D. Kurzmitteilungen

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Barwert einer asymptotischen Rente

1. Einleitung

In der klassischen finanzmathematischen Literatur (vgl. [1] oder [2]) werden verschiedene Rentenarten behandelt. Renten mit analytischen Barwertformeln sind z.B. die konstante Rente, arithmetisch fallende oder arithmetisch wachsende Renten sowie die geometrisch wachsende (bzw. fallende) Rente.
Meines Wissens wurde hingegen bisher keine asymptotisch wachsende Rente betrachtet, obwohl eine solche Eigenschaften aufweist, welche in gewissen Fällen gefragt sein könnten. Zwei Stichworte hierzu mögen genügen. Als erstes sei die *Finanzierung von Projekten* erwähnt. Für ein Projekt wird häufig ein Kredit aufgenommen, welcher im Laufe der Zeit aus dem resultierenden Cashflow des Projektes amortisiert wird. Dieser Cashflow hat oft die Eigenschaft, dass er zu Beginn, wenn die Kosten anfallen, klein ist, dann stark ansteigt und sich schliesslich einer Sättigungsgrenze nähert: er bildet also eine asymptotische Rente. Als zweite, aus der Sicht von Kunden, möglicherweise sinnvolle Anwendung sei eine *Erbrente* erwähnt, die *auf einem Inflationshöhepunkt* zu laufen beginnt. Solange die Inflation noch hoch ist, soll die Rente stark ansteigen. Später soll sie sich einem Grenzcashflow nähern.
Im Prinzip lassen sich asymptotische Renten auf einem PC berechnen. Da der Barwert sich jedoch auf einfachste Weise durch bekannte aktuarielle Grössen ausdrücken lässt, soll die Formel den Lesern als Anregung nicht vorenthalten bleiben.

2. Definition der asymptotischen Rente

Wir beschränken uns auf einen festen Fall, der durch die nachfolgenden Annahmen beschrieben ist. Eine Erweiterung etwa auf logistische Zinssätze (vgl. [1] S. 130) im Sinne von Stoodley ist denkbar.

Annahmen

(1) Vorschüssige jährliche Rentenzahlungen. Die erste Zahlung habe den Wert 1.
(2) Die Rente habe eine Laufzeit von n Jahren (n ganz).
(3) Nach unendlich langer Zeit wäre die Auszahlung auf G angewachsen.
(4) Nach NH Jahren ist sie auf $1 + \frac{G-1}{2}$ angewachsen.
Nach $2 \cdot NH$ Jahren auf $1 + \frac{3}{4}(G-1)$ usw.
(Die Auszahlung nähert sich also asymptotisch dem Wert G, mit einer Halbwertszeit von NH).
(5) Für die Berechnung des Barwertes werde die geometrische Verzinsung mit flacher Zinskurve vorausgesetzt.

Berechnung des Barwertes

Bezeichnungen

G	=	asymptotisches Grenzniveau für die Rentenzahlung.
NH	=	Halbwertszeit für Anstieg der Rentenzahlung.
p	=	$\sqrt[NH]{2}$
x	=	$(G-1)(p-1)$
i	=	Zinssatz für geometrische Diskontierung.
v	=	$\frac{1}{1+i}$ (Abzinsungsfaktor).
$\ddot{a}_{\overline{n}\rceil}$	=	Barwert der n Jahre laufenden, vorschüssig zahlbaren Einheitsrente. Falls erforderlich, wird der Zinssatz beigefügt, um Verwechslungen vorzubeugen.

Auszahlungen der Rente

$$1, \quad 1+\frac{x}{p}, \quad 1+\frac{x}{p}+\frac{x}{p^2}, \quad 1+\frac{x}{p}+\frac{x}{p^2}+\frac{x}{p^3}, \quad \cdots$$

Man verifiziert sofort, dass die Auszahlungen die Bedingungen für die Halbwertszeit erfüllen.

Barwert

$$BW = 1 + v\left(1 + \frac{x}{p}\right) + \cdots + v^{n-1}\left(1 + \frac{x}{p} + \cdots + \frac{x}{p^{n-1}}\right)$$

$$= \ddot{a}_{\overline{n}|} + \frac{x}{p-1}\left\{v\frac{p-1}{p} + \cdots + v^{n-1}\frac{p^{n-1}-1}{p^{n-1}}\right\}$$

$$= \ddot{a}_{\overline{n}|} + \frac{x}{p-1}\left\{v + v^2 + \cdots + v^{n-1} - \frac{v}{p} - \left(\frac{v}{p}\right)^2 - \cdots - \left(\frac{v}{p}\right)^{n-1}\right\}$$

v/p fassen wir als Abzinsungsfaktor zu einem unbekannten Zinssatz j auf:

$$w = \frac{1}{1+j} = \frac{v}{p} = \frac{1}{p}\frac{1}{1+i}$$

somit gilt

$$1 + j = p(1 + i)$$

oder

$$j = p(1 + i) - 1$$

Mit diesem Hilfszinssatz j und der Tatsache, dass $x/(p-1) = G - 1$ ist, lässt sich der Barwert der vorschüssig zahlbaren asymptotischen Rente schreiben als:

$$BW = \ddot{a}_{\overline{n}|} + (G-1)\{\ddot{a}_{\overline{n}|} - \ddot{a}^{(j)}_{\overline{n}|}\}$$

Der Barwert lässt sich somit auf einfachste Weise durch die klassischen Einheitsrentenbarwerte darstellen.

Beispiel

$$N = 10, \quad NH = 3, \quad G = 2, \quad i = 0,1$$

Daraus erhält man $p = 1,259921$

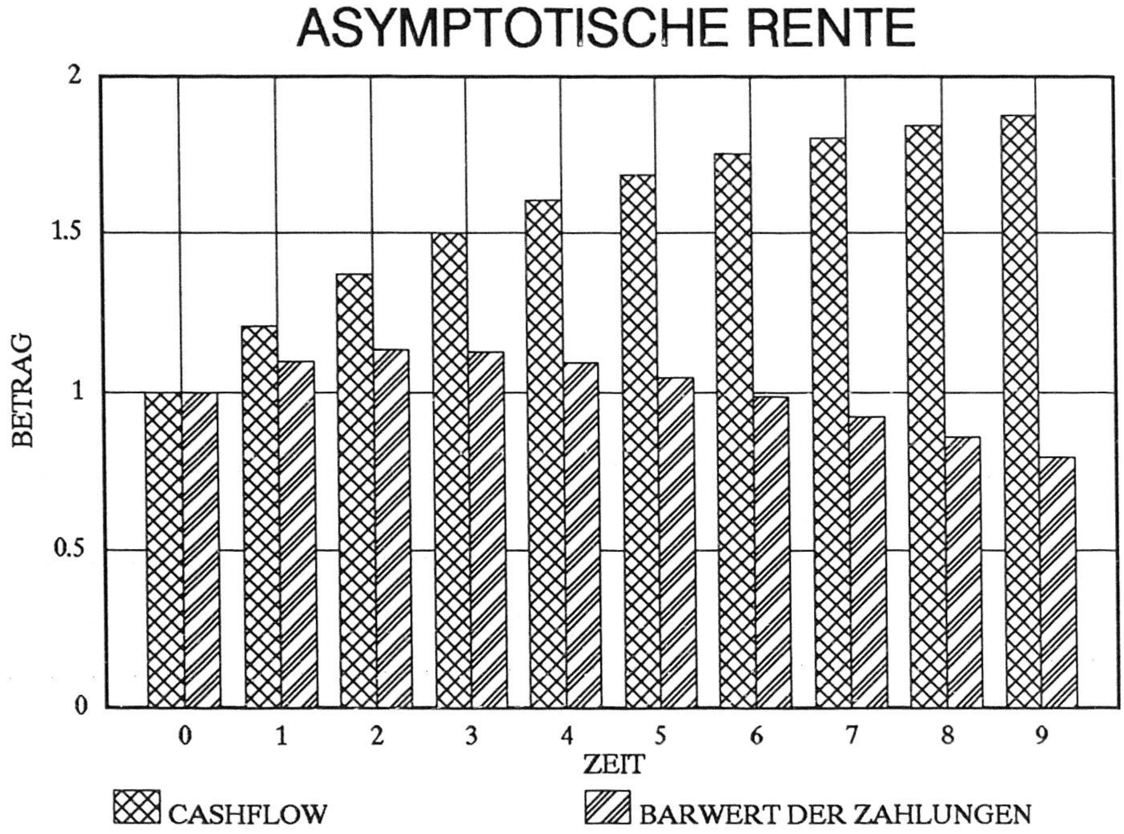
$$j = 0,385913$$

$$\ddot{a}_{\overline{10}|} = 6,759024$$

$$\ddot{a}^{(j)}_{\overline{10}|} = 3,453827$$

Somit ist der Barwert:

$$BW = \underline{10{,}0642}$$



3. Verallgemeinerung

Da die Volkswirtschaften längerfristig stets einer Inflation unterlagen, sollte das Modell derart verallgemeinert werden, dass wir den Cashflow mit den Potenzen eines Wachstumsfaktors $(1+g)$ multiplizieren.
Unser neuer Cashflow lautet daher:

$$1, \qquad (1+g)\left(1+\frac{x}{p}\right), \qquad (1+g)^2\left(1+\frac{x}{p}+\frac{x}{p^2}\right), \quad \cdots$$

Wir berechnen nach dem bei geometrisch wachsenden Renten üblichen Verfahren zwei Hilfszinssätze y und k:

$$y = \frac{1+i}{1+g} - 1$$

$$k = p(1+y) - 1$$

und erhalten für den Barwert dieses verallgemeinerten Cashflows wiederum:

$$BW = \ddot{a}_{\overline{n}|}^{(y)} + (G-1)\left\{\ddot{a}_{\overline{n}|}^{(y)} - \ddot{a}_{\overline{n}|}^{(k)}\right\} \qquad (*)$$

Bemerkungen

1. Die Formel (*) eignet sich als Dividendendiskontierungsmodell für die Bewertung von Aktien. Sie verallgemeinert die Formel von Gordon – Shapiro im Falle $n = \infty$.
2. In der finanztheoretischen Literatur werden häufig zwei verschiedene Typen von Dividendendiskontierungsmodellen verwendet: Für theoretische Untersuchungen wird die Formel von Gordon – Shapiro (Barwert der ewigen, geometrisch wachsenden Rente) benutzt, weil sie sehr einfach ist, während bei praktischen Anwendungen Renten mit stückweise konstanten Wachstumsraten (vgl. [3], S. 416) bevorzugt werden, da diese die Realität besser beschreiben. (*) vereinigt bis zu einem gewissen Grade die Vorteile beider Typen: das Wachstum bleibt stetig, ohne dass der Realitätsbezug allzusehr darunter leidet.
3. Die Verallgemeinerung (*) beschreibt auch ein Marktpotential (Barwert) im Falle einer allmählichen Marktsättigung in einer inflationären Umgebung.

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Literaturverzeichnis

[1] *Mc Cutcheon, J.J./Scott, W.F.:;* An Introduction to the Mathematics of Finance. Heinemann, London.
[2] *Caprano, E./Gierl,A.:* Finanzmathematik. Verlag Vahlen, München.
[3] *Jacob, N.L./Pettit, R.R.:* Investments. Irwin, Homewood, Illinois.

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Rating of a Special Stop Loss Cover

1 Introduction

During the past 35 years the problem of calculating the premium of a stop loss reinsurance cover was one of the most frequently discussed topics of the mathematical risk theory. Already the Swiss *Ammeter* (1955) gave mathematical formulas for rating that treaty. Some years earlier *Vajda* (1951) published an interesting mathematical study, which he continued in the year 1955 (see *Vajda* (1955)). Some years later several authors derived rating methods by applying certain approximation formulas for the distribution function of the total claims amount (see e.g. *Bohman* et al. (1963/64), *Berger* (1972)). They got handy procedures since the stop loss premium can be represented in a simple form as a function of the distribution function of the total claims amount. This relationship was also used lateron for deriving handy, recursive rating methods. The Canadian Panjer published in 1980 such a recursive procedure (see *Panjer* (1980)), that was generalised afterwords by several other researchers (see e.g. *Sundt* et al. (1982), *Willmot* et al. (1987)). Certain modifications were derived e.g. for the special situation of life insurance (see e.g. *Kremer* (1989)). As alternative to these recursive methods one developed a method based on the well known Fast-Fourier method for computing the distribution function of the total claims amount (see e.g. *Bühlmann* (1984), *Hürlimann* (1986)). All these methods have the aim to calculate the stop loss premium as exact as possible. Since two centuries there exists also a different type of procedures. This type of methods has the aim of giving conservative premium estimates, i.e. upper bounds to the unknown, exact premium. Worth mentioning are especially the contributions of *Bowers* (1969), *Bühlmann* et al. (1974), *De Vylder* et al. (1982a/1982b/1983), *Taylor* (1977)). In these publications handy, as well as fairly general premium bounds are derived by applying certain inequalities or results of the optimisation theory. So far, a lot of different results and methods for calculating or estimating the stop loss premiums exist. Many main problems seem to be solved, the different approaches are fairly well developed. Nevertheless for certain more special situations one still can give some new results. Such a more special set-up is investigated in the following notes. An until now unknown premium formula is derived and discussed.

2 The stop loss treaty

Everything is based on the probability space $(\Omega, \triangle, P)$. Consider a collective of insurance risks, producing claims each year. The number of claims per year let be described by the random variable N, the corresponding claims sizes by the random variables

$$X_1, \; X_2, \; X_3, \; \ldots .$$

With this notation the total claims amount of the collective is given by the random variable:

$$S = \sum_{j=1}^{N} X_j,$$

with the corresponding distribution function G defined as

$$G(s) = P(S \leq s).$$

The stop loss treaty with priority L is defined by the claims amount R, taken by the reinsurer:

$$R = \max(S - L, 0).$$

The reinsurer takes that part of the total claims amount S, that exceeds the priority L. The *net premium* v of that treaty is defined as the expectation of R, i.e.

$$v = E(R).$$

This quantity can in general be calculated with the formula:

$$v = \int_{[L,\infty)} (s - L) G(ds). \tag{2.1}$$

The *risk premium* is defined as the net premium plus a security loading σ. The *security loading* σ can be calculated according to the standard deviation principle (see e.g. *Reich* (1985)), i.e.:

$$\sigma = \lambda \cdot (\mathrm{Var}(R))^{1/2},$$

where λ is a given coefficient and $\mathrm{Var}(R)$ the variance of the random variable R. Having given the coefficient λ the calculation of the security loading reduces to the calculation of the variance $\mathrm{Var}(R)$. One has the formula:

$$\left(\frac{\sigma}{\lambda}\right)^2 = \int_{[L,\infty)} (s-L)^2 G(ds) - v^2. \tag{2.2}$$

A short and embracing introduction into the mathematics of the stop loss net premiums can be found e.g. in the new booklet *Kremer* (1988).

3 The special set-up

More special assumptions are taken in the present investigation. Suppose that:

(a) The claims number distribution satisfies with certain constants $a > 0$, $b < 1$ the linear recursion

$$(k+1) \cdot P(N = k+1) = (a + b \cdot k) \cdot P(N = k) \tag{3.1}$$

for $k = 0, 1, 2, \ldots$, starting with $p_O = P(N = 0)$.

(b) The claims sizes are almost surely equal to a given, known amount x, i.e.

$$X_j = x \quad \text{almost surely} \quad \text{for all} \quad j = 1, 2, 3, \ldots .$$

One should note that the recursion (3.1) gives the

1. *Poisson distribution* for the choice $a + b \cdot k = \lambda$,
2. *binomial distribution* for the choice $a + b \cdot k = (q/p) \cdot (r - k), q = 1 - p$
3. *negative binomial distribution* for the choice $a + b \cdot (k - 1) = q \cdot (r + k)$

(see e.g. *Johnson* et al. (1969)).

A similar recursion also was used by *Panjer* (1981), for deriving his well known results. Obviously the recursion (3.1) allows the recursive calculation of the probabilities $P(N \leq k)$ for $k = 0, 1, 2, \ldots$. More concretely one has

$$P(N \leq k+1) = P(N \leq k) + \frac{(a + b \cdot k)}{(1+k)} \cdot P(N = k), \tag{3.2}$$

starting with $P(N \leq 0) = P(N = 0) = p_O$. Furthermore one has the following results:

$$E(N) = \frac{a}{(1-b)} \tag{3.3}$$

$$\mathrm{Var}(N) = \frac{a}{(1-b)^2} \tag{3.4}$$

which lateron become important for calculating or estimating the unknown parameters a and b.
According to the assumption (b) the total claims amount S is almost surely nothing else but:

$$S = N \cdot x.$$

The assumption (a) is already fairly general, since it covers at least three claims number distributions of great practical and theoretical importance. The assumption (b) clearly is quite restrictive, compared with the more general conditions of the former papers (see e.g. *Panjer* (1980), *Hürlimann* (1986), *Bowers* (1969)). Howbit, the set-up fits to the very practical situation, that the collective consists of policies insuring a lot of (identical) objects, each defining a possible claims size of the amount x. The reinsurer does not know the number of objects, but only gets information on the claims number per year. Such situations sometimes are given in practice.

4 The premium formula

For stating the main result, some additional notation is needed. According to the condition (b) the priority of the stop loss treaty will be fixed as a multiple of the possible claims size x, i.e. one has with an integer m that:

$$L = m \cdot x.$$

Denote with β the value:

$$\beta = \frac{E(N) \cdot x}{L},$$

implying with (3.3) that:

$$\beta = \left(\frac{a}{1-b}\right) \cdot \left(\frac{1}{m}\right). \tag{4.1}$$

Like mentioned suppose that the coefficient λ is given. Then the calculation of the risk premium $(v + \sigma)$ is nothing else but the calculation of the moments $E(R)$ and $\operatorname{Var}(R)$, given by the formulas (2.1), (2.2). One has the following result:

Theorem

Suppose the conditions of the section 3 and take the above notations. One gets as formulas for the net premium and the security loading of the stop loss cover with priority L:

$$v = L \cdot K_m(a, b)$$

$$\left(\frac{\sigma}{\lambda}\right) = L \cdot [J_m(a, b) - K_m(a, b)^2]^{1/2},$$

with the functions $K_m(.,.)$, $J_m(.,.)$ defined according

$$K_m(a, b) = \alpha \cdot P(N = m) - (1 - \beta) \cdot (1 - P(N \leq m - 1)) \tag{4.2}$$

$$J_m(a, b) = \left[\beta \cdot \left(1 + \frac{b}{a}\right) - 1\right] \cdot \alpha \cdot P(N = m)$$

$$+ \left[\left(\beta \cdot \left(1 + \frac{1}{a}\right) - 2\right) \cdot \beta + 1\right] \cdot (1 - P(N \leq m - 1)), \tag{4.3}$$

where:

$$\alpha = (1 - b)^{-1}.$$

Proof

(a) The assumption (3.1) implies for positive integer m:

$$\sum_{k=m}^{\infty} (k+1) \cdot P(N = k+1) = a \cdot P(N \geq m) + b \cdot \sum_{k=m}^{\infty} k \cdot P(N = k).$$

Adding $m \cdot P(N = m)$ on both sides and rearranging yields:

$$\sum_{k=m}^{\infty} k \cdot P(N = k) = (1 - b)^{-1} \cdot (a \cdot P(N \geq m) + m \cdot P(N = m)) \tag{4.4}$$

and this at once:

$$\begin{aligned}
&\sum_{k=m}^{\infty} (x \cdot k - L) \cdot P(N = k) \\
&= L \cdot \left(\left(\frac{x \cdot m}{L} \right) \cdot \left(\frac{a}{(1 - b) \cdot m} - 1 \right) \cdot P(N \geq m) + \frac{P(N = m)}{(1 - b)} \right) \\
&= L \cdot (\alpha \cdot P(N = m) + (\beta - 1) \cdot P(N \geq m)),
\end{aligned}$$

because of (4.1) and the definitions of m and α.

(b) Again the assumption (3.1) implies for positive integer m:

$$\begin{aligned}
&\sum_{k=m}^{\infty} (k + 1)^2 \cdot P(N = k + 1) \\
&\quad = a \cdot \sum_{k=m}^{\infty} (k + 1) \cdot P(N = k) + b \cdot \sum_{k=m}^{\infty} k \cdot (k + 1) \cdot P(N = k).
\end{aligned}$$

Adding $m^2 \cdot P(N = m)$ on both sides and rearranging gives with (4.4):

$$\begin{aligned}
&\sum_{k=m}^{\infty} k^2 \cdot P(N = k) \\
&= (1 - b)^{-2} \cdot (a \cdot (a + 1) \cdot P(N \geq m) + m \cdot (a + b + m \cdot (1 - b)) \cdot P(N = m)).
\end{aligned}$$

With this equality and (4.4) one can easily derive:

$$\begin{aligned}
\sum_{k=m}^{\infty} (x \cdot k - L)^2 \cdot P(N = k) = L^2 \cdot &\left[\left(\beta \cdot \left(1 + \frac{b}{a} \right) - 1 \right) \cdot \alpha \cdot P(N = m) \right. \\
&\left. + \left(\left(\beta \cdot \left(1 + \frac{1}{a} \right) - 2 \right) \cdot \beta + 1 \right) \cdot P(N \geq m) \right]
\end{aligned}$$

using the definitions of m, α, β and (4.1).

(c) Under the given special model assumptions one has that:

$$\sum_{k=m}^{\infty}(x \cdot k - L) \cdot P(N = k) = \int_{[L,\infty)} (s - L)G(ds)$$

$$\sum_{k=m}^{\infty}(x \cdot k - L)^2 \cdot P(N = k) = \int_{[L,\infty)} (s - L)^2 G(ds).$$

This implies together with parts (a) and (b) and (2.1), (2.2) the statement of the theorem. □

Remark 1: Take the special situation that $(a + b \cdot k) = \lambda$, i.e. the claims number is Poisson-distributed. Under these very special model assumptions, the specialized results of the theorem are clearly well known since a long time. Choose in addition $m = E(N)$. Then the formula (4.2) reduces to:

$$K_m(a, b) = P(N = m).$$

One gets a result in *Benktander* (1977, page 34) as special case.

Remark 2: For the assumption (a) and some weaker conditions than (b) (see section 3) one can give recursions for the net premium v, which are similar to those given by *Panjer* (1980). Clearly one can apply these recursions to the present more special set-up. However the result of the above theorem is more elegant for the present set-up, since it gives a nonrecursive, direct premium formula.

Remark 3: The functions $K_m(a, b)$, $J_m(a, b)$ do not directly depend on x, but only on a, b, m and β. For practical application one can tabulate these functions for different values of a, b and m. Note that m is nothing else but the ratio of the priority and the claims size. Furthermore β is nothing else but ratio of the mean total claims amount and the priority.
The practical importance of the result of the theorem relies on the fact that one does not need to restrict on one of the special model assumptions, i.e. on the Poisson-, negative binomial- and binomial claims number distributions. One can directly apply the general premium formula to *premium rating*.

5 The rating procedure

Suppose one has the claims numbers

$$N_1, N_2, N_3, \dots, N_n$$

of some past years. These random variables are assumed to be stochastically independent and each to be distributed according to the model defined in the condition (a). Based on this claims experience one likes to calculate the risk premium of the stop-loss treaty. How can one proceed?

One simply has to estimate the unknown parameters a, b, p_O from the given claims experience, leading to he corresponding estimators $\widehat{a}, \widehat{b}, \widehat{p}_O$. With these one can calculate an estimator $\widehat{\beta}$ for β by simply inserting $\widehat{a}, \widehat{b}$ into the right hand side of (4.1). With (3.1), (3.2) one can calculate recursively the probabilities $P(N = m)$ and $P(N \leq m - 1)$ with the estimators $\widehat{a}, \widehat{b}, \widehat{p}_O$ inserted for a, b, p_O. Inserting all these quantities into the right hand side of (4.2), (4.3), gives the $K_m(\widehat{a}, \widehat{b})$, $L_m(\widehat{a}, \widehat{b})$ and finally the *calculated risk premium:*

$$L \cdot [K_m(\widehat{a}, \widehat{b}) + \lambda \cdot (J_m(\widehat{a}, \widehat{b}) - K_m(\widehat{a}, \widehat{b})^2)^{1/2}],$$

where the coefficient λ is given.

All these steps define the proposed procedure for rating the risk premium of the stop loss cover. The only point that should be clarified, is how to calculate suitable estimators $\widehat{a}$, $\widehat{b}$, $\widehat{p}_O$. The simplest approach for giving $\widehat{a}$, $\widehat{b}$ is taking the so called *moment estimators* (see e.g. *Lehmann* (1983)). These are based on the equations (3.3), (3.4). One estimates the moments $E(N)$ and $\mathrm{Var}(N)$ by:

$$mn = \left(\frac{1}{n}\right) \cdot \sum_{i=1}^{n} N_i$$

$$sn = \left(\frac{1}{n-1}\right) \cdot \sum_{i=1}^{n} (N_i - mn)^2,$$

inserts them into the left hand side of (3.3), (3.4) and calculates the solutions $a = \widehat{a}$, $b = \widehat{b}$ of the resulting system. This results in the simple estimators:

$$\widehat{a} = \frac{mn^2}{sn}, \qquad \widehat{b} = \frac{1 - mn}{sn}.$$

A more refined procedure for estimating a, b is given in *Ord* (1967). Finally the p_O has to be estimated with a suitable estimator $\widehat{p}_O$. In case one has a long sequence of past claims numbers, such that some N_i take on the value zero, one can take:

$$\widehat{p}_O = \left(\frac{1}{n}\right) \cdot | \{i : N_i = 0\} | .$$

Anyway, the probabilities $P(N = k)$ with $k = 0, 1, 2, \ldots$ have to sum up to one. This side condition can be used to determine the estimator $\widehat{p}_O$ with an iterative numerical procedure, when already having the estimators $\widehat{a}$, $\widehat{b}$.

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References

Ammeter, H. (1955): the calculation of premium rates for excess-of- loss and stop-loss reinsurance treaties. Nonproportional reinsurance. Bruxelles.

Benktander, G. (1977): On the rating of a special stop loss cover. Astin Bulletin.

Berger, G. (1972): Integration of the normal power approximation. Astin Bulletin.

Bohman, H. and *Esscher, F.* (1963/64): Studies in risk theory with numerical illustration concerning distribution function and stop-loss premium. Part 1–2. Scandinavian Actuarial Journal.

Bowers, N.L. (1969): An upper bound on the stop-loss net premium. Transactions of the Society of actuaries.

Bühlmann, H. (1984): Numerical evaluation of the compound Poisson distribution: Recursion or Fast-Fourier transform? Scandinavian Actuarial Journal.

Bühlmann, H., Gagliardi, B., Gerber, H.U. and *Straub, E.* (1974): Some inequalities for stop-loss premiums. Mitteilungen der Vereinigung Schweizerischer Versicherungsmathematiker.

De Vylder, F. and *Goovaerts, M.J.* (1982a): Analytic best upper bounds on stop-loss premiums. Insurance: Mathematics and Economics.

De Vylder, F. and *Goovaerts, M.J.* (1982b): Upper and lower bounds on stop-loss premiums in case of known expectation and variance of the risk variable. Mitteilungen der Vereinigung Schweizerischer Versicherungsmathematiker.

De Vylder, F. and *Goovaerts, M.J.* (1983): Best bounds on the stop-loss premium in case of known range, expectation, variance and mode of the risk. Insurance: Mathematics and Economics.

Hürlimann, W. (1986): Error bounds for stop-loss premiums calculated with the Fast-Fourier-transform. Scandinavian Actuarial Journal.

Johnson, N.I. and *Kotz, S.* (1969): Discrete distributions. John Wiley, New York.

Kremer, E. (1988): Net premiums of nonproportional reinsurance covers in nonlife insurance. Verein zur Förderung der angewandten mathematischen Statistik und Risikotheorie, e.V., Hamburg.

Kremer, E. (1989): Stochastic life insurance mathematics. Verein zur Förderung der angewandten mathematischen Statistik und Risikotheorie, e.V., Hamburg.

Lehmann, E.L. (1983): Theory of point estimation. John Wiley, New York.

Ord, J.K. (1967): Graphical methods for a class of discrete distributions. Journal of the Royal Statistical Society.

Panjer, H.H. (1980): The aggregate claims distribution and stop-loss reinsurance. Transactions of the Society of actuaries.

Panjer, H.H. (1981): Recursive evaluation of a family of compound distributions. Astin Bulletin.

Sundt, B. and *Jewell, W.S.* (1981): Further results on recursive evaluation of compound distributions. Astin Bulletin.

Taylor, G.C. (1977): Upper bounds on stop-loss premiums under the constraint on claim size distribution. Scandinavian Actuarial Journal.

Vajda, S. (1951/55): Analytic studies in stop-loss reinsurance. Parts 1–2. Scandinavian Actuarial Journal.

Willmot, G.E. and *Panjer, H.H.* (1987): Difference equation approaches in evaluation of compound distributions. Insurance: Mathematics and Economics.