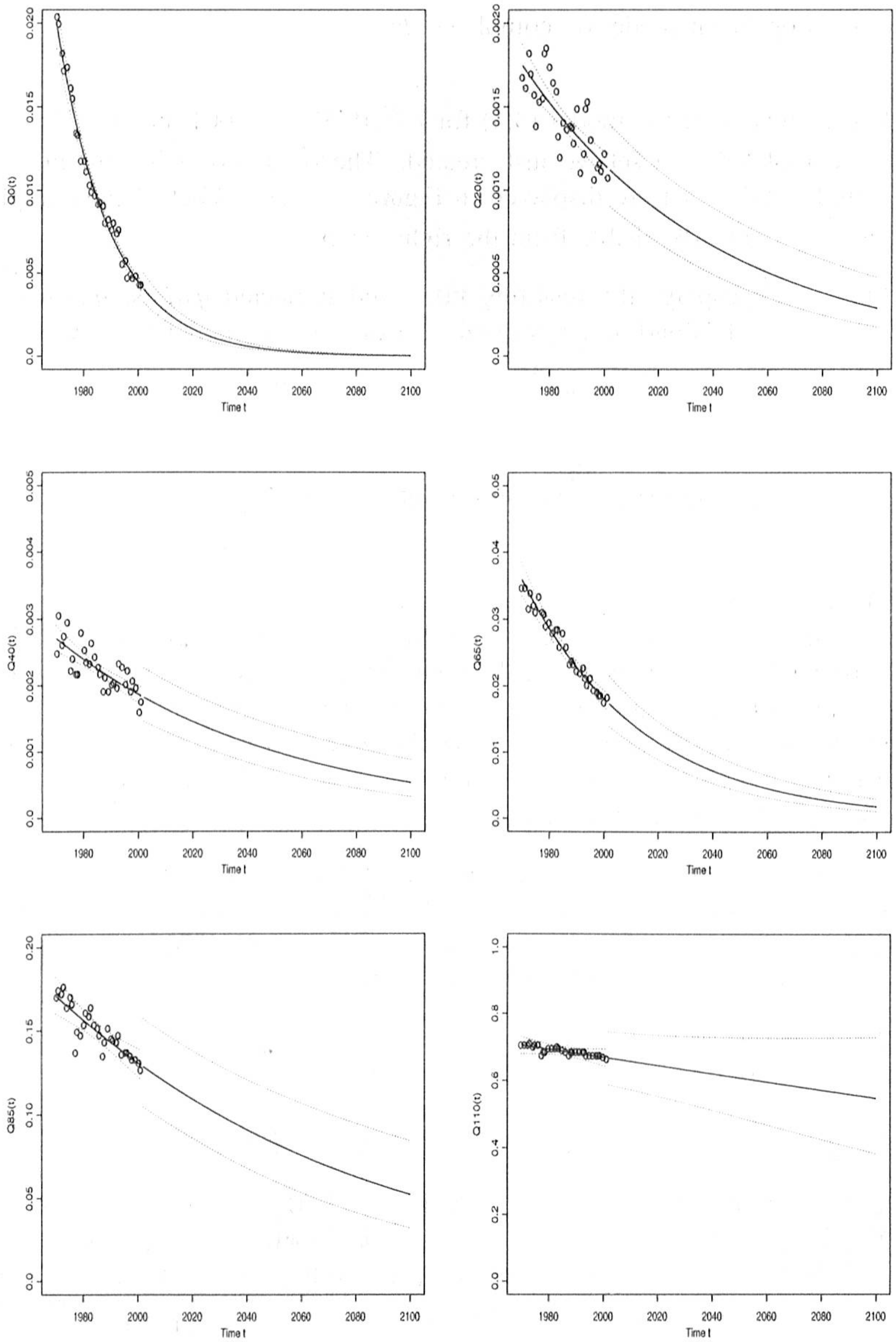

Figure 3.8: Fitted and projected $q_0(t)$'s, $q_{20}(t)$'s, $q_{40}(t)$'s, $q_{65}(t)$'s, $q_{85}(t)$'s and $q_{110}(t)$'s obtained from the the model (3.4) with completed data.

3.3.3 Log-linear model on completed data

Let us now re-fit the model (3.4) for $t = 1970, \ldots, 2001$ and $x = 0, \ldots, 125$ on the completed data set we just created. The $\widehat{\alpha_x}$'s and $\widehat{\beta_x}$'s computed with the completed data set are displayed in Figure 3.7. The effect of the completion on the $\widehat{\beta_x}$'s is clearly visible from the right panel.

Figure 3.8 displays the resulting fitted and projected $q_0(t)$'s, $q_{20}(t)$'s, $q_{40}(t)$'s, $q_{65}(t)$'s, $q_{85}(t)$'s and $q_{110}(t)$'s obtained from the the model (3.4) with completed data.

4 Controlling the asymptotic level of mortality

Since $\widehat{\beta_x} < 0$ for all ages x, we have that $\widehat{\mu_x(t)} \to 0$ for all ages as $t \to +\infty$ yielding infinite expected remaining lifetimes at every age. This is clearly not reasonable. In order to avoid this deficiency, we could resort to the concept of "optimal life table" used by demographers. The age pattern of mortality reflected by the optimal life table is meant as the limit to which mortality improvements can lead. Let q_x^∞ denote the limit death probability at age x, and μ_x^∞ the corresponding force of mortality.

For long-term predictions, it appears reasonable to control the asymptotic level of mortality using the μ_x^∞ proposed by DUCHÊNE & WUNSCH (1988). Figure 4.1 displays the limit q_x^∞, together with the $q_x(2050)$'s and $q_x(2100)$'s obtained before. The limit life table seems to be attained around 2100.

The model is now

$$\ln\left(\widehat{\mu_x(t)} - \mu_x^\infty\right) = \alpha_x + \beta_x t_c + \epsilon_{xt} \quad \text{with } \epsilon_{xt} \text{ iid } \mathcal{N}or(0, \sigma^2) \tag{4.1}$$

and it is fitted to completed data with $t = 1970, \ldots, 2001$ and $x = 0, \ldots, 125$. Note that in DUCHÊNE & WUNSCH (1988) optimal life tables, $\mu_x^\infty = 0$ for $x \leq 34$ and $\mu_x^\infty \approx 0$ for $x \leq 70$. The resulting α_x and β_x are displayed in Figure 4.2. From age 85, we observe the impact of the limit life table.

Fitted and projected $q_0(t)$'s, $q_{20}(t)$'s, $q_{40}(t)$'s, $q_{65}(t)$'s, $q_{85}(t)$'s, $q_{110}(t)$'s obtained from the the model (3.4) with completed data and limit life table are diplayed in Figure 4.3.

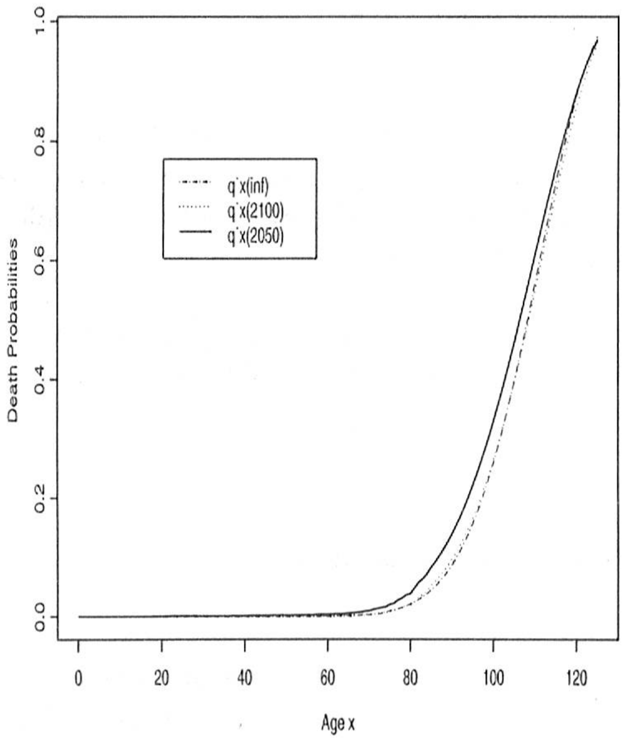

Figure 4.1: DUCHÊNE & WUNSCH (1988)'s limit q_x^{∞} compared with projected $q_x(2050)$ and $q_x(2100)$.

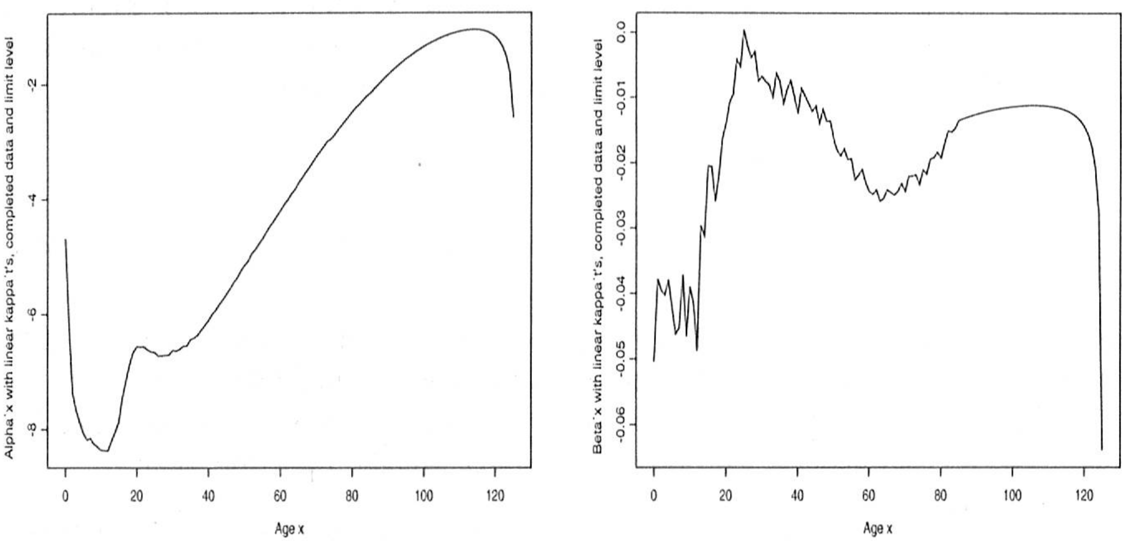

Figure 4.2: Graph of the $\widehat{\alpha_x}'s$ (left) and $\widehat{\beta_x}'s$ (right) involved in the model (4.1).

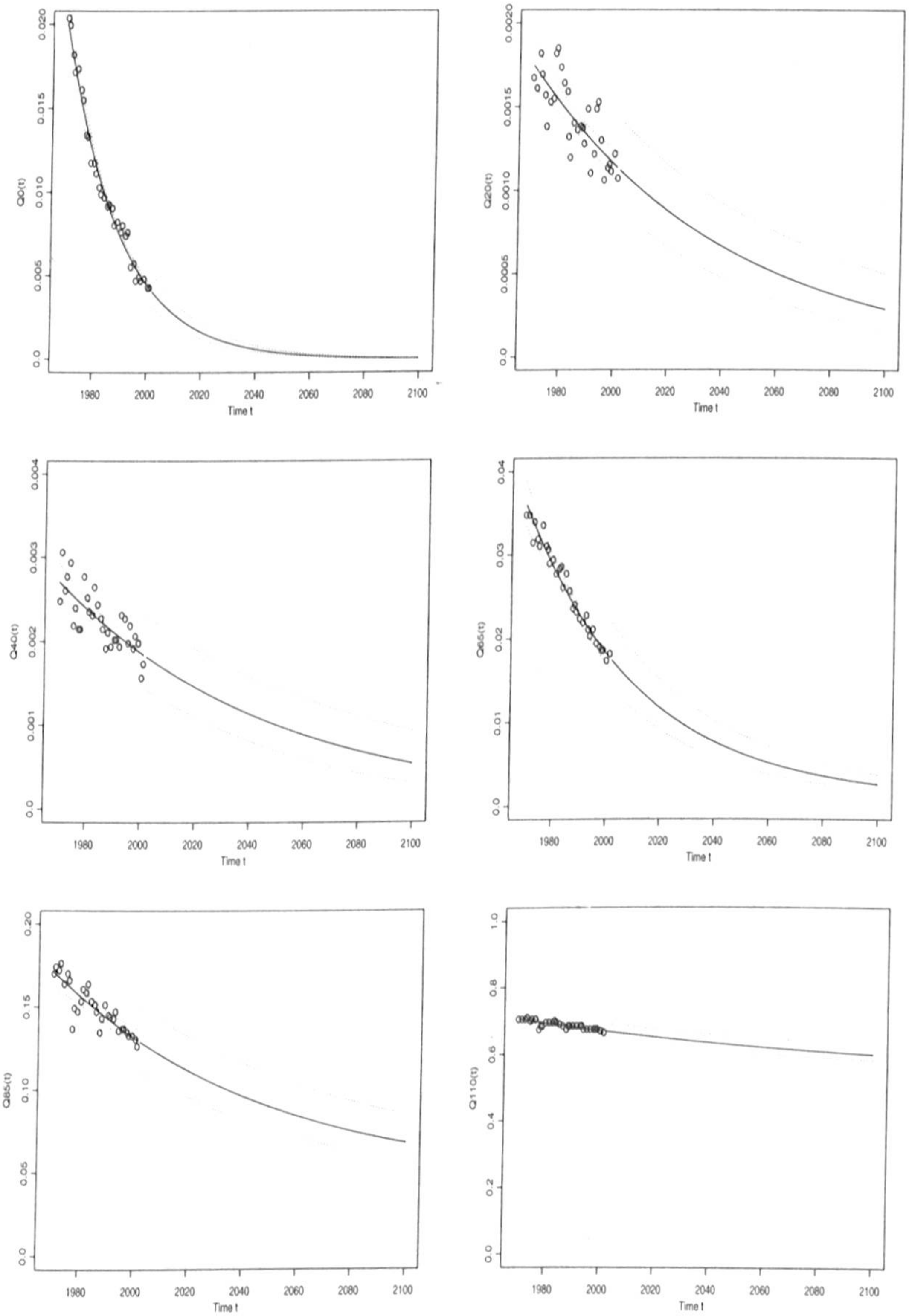

Figure 4.3: Fitted and projected $q_0(t)$'s, $q_{20}(t)$'s, $q_{40}(t)$'s, $q_{65}(t)$'s, $q_{85}(t)$'s, $q_{110}(t)$'s obtained from the the model (3.4) with completed data and limit life table.

5 Conclusion

Longevity has improved consistently and significantly over the last century. This trend appears to be continuing from now, and its seems reasonable to build life tables incorporating further improvements into the future.
As a consequence, annuities will become more and more expensive, especially with continually improving mortality in conjunction with low inflation environment and low interest rates. Despite this, mortality is too often considered as being of secondary importance to financial factors. This is a dangerous view to take, and it is nowadays important that mortality rates are given appropriate considerations in assessing long-term life benefits to the elderly.
In light of baby boom cohorts near retirement, of possible reforms of public pension regimes and the shift from defined benefit to defined contribution private pension plans, an increased interest in individual annuity products can be expected in the future. Several European governments also envisage to shift (at least partially) from a pure pay-as-you-go to funding methods for public pensions. This evolutions poses numerous challenges to private insurance companies. Managing longevity risk is one of them.
The approach suggested in this paper describes how the Lee-Carter model can be "linearized". It is then easily implemented for pricing and reserving in life insurance. Reference life tables controlling the asymptotic level of mortality can also be taken into account, and a simple and powerful method to close the life table has been proposed.

Acknowledgements

The experts of the Belgian Bureau fédéral du Plan and Institut National de Statistique, both based in Brussels, are gratefully acknowledged for having provided the mortality statistics as well as for the stimulating discussions during the meetings of the Working Group "Projected life tables for Belgium" (especially, Micheline Lambrecht and Leïla Bellamammer).
The authors thank the support of Belgian Government under the "Projet d'Action de Recherches Concertées" 04/09-320.

References

BROUHNS, N., DENUIT, M., & VERMUNT, J.K. (2002a). A Poisson log-bilinear approach to the construction of projected life tables. Insurance : Mathematics & Economics 31, 373–393.

BROUHNS, N., DENUIT, M., & VERMUNT, J.K. (2002b). Measuring the longevity risk in mortality projections. Bulletin of the Swiss Association of Actuaries, 105–130.

BUETTNER, T. (2002). Approaches and experiences in projecting mortality patterns for the oldest-old. North American Actuarial Journal 6, 14–25.

COALE, A., & GUO, G. (1989). Revised regional model life tables at very low levels of mortality. Population Index 55, 613–643.

COALE, A., & KISKER, E.E. (1990). Defects in data on old age mortality in the United States: New procedures for calculating approximately accurate mortality schedules and life tables at the highest ages. Asian and Pacific Population Forum 4, 1–31.

DENUIT, M., DEVOLDER, P., & GODERNIAUX, A.-C. (2004). Securitization of longevity risk: Pricing survival bonds with Wang transform in the Lee-Carter framework. Working Paper 04-08, Institut des Sciences Actuarielles, Université Catholique de Louvain, Louvain-la-Neuve, Belgium.

DUCHÊNE, J. & WUNSCH, G. (1988). From the demographer's cauldron: single decrement life tables and the span of life. Genus 3, 1–17.

FELIPE, A., GUILLÉN, M. & PEREZ-MARIN, A.M. (2002). Recent mortality trends in the Spanish population. British Actuarial Journal 8, 757–786.

HORIUCHI, S., & WILMOTH, J. (1998). Deceleration in the age pattern of mortality at older ages. Demography 35, 391–412.

LEE, R.D. (2000). The Lee-Carter method of forecasting mortality, with various extensions and applications. North American Actuarial Journal 4, 80–93.

LEE, R.D., & CARTER, L. (1992). Modelling and forecasting the time series of US mortality. Journal of the American Statistical Association 87, 659–671.

LINDBERGSON, M. (2001). Mortality among the elderly in Sweden 1988-97. Scandinavian Actuarial Journal, 79–94.

MCDONALD, A.S., CAIRNS, A.J.C., GWILT, P.L., & MILLER, K.A. (1998). An international comparison of recent trends in mortality. British Actuarial Journal 4, 3–141.

PITACCO, E. (2004). Survival models in a dynamic context: a survey. Insurance: Mathematics & Economics 35, 279–298.

RENSHAW, A., & HABERMAN, S. (2003). Lee-Carter mortality forecasting: a parallel generalized linear modelling approach for England and Wales mortality projections. Applied Statistics 52, 119–137.

SITHOLE, T.Z., HABERMAN, S., & VERRALL, R.J. (2000). An investigation into parametric models for mortality projections, with applications to immediate annuitants and life office pensioners' data. Insurance: Mathematics & Economics 27, 285–312.

TABEAU, E., VAN DEN BERGH JETHS, A., & HEATHCOTE, C. (eds) (2001). Forecasting Mortality in Developed Countries. Kluwer Academic Publishers.
WILMOTH, J.R. (2000). Demography of longevity: past, present and future trends. Experimental Gerontology 35, 1111–1129.

Michel Denuit
Institut des Sciences Actuarielles &
Institut de Statistique
Université Catholique de Louvain
Louvain-la-Neuve
Belgium

Anne-Cécile Goderniaux
ISI Pierrard
Haute École Blaise Pascal
Virton
Belgium

Abstract

This paper proposes a simple and powerful method for generating projected life tables in a dynamic mortality environment. Yearly death probabilities are first extended to old ages, using a constrained log-linear regression model involving age and its squared value. Assuming a further continuation of the stable pace of mortality decline, a linear version of the classical Lee-Carter model is then applied to the forecasting of the gender- and age-specific mortality rates applying to the Belgian population.

Résumé

Cet article propose une méthode à la fois simple et efficace de construction de tables de mortalité prospectives lorsque la mortalité évolue. Les probabilités annuelles de décès sont tout d'abord extrapolées aux âges élevés, à l'aide d'un modèle de régression log-linéaire contraint, faisant intervenir l'âge et son carré. En supposant que l'évolution passée de la mortalité se poursuivra dans le futur, une variante linéaire du modèle de Lee-Carter est ensuite appliquée à la prévision des taux de mortalité par âge et par sexe pour la population belge.

Zusammenfassung

Im Artikel wird eine einfache und leistungsfähige Methode zur Bestimmung künftiger Sterbetafeln unter sich verändernden Sterblichkeiten vorgestellt. Die jährlichen Sterbewahrscheinlichkeiten werden zuerst auf hohe Alter extrapoliert. Dies geschieht mit Hilfe eines Constrained Log-Linear-Regression-Modells, unter Verwendung vom Alter und dessen Quadrat. Unter der Annahme, dass sich die Sterblichkeit im selben Mass wie bisher vermindert, wird eine lineare Version des klassischen Lee-Carter Modells verwendet, um geschlechts- und altersabhängige Sterblichkeitsraten für die belgische Bevölkerung zu prognostizieren.