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C. Literatur-Rundschau

Divers classiques des sciences actuarielles ont été récemment réimprimés ou réédités. Il s'agit entre autres de

Hans Bühlmann, *Mathematical Methods in Risk Theory,* Grundlehren der Mathematischen Wissenschaften 172, ISBN 3-540-61703-5, Springer Verlag, Fr. 79.50. Réimpression de l'ouvrage paru en 1970.

Erwin Straub, *Non-Life Insurance Mathematics,* ISBN 3-540-18787-1, Springer Verlag, Fr. 95.50. Réimpression de l'ouvrage paru en 1988.

Hans U. Gerber, *An introduction to Mathematical Risk Theory,* S. S. Huebner Foundation, $ 18.95. Réimpression de l'ouvrage paru en 1979.

Hans U. Gerber, *Life Insurance Mathematics,* Springer Verlag, Fr. 69.–. Troisième édition de l'ouvrage paru en 1990.

Newton L. Bowers, Hans U. Gerber, James C. Hickman, Donald A. Jones, Cecil J. Nesbitt, *Actuarial Mathematics,* The Society of Actuaries, Deuxième édition de l'ouvrage paru en 1986.

M.-T. Kohler

D. R. Dannenburg, R. Kaas, M. J. Goovaerts. *Practical actuarial credibility models.* Institute of Actuarial Science and Econometrics, University of Amsterdam, 1996, 157+xi pp.

The present book gives an introduction to credibility theory primarily intended for preparing university actuarial exams.
In Chapter 1 the authors give a motivating introduction based on the Bühlmann model. Chapter 2 is devoted to the Bühlmann-Straub model, which is extended to a hierarchical model in Chapter 3. A more general treatment of credibility estimators is given in Chapter 4. Chapter 5 discusses credibility estimators and Bayes estimators. This discussion is performed mainly within a frequentist framework; the subjectivist approach is only briefly mentioned. The remaining chapters are devoted to specific models; Chapter 6 treats two-way crossed classification, Chapter 7 IBNR problems, and Chapter 8 regression.

As seen from the previous paragraph, before discussing credibility theory in general, the readers are motivated through some specific models. After the general discussion in Chapters 4 and 5 one returns to specific models as special cases of the general theory. Although this approach will inevitably lead to some repetition, it is pedagogically reasonable. It would be too hard on the students to throw them into the general form of normal equations etc. without preparation. However, it is a question whether it is necessary to present as much as three specific models before discussing credibility models in general. The reviewer believes that an extended presentation of the Bühlmann model would suffice.

The reviewer finds it reasonable to include the subjects covered in the book in an introductory course on credibility theory. The mathematical level is not too demanding, and the presentation is clear. Often the authors give thorough and clarifying motivations before proving a result. In particular for formulae and mathematical assumptions the motivations are good. However, perhaps more emphasis could have been given to the modeling aspect. The book gives numerical examples for most of the major models presented. However, to exaggerate a bit, one has the feeling that the attitude is that here we have a dataset; now let us see what figures we obtain when applying our present model to these data. The reviewer would have liked a more thorough discussion on the appropriateness of that particular model in that particular situation with emphasis on the fact that describing reality with a mathematical model always has to be a compromise between realism and mathematical convenience. What is realistic; where do we sacrifice realism for mathematical convenience? On p. 104 it is said in connection with a numerical example that "the weights w_{ijt} are all equal to one, ..." The reviewer would have preferred if it had been said that they *were assumed to be equal* to one. Model assumptions very rarely, if ever, hold, and in particular in a book intended to educate students, one should make the students conscious that assumptions are just assumptions.

As stated above, the authors mention only briefly the subjectivist Bayesian approach. The reviewer would have preferred a more extensive treatment of this topic. In some lines of insurance one often has so much data that one can afford to be a frequentist. However, in other situations one has to set a premium with scarce objective data. Here one has to apply subjective judgement, and the subjectivist Bayesian approach enables us to formalise this judgement mathematically. In this framework, credibility theory is appealing as we obtain an estimator in the shape of a weighted average of objective observations and subjective judgement. It was within this framework Jewell introduced hierarchical credibility models. It

may therefore seem a bit confusing when the authors name a hierarchical model in a frequentist framework Jewell's hierarchical model.

As the authors have presented both homogeneous and inhomogeneous credibility estimators in the Bühlmann-Straub model and then introduce a hierarchical extension of that model, it would have been interesting to point out that if they in the hierarchical model let the between-sector variance approach zero, then they obtain the inhomogeneous Bühlmann-Straub estimator, whereas they obtain the homogeneous estimator by letting that variance approach infinity.

On p. 19 the authors say in connection with the Bühlmann-Straub model that homogeneous credibility estimators are more suitable for practical use than inhomogeneous estimators as they contain a built-in estimator of the over-all mean. The reviewer would rather make the weaker statement that the homogeneous estimator is of interest as it motivates an estimator for the over-all mean. Let us make a parallel with life assurance based on Makeham's mortality law. This law contains three parameters. When setting premiums for a portfolio, you do not base your premiums on estimates of the Makeham parameters from your present portfolio; you use estimates found earlier from other populations. The reviewer finds that this is also a natural approach in credibility theory. One should see the credibility estimators (possibly containing unknown parameters) and parameter estimation as two separate issues.

The authors define the homogeneous credibility estimator as the best linear unbiased estimator. Unfortunately they do not discuss further what they mean by unbiasedness in this connection. Should it be for one specific value of the parameters or for a larger, specified set of parameter values? This issue creates no problem in models like the Bühlmann-Straub model where all the observations have the same mean as the estimand. Here the unbiasedness constraint is in both cases that the coefficients of the credibility estimator sum to one. However, it becomes more controversial in the general set-up of Chapter 4. Here the authors implicitly assume that the unbiasedness should be for one particular value of the parameters, and then the credibility estimator can depend on the means of the observations and the estimand. But then the above mentioned argument for using homogeneous estimators is no longer valid; we still have all the parameters of the inhomogeneous estimator, and in addition, we have obtained a more complicated estimator that demands more data and has a larger expected quadratic loss. In the referee's opinion, homogeneous credibility estimators make sense only in situations where the parameters of the unconditional means are allowed to vary in such a way that there exists a linear combination of the observations whose mean is equal to the mean of the estimand for all values of the parameters.

In the foreword the authors say, "In the authors' opinion, credibility is currently taught in a needlessly complicated way". They claim that the usual use of a random θ related to the risk is difficult to understand, and that "it is much easier to formulate the credibility models by decomposing the risk variables additively into independent variance components". This means that one has to impose more restrictive assumptions, but it is argued that no generality is lost as the credibility estimators are determined by only first and second order moments. The reviewer has never understood that the assumption of a random θ should be so complicated that it ought to be replaced with something else, and he finds the variance component models more difficult to understand.

To illustrate, let us consider the simple Bühlmann model. Let X_j denote the total claim amount from a motor insurance policy in the jth year it is in force. As there are individual differences between the policies, it is assumed that the characteristics that are not captured through the rating structure, are represented by an unknown, random risk parameter θ. We assume that the X_j's are conditionally independent and identically distributed given θ.

In the book X_j is expressed as

$$X_j = m + \Xi + \Xi_j,$$

where in the traditional formulation

$$m = \mathrm{E}X_1 \qquad \Xi = \mathrm{E}[X_1|\theta] - m \qquad \Xi_j = X_j - \Xi,$$

and the additional assumption that the Ξ_j's are mutually independent and identically distributed and independent of Ξ is imposed. The reviewer finds this formulation more complicated to understand. Furthermore, although one could argue that for the deduction of credibility estimators the additional assumption imposes no loss of generality, the fact that this assumption is made, leads to the question of its realism. One consequence of the assumption is that the conditional variance of X_j given θ is independent of θ. In some cases this may be realistic, but not always. Let us for instance assume that mileage is a major contributor to the characteristics captured in θ. Then it is natural to assume that the claim amounts of a policy with a high mileage would have not only a large conditional mean, but also a large conditional variance. In the traditional formulation of the model, such questions are not relevant.

The reviewer agrees that in some cases the variance component formulation of credibility models leads to simplifications in deductions. In such cases he would have formulated the model in the traditional way, but for the deductions used the variance component formulation, arguing that the additional assumptions im-

pose no loss of generality as the structure of the first and second order unconditional moments is still the same.

In addition to the variance component model, the authors advocate use of linearity and iterativity. These three tools are most successfully applied in the treatment of the two-way crossed classification model, which is what the reviewer liked best in the book. On the other hand, he was disappointed by the treatment of the Hachemeister model. Here the authors restrict to the case with two regressors. Both the formulation of the model and the deductions would have been much simpler under more general assumptions when using traditional methods. One gets the uncomfortable feeling that it has been more important for the authors to show that their tools are applicable also in the regression model, than to obtain simplicity. A striking indication of this is when they on p. 127 argue that they will not deduce homogeneous credibility estimators in the regression model as that is not possible with their techniques. It is not too complicated with traditional techniques.

On p. 2 it is said that the greatest accuracy credibility theory was developed by Bühlmann in the 1960's. As this may give the impression that the theory is much younger than it really is, the reviewer finds it appropriate to point out that the paper by Whitney (1918) is usually considered as the first paper on greatest accuracy credibility theory, and that the terms "greatest accuracy credibility" and "limited fluctuation credibility" were applied by Bailey (1943).

The reviewer has profited from reading the book. He has learnt from the things that he liked, and also from the things that he did not like, as they forced him to think through them. There is no doubt that he would be influenced by the book if he were to give a course on credibility theory. However, he would not use it as textbook for the course as there are too many controversial issues. He strongly recommends the book to all teachers of credibility theory, but would be more reluctant to recommend it to their students.

References

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Whitney, A. W. (1918). The theory of experience rating. *Proceedings of the Casualty Actuarial Society* IV, 274–292.

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