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Loss-Reserving by Kernel Regression

1   Introduction

In the following one of the main problems of nonlife insurance mathematics is reconsidered, i.e. how to calculate loss reserves. In principal this problem reduces to the prediction of the future development of yet unsettled claims. In the last few years a vast body of papers appeared in actuarial journals treating the problem of loss reserving in nonlife insurance, worth mentioning are e.g. the articles of *Bühlmann* et al. (1980), *De Jong / Zehnwirth* (1983), *De Vylder* (1978), *Kremer* (1984), *Linnemann* (1980), *Straub* (1971), *Taylor* (1977), *Verbeek* (1972) and the actuarial surveys of *Van Eeghen* (1981) and *Taylor* (1986). The papers contain many different approaches, some authors use socalled separation techniques (see *Linnemann* (1980), *Taylor* (1977), *Verbeek* (1971)), some others apply credibility methods (see *De Vylder* (1982), *Straub* (1971)), recently also time series methods were adopted (see *De Jong* and *Zehnwirth* (1983), *Lemaire* (1981) and *Kremer* (1984)). Some methods were tested and applied on empirical data, see e.g. *Taylor* (1981), *Pater* (1987), and found to be adequate for practical determination of loss reserves. Though at the present state there exist already quite many different methods, the author thought again about that topic and recognized an interesting connection between loss reserving and nonparametric kernel-regression estimation, a topic discussed extensively in journals on Mathematical Statistics during the past twenty-five years (see *Devroye* (1981), *Gleblicki* (1984), *Nadaroya* (1964)). This noticed correspondence led the author to write this further paper, presenting a new loss reserving or better predicting approach, based on modified nonparametric estimation methods.

2   The Loss Reserving Problem

Let Y_{ij}, $j = 1, \ldots, m$, $i = 1, \ldots, m$ be (nonnegative) random variables on a probability space $(\Omega, \mathscr{A}, P)$, Y_{ij} denoting the claims number or the claims size (per claim) of a collective of risks in the development year j and in respect of the accident year i. Known is only the triangle

$$Y_\nabla = (Y_{ij}, \ j = 1, \ldots, m - i + 1, \ i = 1, \ldots, m),$$

representing the past claims development. The problem consists in calculating or estimating the unknown loss reserves for the accident years $i = 2, \ldots, m$. For this one has to calculate or estimate the future growth

$$R_i(Y_\nabla) = Y_{im} - Y_{i,m-i+1}, \tag{2.1}$$

for each accident year $i = 2, \ldots, m$. In case of claims numbers one speaks of the IBNR-reserving problem, in case of claim sizes of the IBNER-reserving problem. Here IBNR or IBNER are abbreviations for 'incurred but not reported' or 'incurred but not enough reserved'. Replacing in case of claim numbers Y_{ij} by N_{ij}, in case of claim sizes Y_{ij} by S_{ij}, the *loss reserve* for the IBNR-claims of accident year i is given by

$$R_i(N_\nabla) \cdot S_{im} \tag{2.2}$$

and the loss reserve for the IBNER-claims of the accident year i by

$$R_i(S_\nabla) \cdot N_{i,m-i+1}. \tag{2.3}$$

The total loss reserve for the accident year no. i then is given by the sum of the reserves for the IBNR-claims and the IBNER-claims.

Obviously claims reserving, in the above sense, reduces to predicting the unknown Y_{ij}, $j = m - i + 2, \ldots, m$, $i = 2, \ldots, m$ from the known triangle Y_∇. Fortunately concepts and ideas of the Mathematical Statistics can be adapted to this situation, more concretely methods of the estimation and prediction theory (see e.g. *Lehmann* (1983) and *Granger/Newbold* (1977)). As already mentioned in the introduction many adequate actuarial methods, which are modifications of corresponding mathematical-statistical methods, do yet exist and are successfully applied in the insurance practice. In the sequel a further new one is presented.

3   The Optimal Predictions

Denote by L_2 the set of all square-integrable random variables X, defined on a fixed probability space $(\Omega, \mathscr{A}, P)$. By identifying X with the equivalence class of all $\widetilde{X}$ with

$$\widetilde{X} = X \qquad \text{almost surely},$$

the L_2 becomes a Hilbert space with scalar product and norm respectively

$$\langle X, Y \rangle = E(X \cdot Y),$$
$$\| X \|_2 = (E(X^2))^{1/2}$$

for X, Y of the L_2. We denote by M_{∇} the set of all Z of the L_2, depending measurably on Y_{∇}, this means that there is a measurable function g on the $m \cdot (m+1)/2$-dimensional real space with

$$Z = g(Y_{ij}, \ j = 1, \ldots, m-i+1, \ i = 1, \ldots, m).$$

M_{∇} is the class of all predictors from the triangle Y_{∇}. It is obvious to define the *optimal forecast* (or prediction) of Y_{ij} from the triangle Y_{∇} as the unique element $\widehat{Y}_{ij} \in M_{\nabla}$ satisfying

$$\| Y_{ij} - \widehat{Y}_{ij} \|_2 \leq \| Y_{ij} - Z \|_2$$

for all $Z \in L_2$, i.e. as the (orthogonal) projection of Y_{ij} on the closed linear subspace M_{∇} of the L_2. As wellknown this projection can be represented as a conditional expectation operator, more concretely

$$\begin{aligned}
\widehat{Y}_{ij} &= E(Y_{ij} \mid Y_{\nabla}) \\
&= E(Y_{ij} \mid Y_{kl}, \ l = 1, \ldots, m-k+1, \ k = 1, \ldots, m)
\end{aligned} \tag{3.1}$$

(compare e.g. Theorem 2.15 in *Kremer* (1985)). On the additional assumption that the row vectors

$$(Y_{i1}, \ldots, Y_{im}), \qquad i = 1, \ldots, m \tag{3.2}$$

are stochastically independent,

one has the more simple formula for the optimal prediction:

$$\widehat{Y}_{ij} = E(Y_{ij} \mid Y_{il}, \ l = 1, \ldots, m-i+1). \tag{3.3}$$

According to the above written, the loss reserving problem reduces simply to the determination of the conditional expectations given in (3.1) or (3.3) respectively.

4   Estimating the Optimal Predictions

Without additional assumptions no simple explicit formula can be given for the conditional expectations of (3.1) and (3.3). In a former article the author took a nonstationary, autoregressive model for the development of the Y_{ij}, $j = 1, 2, \ldots, m$, i.e.

$$Y_{ij} = \sum_{l=1}^{p} a_{jl} \cdot Y_{i,j-l} + b_{ij} + e_{ij}, \qquad i = 1, \ldots, m \quad j = 1, \ldots, m \qquad (4.1)$$

with real parameters a_{jl}, b_{ij} and random error terms e_{ij} (see *Kremer* (1984)) and gave in his Theorem 1 simple recursions for the conditional expectation or optimal prediction (3.3). As variants of the classical least squares estimation practicable estimation methods were given in his Theorem 2 for the unknown parameters a_{jl}, b_{jl} (see also *Pater* (1987)). Instead of assuming a parametric model, e.g. something like (4.1), let us only assume that for a given $p \geq 1$:

> Y_{ij} depends only through the
>
> $Y_{i,m-i-p+2}, \ldots, Y_{i,m-i+1}$ (4.2)
>
> from the $Y_{i1}, \ldots, Y_{i,m-i+1}$
>
> for $j = m - i + 2, \ldots, m$ and $i = 2, 3, \ldots, m$.

Then (3.3) simply reduces to the predictor

$$\widehat{Y}_{ij} = E(Y_{ij} \mid Y_{i,m-i-p+2}, \ldots, Y_{i,m-i+1}) \qquad (4.3)$$

for $j = m - i + 2, \ldots, m$ and $i = 2, \ldots, m - p + 1$.

Now, how to estimate this slightly more simple conditional expectation without any additional parametric assumption?

For this one can use ideas of a special field of nonparametric estimation theory, the socalled *nonparametric regression estimation*. For adapting we assume in addition to (3.2) and (4.2) that

> one has given (random or nonrandom) variables A_i, B_i, $i = 1, 2, \ldots, m$ such, that for the transformed variables
>
> $$X_{ij} = \frac{(Y_{ij} - B_i)}{A_i}$$
>
> the random vectors
>
> $$(X_{i1}, \ldots, X_{im}), \qquad i = 1, 2, \ldots, m$$
>
> are identically distributed.   (4.4)

These transformations represent possible trend effects in the different accident years, which have to be eliminated in advance. Besides this, in case of claim sizes one has to inflation adjust all claims in advance, leading to the above random variables Y_{ij}. Assuming for the X_{ij} in place of the Y_{ij} the conditions (3.2) and (4.2) (which for nonrandom A_i, B_i clearly carry over), obviously the optimal predictions of the unknown X_{ij} from the triangle

$$X_\nabla = (X_{ij}, \quad j = 1,\ldots,m-i+1, \quad i = 1,\ldots,m)$$

are

$$\widehat{X}_{ij} = E(X_{ij} \mid X_{i,m-i-p+2},\ldots,X_{i,m-i+1}) \tag{4.5}$$

for $j = m-i+2,\ldots,m$ and $i = 2,\ldots,m-p+1$.

If the A_i, B_i are stochastically independent of the X_{ij}, $j = m-i-p+2,\ldots,m-i+1$ (which in case of nonrandom A_i, B_i clearly is satisfied), the predictions $\widehat{Y}_{ij}$ of (4.3) obviously can be computed from predictions $\widehat{X}_{ij}$ of (4.5) according

$$\widehat{Y}_{ij} = A_i \cdot \widehat{X}_{ij} + B_i. \tag{4.6}$$

Consequently it remains to give a formula or an approximate formula for the predictions (4.5) on the assumptions (3.2), (4.2) and (4.4) with the X_{ij} instead of the Y_{ij}. In order to get a good approximation procedure, we extend the above setting a little bit. We assume that we have some more complete claims developments of past years indexed by $i = -n, -n+1,\ldots,0$. This means we have in addition the random variables

$$Y_{i1},\ldots,Y_{im}, \quad i = -n, -n+1,\ldots,0$$

with the claims amount or claims number Y_{ij} of the j-th development year with respect to the i-th accident year. We assume (4.4) for the whole set of claims data, with accident year index running through the values $i = -n,\ldots,m$. Finally (3.2) is supposed for the whole set of transformed data

$$(X_{i1},\ldots,X_{im}), \quad i = -n,\ldots,m$$

(clearly for nonrandom A_i, B_i this follows from the same statement for the original data Y_{ij}).

In this setting approximate formulas for (4.5) can be given by the use of *kernel regression estimators* of order p. We take a nonnegative function K on the

p-dimensional real space of row vectors into the real line such, that there exist constants c_1, c_2 and $r > 0$ with

$$c_1 \leq K(x) \leq c_2,$$

holding on the circle $\{x : \|x\| \leq r\}$ of the p-dimensional space of row vectors. Here and in the following $\| \cdot \|$ denotes the euklidean norm. Furthermore choose a sequence $(h_n)_{n \geq 1}$ with

$$h_n \to 0, \quad \text{for } n \to \infty.$$

With this notation define for $i = 2, \ldots, m - p + 1$ and $j = m - i + 2, \ldots, m$ the following functions μ_{ij} on the p-dimensional real space of row vectors $x = (x_1, \ldots, x_p)$ according

$$\mu_{ij}(x) = \frac{\sum\limits_{l=-n}^{m-j+1} K\left(\left(\frac{x_1 - X_{l,m-i-p+2}}{h_{m-j+n+2}} \right), \ldots, \left(\frac{x_p - X_{l,m-i+1}}{h_{m-j+n+2}} \right) \right) \cdot X_{lj}}{\sum\limits_{l=-n}^{m-j+1} K\left(\left(\frac{x_1 - X_{l,m-i-p+2}}{h_{m-j+n+2}} \right), \ldots, \left(\frac{x_p - X_{l,m-i+1}}{h_{m-j+n+2}} \right) \right)}.$$

The sense of this becomes clear in the following general Theorem, giving the fundamental property of these functions $\mu_{ij}(x)$.

Theorem

Assume for the sequence $(h_n)_{n=1,2,\ldots}$ that one has

$$n \cdot \frac{h_n^p}{\log(n)} \to \infty, \qquad \text{for } n \to \infty.$$

and that (what is satisfied in insurance):

$$|X_{ij}| \leq \bar{x} < \infty, \qquad \text{for all } i \text{ and } j.$$

Then one has

$$\lim_{n \to \infty} (\mu_{ij}(x)) = E(X_{ij} \mid X_{i,m-i-p+2} = x_1, \ldots, X_{i,m-i+1} = x_p)$$

in almost all $x = (x_1, \ldots, x_p)$. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\square$

Proof

We reformulate the statement in a more general way: Let $(X, Y), (X_1, Y_1), \ldots,$ (X_n, Y_n) be independent, identically distributed (row) random vectors in the corresponding $(p + 1)$-dimensional real space with $Y, Y_i \in L_2$. Similar to the definition of μ_{ij} we define m_n on the real space of p-dimensional row vectors according:

$$m_n(x) = \sum_{i=1}^{n} W_{ni}(x) \cdot Y_i$$

for $x = (x_1, \ldots, x_p)$ with the weights

$$W_{ni}(x) = \frac{K\left((x_1 - X_{i1})/h, \ldots, (x_p - X_{ip})/h\right)}{\sum_{i=1}^{n} K\left((x_1 - X_{i1})/h, \ldots, (x_p - X_{ip})/h\right)}$$

for $(X_{i1}, \ldots, X_{ip}) = X_i$ and $K, (h_n)_{n \geq 1}$ declared as above. Obviously this is a generalized version of the above setup of predicting the X_{ij} from the X_{il}, $l = m - i - p + 2, \ldots, m - i + 1$. The statement of the Theorem simply reduces to

$$|m_n(x) - m(x)| \to 0 \qquad \text{a.e. for} \quad n \to \infty, \tag{4.7}$$

for almost all x, with the definition:

$$m(x) = E(Y \mid X_{i1} = x_1, \ldots, X_{ip} = x_p).$$

This statement is nothing else but the Theorem 4.2 in *Devroye* (1981). For sake of completeness the main steps of Devroye's proof are given in a very short style. One has obviously:

$$|m_n(x) - m(x)| \leq \left| \sum_{i=1}^{n} W_{ni}(x) \cdot (Y_i - m(X_i)) \right| +$$

$$+ \sum_{i=1}^{n} W_{ni}(x) \cdot |m(X_i) - m(x)|. \tag{4.8}$$

For given $\varepsilon > 0$ one can give constants c_1, c_2 such that

$$P\left(\left| \sum_{i=1}^{n} W_{ni}(x) \cdot (Y_i - m(X_i)) \right| > \varepsilon \mid X_1, X_2, \ldots, X_n \right)$$

$$\leq c_1 \cdot \exp\left(- c_2 \cdot \sup_i W_{ni}(x) \right) \tag{4.9}$$

and the second term in (4.8) is bounded from above by

$$U_n(x) = \left(\frac{c_2}{c_1}\right) \cdot \sum_{i=1}^{n} |m(X_i) - m(x)| \cdot \left(\frac{1_{A_{in}(x)}}{\sum_{i=1}^{n} 1_{A_{in}(x)}}\right),$$

where $1_{A_{in}(x)}$ is the indicator function of the event

$$A_{in}(x) = \{\|X_i - x\| \le r \cdot h_n\}.$$

One concludes that for given $\varepsilon > 0$ there exist constants c_3, c_4 such, that

$$P\left(|U_n(x) - E(U_n(x))| > \varepsilon\right) \le c_3 \cdot E\left(\exp(-c_4 \cdot N_n(x))\right),$$

where $N_n(x)$ is distributed like $1_{A_{in}(x)}$, i.e. is binomially distributed with parameters n and $p_n(x)$, satisfying:

$$n \cdot \frac{p_n(x)}{\log(n)} \to \infty \qquad \text{for } n \to \infty,$$

for almost all x. One can show that for almost all x:

$$\sum_{n=1}^{\infty} E\left(\exp(-s \cdot N_n(x))\right) < \infty$$

for all $s > 0$, implying with Borel-Cantelli-Lemma that for almost all x

$$U_n(x) - E(U_n(x)) \to 0 \qquad \text{a.e., for } n \to \infty.$$

Since

$$E(U_n(x)) \to 0 \qquad \text{for } n \to \infty$$

in almost all x, one has that the second term on the right hand side of (4.8) converges a.e. for almost all x to zero. Also the right hand side of (4.9) can be bounded with a suitable c_5 by

$$c_1 \cdot \exp(-c_5 \cdot N_n(x)),$$

implying again with the Borel-Cantelli Theorem and Lebesques Theorem, that also the first term on the right hand side of (4.8) converges a.e. to zero for almost all x. This completes the proof of the statement (4.7). $\qquad\square$

According to this result the $\widehat{\widehat{X}}_{ij}$, $i = 2,\ldots,m-p+1$, $j = m-i+2,\ldots,m$, calculated by

$$\widehat{\widehat{X}}_{ij} = \mu_{ij}(X_{i,m-i-p+2},\ldots,X_{i,m-i+1})$$

can be used as *approximations* to the *optimal prediction* $\widehat{X}_{ij}$. With the modification of (4.6)

$$\widehat{\widehat{Y}}_{ij} = A_i \cdot \widehat{\widehat{X}}_{ij} + B_i$$

one has the desired *loss predicting procedure* of forecasting Y_{ij} by $\widehat{\widehat{Y}}_{ij}$.

In the application of this loss predicting procedure the following things have to be considered:

1. In the case $p > 1$ there is a terminating problem for the accident years $i > m-p+1$. For these years one can use functions K defined on lower dimensioned spaces and proceed like above.

2. The question appears, how to choose the sequence $(h_n)_{n\geq 1}$. A criterion for the choice of the h_n, $n \geq 1$ is given in the above Theorem, i.e. choose the sequence such, that $h_n \to 0$ and

    $$n \cdot \frac{h_n^p}{\log(n)} \to \infty, \qquad n \to \infty.$$

3. A lot of freedom in the above general method lies in the choice of the function K, called *Kernel-function*, and the appropriate dimension p of the definition space. In the case $p = 1$ the author got good results with a kernel of the type:

    $$K(x) = |x|^{-1}, \qquad \text{for } x \notin (-\varepsilon,\varepsilon)$$
    $$= \overline{x}, \qquad \text{for } x \in (-\varepsilon,\varepsilon)$$

    where $\overline{x}$ is a comparably large and ε a comparably small positive value. When applying the method, one should try to find an adequate kernel function on some given test data.

For illustration of the above method a simple example is cited.

5 An Example

In the above notation with $m = 3$, $n = 0$ we take the truncated triangle of the claims sizes

$$Y_{0j}, \quad j = 1, 2, \ldots, m$$
$$Y_{ij}, \quad j = 1, \ldots, m - i + 1, \qquad i = 1, 2, \ldots, m.$$

given by:

	$j = 1$	2	3	4
$i = 0$	23.2	33.8	37.3	38.9
1	25.8	37.3	42.9	45.6
2	22.1	30.3	30.7	
3	35.9	43.0		
4	34.9			

Obviously the rows seem to be not identically distributed. We have to transform the data like in (4.4). We choose simply $A_i = Y_{i1}$, $B_i = 0$, i.e. we take:

$$X_{ij} = \frac{Y_{ij}}{Y_{i1}}, \qquad \text{for all } i \text{ and } j,$$

and use a kernel function with $p = 1$, i.e. calculate the approximate prediction according:

$$\widehat{\widehat{X}}_{ij} = \frac{\displaystyle\sum_{l=0}^{m-j+1} K\left((X_{l,j} - X_{l,m-i+1})/h_{m-j+2}\right) \cdot X_{lj}}{\displaystyle\sum_{l=0}^{m-j+1} K\left((X_{l,j} - X_{l,m-i+1})/h_{m-j+2}\right)}$$

for $j = m - i + 2, \ldots, m$, $i = 2, \ldots, m$. According to the remark 2. one can take e.g.

$$h_n = \left(\frac{1}{n}\right)^{1/2}$$

and according to the remark 3.:

$$K(x) = |x|^{-1}, \qquad \text{for } x \notin (-\varepsilon, \varepsilon)$$
$$\qquad\quad = 1000, \qquad \text{for } x \in (-\varepsilon, \varepsilon)$$

with $\varepsilon = 1000^{-1}$. This implies the completed rectangle of the X_{ij}, $i = 0, \ldots, m$, $j = 1, \ldots, m$:

	$j = 1$	2	3	4
$i = 0$	1.0000	1.4569	1.6078	1.6767
1	1.0000	1.4457	1.6628	1.7674
2	1.0000	1.3710	1.3891	1.7170
3	1.0000	1.1978	1.5316	1.7230
4	1.0000	1.3678	1.5532	1.7220

Multiplication of the rows with the corresponding Y_{i1}-values yields the completed lower part of the rectangle of the Y_{ij}-values:

		37.95
	54.98	61.86
47.74	54.21	60.10

These values are basis for giving the loss reserves of section 2. □

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Summary

A new approach to loss reserving is presented in this paper. The statistical concept of nonparametric regression is adapted to the problem of calculating IBNR or IBNER reserves. General prediction formulas for forecasting the unknown future claims development are defined with a given kernel function. The application of the resulting methods is demonstrated in an example.

Zusammenfassung

In der vorliegenden Arbeit wird ein neuer Zugang für die Schätzung von Schadenreserven vorgestellt. Das statistische Konzept der nichtparametrischen Regression wird angewandt auf das Problem der Berechnung von IBNR- und IBNER-Reserven. Allgemeine Formeln zur Voraussage der unbekannten künftigen Schadenentwicklung werden mit einer gegebenen Kernfunktion definiert. Die Anwendung der Methode wird an einem Beispiel erläutert.

Résumé

Une nouvelle approche pour l'évaluation des réserves est présentée dans cet article. Le concept statistique de régression non-paramétrique est adapté au problème du calcul des réserves IBNR et IBNER. Des formules générales pour la prévision de l'évolution future et inconnue des sinistres sont définies avec une fonction noyau donnée. L'application des méthodes résultantes est illustrée par un exemple.