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A Mathematical Insurance Model Including Lapse Rates

Introduction

The purpose of this paper is to investigate the mathematical formulas required to implement a whole life insurance where projected lapse rates are taken into consideration when calculating the premiums and required reserves. The underlying motivations for writing this paper are as follows.

(I) To investigate the behaviour and effects of lapse rates on reserves and profits.

(II) To provide concise and accurate definitions of formulas together with numerical examples which can be reproduced by the reader to elicit understanding and test correctness.

(III) To provide clearness and uniformity in notation because this domain extends traditional insurance mathematical theory.

(IV) To provide a platform upon which investigations of profitability for existing products as well as profit testing for new products may be undertaken in an orderly and documented manner.

(V) To determine a framework and the criterion for the allocation of income and expenditure, allowing for the subsequent analysis of profits with respect to a given whole life contract. Particularly with view to separating out the cost from the basic benefits when analysing for profit or administering an existing product.

Although the formulas and examples presented in this paper are based on a specific product, and in part follow actuarial practice in Canada, this work is designed to have more general applications to product design and profit testing.

1 Notation

Since the argument presented in this paper is independent of entry age x, this value is fixed at outset and very soon dropped from sight altogether. The normal termination age of an insurance contract is s, and the resulting interval of insurance $[x, s[$ is broken into $n = s - x$ intervals $[t, t+1[$ representing the $(t+1)^{\text{th}}$ year of insurance $(t = 0, \ldots, n-1)$. The following simplifying assumptions are made

(I) Policy year and calendar year coincide viz.
  - Beginning of $(t+1)^{\text{th}}$ year of insurance is 00:00:00 hours, 1st of January.
  - End of $(t+1)^{\text{th}}$ year of insurance is 23:59:59 hours, 31st December.

(II) Premiums are paid at the beginning of the year.

(III) Benefits are paid at the end of the year.

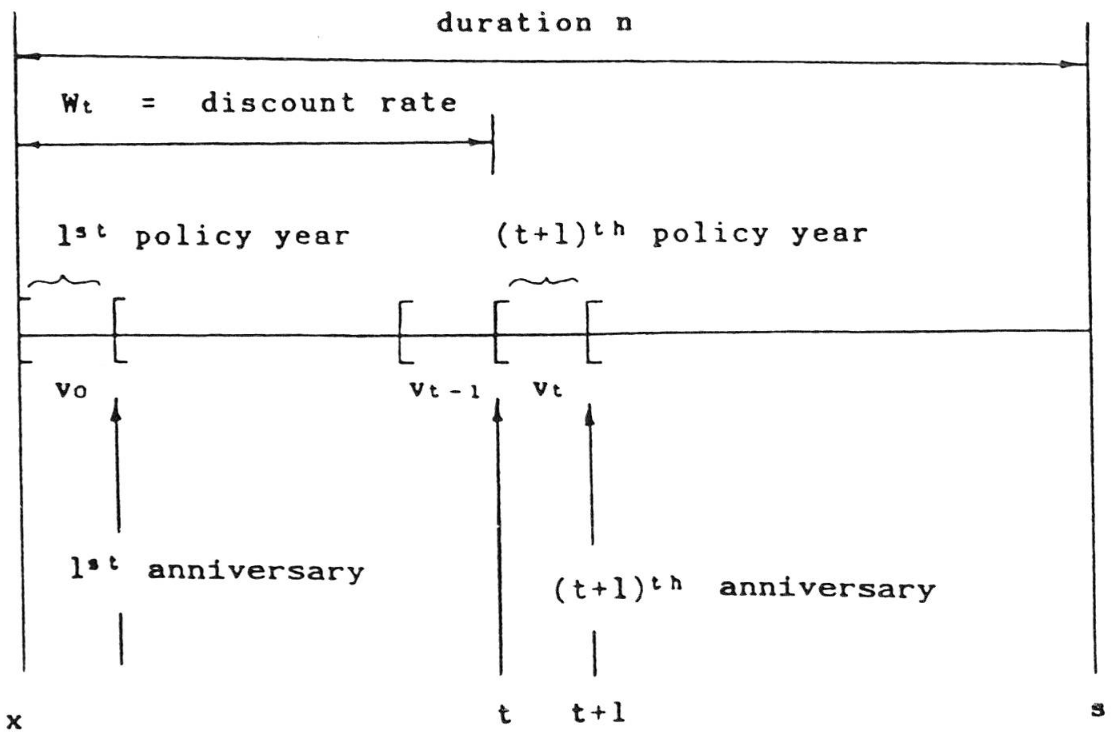
(IV) The discount factor v_t, $(0 < v_t \leq 1)$, remains constant throughout the year.

(V) The period of insurance is at least one year (i.e. $n \geq 1$).

Since the interest rates vary from year to year, it is convenient to denote the discount factor for the interval $[0, t]$ by

$$W_t = v_0 v_1 \cdots v_{t-1}, \qquad t = 1, \ldots, n.$$

It is useful to define $W_0 = 1$. See diagram.

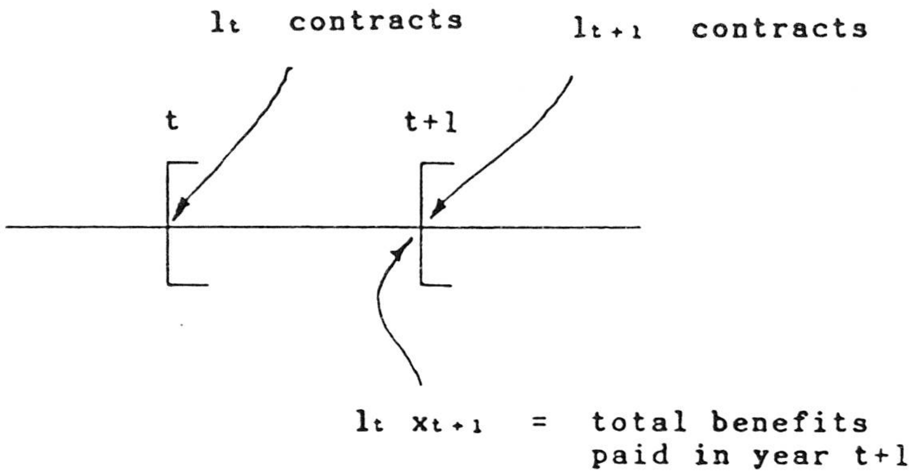


2 A Mathematical Model

Before considering the specific but well-known product categories such as term life, whole life, etc., a generalized discrete insurance model is developed. This brings with it a certain notational convenience, conceptual clarity and a device for the generic treatment of formulas and results.
The two basic ingredients for such a life-insurance model are described below:

(I) An initial portfolio of l_0 contracts which will diminish year by year until all contracts terminate at the end of the n^{th} policy year. This gives rise to a sequence $l_0 \geq l_1 \geq l_2 \geq, \dots, \geq l_{n-1} > 0$ of numbers, where l_t denotes the number of "live" contracts left in the portfolio at time t (i.e. the beginning of year $t+1$). For the purpose of developing the theory, one may assume without loss of generality that $l_{n-1} > 0$, for otherwise one may just shorten the duration n until the condition is satisfied.

(II) The numbers $x_1, x_2, \dots, x_n$ where $l_t x_{t+1}$ is the total cost at point $t+1$ of all benefits paid at the end of year $t+1$. In other words $x_{t+1} v_t$ is the yearly premium at point t per policyholder to cover all benefits incurred during the year.



Intrinsic in a mathematical model is the assumption that the course of events of a portfolio follows exactly the "valuation" assumptions used (Ablauf gemäss Grundlagen 1. Ordnung). The analysis of profits (or losses) occurring due to deviations of these assumptions will be dealt with later.
The present actuarial value I_t at point t for this insurance is given by

$$I_t = \sum_{u=0}^{n-t-1} \frac{l_{t+u}}{l_t} x_{t+u+1} \frac{W_{t+u+1}}{W_t} \qquad t = 0, \dots, n-1 \tag{2.1}$$

The special case where $x_{t+1} = 1/v_t$ is denoted by a_t and represents an annuity:

$$a_t = \sum_{u=0}^{n-t-1} \frac{l_{t+u}}{l_t} \frac{W_{t+u}}{W_t} \qquad t = 0, \dots, n-1 \tag{2.2}$$

Let $P = P^I = I_0/a_0$ be the annual net premium and $V_t = V_t(I) = I_t - Pa_t$ the net premium reserve.

Lemma 1.

Given the sequences $l_0 \geq l_1 \geq l_2 \geq \cdots \geq l_{n-1} > 0$ and $x_1, x_2, \dots, x_n$, then the following are true

a) $$I_t = \begin{cases} v_t \left(\frac{l_{t+1}}{l_t} I_{t+1} + x_{t+1} \right) & t = 0, \dots, n-2 \\ v_{n-1} x_n & t = n-1 \end{cases} \tag{2.3}$$

b) $$a_t = \begin{cases} v_t \frac{l_{t+1}}{l_t} a_{t+1} + 1 & t = 0, \dots, n-2 \\ 1 & t = n-1 \end{cases} \tag{2.4}$$

c) $$v_t \frac{l_{t+1}}{l_t} V_{t+1} = \begin{cases} P^I - x_{t+1} v_t + V_t & t = 0, \dots, n-2 \\ 0 & t = n-1 \end{cases} \tag{2.5}$$

d) If $x_t \geq 0$ all t and $x_t > 0$ some t, then $P > 0$.

Proof:

a) From the definition of I_t, $I_{n-1} = v_{n-1} x_n$. For $t = 0, \dots, n-2$,

$$\begin{aligned} I_t &= \left(\sum_{u=0}^{n-t-1} \frac{l_{t+1+u}}{l_t} x_{t+1+u+1} \frac{W_{t+2+u}}{W_t} \right) + x_{t+1} v_t \\ &= v_t \frac{l_{t+1}}{l_t} \left(\sum_{u=0}^{n-t-1} \frac{l_{t+1+u}}{l_{t+1}} x_{t+1+u+1} \frac{W_{t+1+u+1}}{W_{t+1}} \right) + x_{t+1} v_t \\ &= v_t \left(\frac{l_{t+1}}{l_t} I_{t+1} + x_{t+1} \right) \end{aligned}$$

b) Let $x_{t+1} = 1/v_t$ in (a) above.

c) $V_{n-1} = I_{n-1} - P = v_{n-1}x_n - P$. For $t = 0, \dots, n-2$,

$$\begin{aligned} V_t &= I_t - Pa_t \\ &= v_t\left(\frac{l_{t+1}}{l_t} I_{t+1} + x_{t+1}\right) - P\left(v_t \frac{l_{t+1}}{l_t} a_{t+1} + 1\right) \\ &= v_t \frac{l_{t+1}}{l_t} V_{t+1} - P + x_{t+1} v_t, \end{aligned}$$

hence the result.

d) Since $l_{t+u}/l_t \geq 0$, $v_t > 0$ and $W_{t+u}/W_t > 0$ all t, it follows that $I_0 > 0$, thus $P = I_0/a_0 > 0$.

Traditionally, the numbers $l_0, l_1, \dots, l_{n-1}$ are determined uniquely by the (select) mortality rates for an entry age x. However, the inclusion of lapse rates into the model gives an additional cause for the premature termination of a contract. Suppose for example that the actual course of events for a portfolio size follows the sequence $l_0 = l'_0 \geq l'_1 \geq \cdots \geq l'_{n-1} > 0$. One may define

$$a'_t = \sum_{u=0}^{n-t-1} \frac{l'_{t+u}}{l'_t} \frac{W_{t+u}}{W_t} \qquad t = 0, \dots, n-1 \tag{2.6}$$

$$I'_t = V_t + Pa'_t \qquad t = 0, \dots, n-1 \tag{2.7}$$

In particular $I'_{n-1} = V_{n-1} + P = v_{n-1}x_n$. Define x' by the formula

$$v_t x'_{t+1} = \begin{cases} I'_t - v_t \dfrac{l'_{t+1}}{l'_t} I'_{t+1} & t = 0, \dots, n-2 \\ v_{n-1}x_n & t = n-1 \end{cases} \tag{2.8}$$

The second part of this formula ensures that $I'_{n-1} = v_{n-1}x'_n$. This fact together with the first part of this formula gives:

$$I'_t = \sum_{u=0}^{n-t-1} \frac{l'_{t+u}}{l'_t} x'_{t+u+1} \frac{W_{t+u+1}}{W_t} \tag{2.9}$$

Thus I'_t is an insurance of the form as defined in (2.1) which, according to (2.7), has the same reserves as I_t, that is $V'_t = V_t$ all t. In other words, the two insurances (x, l) *and* (x', l') *are equivalent in the sense that they result in the same annual premium and actuarial reserve.* If one insists further that $I = I'$, then equation (2.7) combined with (2.4) forces $l'_t = l_t$ all t, from which follows $x'_t = x_t$ all t (via 2.8).

The following example illustrates two equivalent insurances for which I is not identically equal to I'.

Example 1. $P = 177.97$

t	i	v_t	l_t	x_t	I_t	a_t	V_t
0	1.0 %	0.990	100.0	0.00	589.037	3.310	0.000
1	8.0 %	0.926	90.0	0.00	661.030	2.592	199.723
2	6.0 %	0.943	80.0	50.00	746.902	1.934	402.648
3	4.0 %	0.962	60.0	200.00	788.955	1.321	553.942
4	4.0 %	0.962	20.0	500.00	961.538	1.000	783.567
5				1000.00			

			l'_t	x'_t	I'_t	a'_t	V'_t
0			100.0	0.00	640.019	3.596	0.000
1			85.0	9.99	748.745	3.085	199.723
2			80.0	28.95	828.429	2.392	402.648
3			70.0	130.76	854.145	1.687	553.942
4			50.0	201.50	961.538	1.000	783.567
5				1000.00			

It is easy to see that the equivalence $(x, l) \sim (x', l')$ forms a mathematical equivalence relation whose identity is the equivalence class of (x, l) where $x_t = 0$ all t. Note that $(x, l) = 0$ implies (via 2.5) that $x_t = 0$ all t. This gives a way of adding or subtracting two insurances (x, l) and (x', l'). We have shown above that there exists a y such that $(y, l) \sim (x', l')$. So we can define the sum $(x, l) + (x', l')$ as the equivalence class of $(y + x, l)$. Because premiums and capital reserves behave additively under this addition, it follows that this addition is well defined. In this way the equivalence classes form a vector space.

3 Specific Models

Until now the model has included the concepts of benefit paid and premium required to cover those benefits. The question now arises what benefits are paid and who are the beneficiaries? Some additional notation is required at this point.

Clearly in each year $[t, t+1[$,

$$l_t - l_{t+1} = d_t = l_t q_t, \qquad t = 0, \dots, n-1$$

contracts terminate giving rise to lapse mortality quotient q_t. Note that l_n, the number of contracts which arrive at the end of the contract period without incurring any benefits is introduced and satisfies: $l_{n-1} \geq l_n \geq 0$. The number d_t consists of $d_t^d = l_t q_t^d$ deaths and $d_t^w = l_t w_t$ lapses, thus q_t^d (where $q_t^d < 1$ all t) is the select mortality rate for an entry age x which is obtained from outset from some recognized mortality table. The lapse rates w_t, however, are guesswork based on the prior experience of a business portfolio. Finally, define the "multiplicative" lapse rate q_t^w according to the formula $(1 - q_t) = (1 - q_t^w)(1 - q_t^d)$.
To summarize, during the interval $[t, t+1[$:

q_t^d	$(= q_{[x]+t})$	= select mortality rate.	(3.1)
q_t^w		= multiplicative lapse rate.	(3.2)
$(1 - q_t) = (1 - q_t^d)(1 - q_t^w)$		= contract survival rate.	(3.3)
w_t	$= q_t^w(1 - q_t^d)$	= additive lapse rate.	(3.4)
d_t^w	$= w_t l_t$	= number of lapses in $[t, t+1[$.	(3.5)
d_t^d	$= q_t^d l_t$	= number of deaths in $[t, t+1[$.	(3.6)
q_t	$= q_t^d + w_t$	= lapse / mortality quotient.	(3.7)
d_t	$= d_t^w + d_t^d = l_t q_t = l_t - l_{t+1} =$ number of contracts which have terminated in $[t, t+1[$.		(3.8)

Note: Since $l_{t+1} = l_t(1 - q_t) = l_t(1 - q_t^w)(1 - q_t^d)$, then $l_{n-1} > 0$ implies $q_t^w < 1$ all $t \leq n-2$.
Having defined the beneficiaries for the period $[t, t+1[$ as:

(I) The $q_t^d l_t$ contracts which have "died", each with death benefit S_{t+1}, and

(II) The $w_t l_t$ contracts which have lapsed, each with cash value benefit C_{t+1},

one obtains $x_{t+1} = d_t^d S_{t+1} + w_t C_{t+1}$, $t = 0, \dots, n-1$. This gives rise to the *n-year endowment insurance with lapse rates.* (Gemischte Versicherung mit Stornowahrscheinlichkeiten):

$$G_t = \sum_{u=0}^{n-t-1} \frac{l_{t+u}}{l_t} x_{t+u+1} \frac{W_{t+u+1}}{W_t}$$

$$= \sum_{u=0}^{n-t-1} \frac{d_{t+u}^d S_{t+u+1} + d_{t+u}^w C_{t+u+1}}{l_t} \frac{W_{t+u+1}}{W_t} \qquad t = 0, \dots, n-1 \tag{3.9}$$

The n-year term annuity is defined as

$$a_t = \sum_{u=0}^{n-t-1} \frac{l_{t+u}}{l_t} \frac{W_{t+u}}{W_t} \tag{3.10}$$

As before:

$G_0 =$ Single net premium.
$P = G_0/a_0 =$ net premium.
$a_t = n$-year annuity.
$V_t = G_t - Pa_t =$ net premium reserve.

Using various combinations of lapse rates, death benefits and cash values, one can obtain the traditional insurance models. This is illustrated below. A comparison is given to the traditional formulas used as defined in *Wolff* (Springer 1970).

Example 2. n-year Pure Endowment (Erlebensfall):

Let

$$q_t^w = \begin{cases} 0 & t = 0, \dots, n-2 \\ 1 & t = n-1 \end{cases}$$

$$C_t = 0 \qquad t = 1, \dots, n-1$$

$$C_n > 0$$

$$S_t = 0 \qquad t = 0, \dots, n-1$$

Thus

$$\begin{aligned} G_t = G_t^e &= \frac{l_{n-1}}{l_t} w_t \frac{W_n}{W_t} C_n \\ &= \frac{l_{n-1}}{l_t} (1 - q_t^d) \frac{W_n}{W_t} C_n \\ &= \frac{l_n' W_n}{l_t' W_t} C_n = {}_{n-t}E_{[x]+t}\, C_n, \end{aligned}$$

where l_t' is the traditional l_t calculated using mortality only.

Example 3. *n-year Term Life* (Temporärer Todesfall):

Let

$$q_t^w = 0 \qquad t = 0, \dots, n-1$$
$$C_t = 0 \qquad t = 1, \dots, n$$

Thus

$$G_t = G_t^a = \sum_{u=0}^{n-t-1} \frac{d'^d_{t+u} S_{t+u+1}}{l'_t} \frac{W_{t+u+1}}{W_t}$$
$$= {}_{|n-t}A_{[x]+t}$$

The last equality holds only in the special case where $S_t = 1$ and $v_t = v$ all t.

Example 4. *n-year Endowment* (Gemischte):

Let

$$q_t^w = \begin{cases} 0 & t = 0, \dots, n-2 \\ 1 & t = n-1 \end{cases}$$
$$C_t = 0 \qquad t = 1, \dots, n-1$$
$$C_n > 0$$

(that is, no lapses occur until $t = n$ when every survivor collects the sum C_n.) Thus, as above $G_t = G_t^g = G_t^a + G_t^e = A_{[x]+t,n-t}$. As before, the last equality holds only when $C_n = 1$ and $S_t = 1$ all t.
In all the Examples 2, 3 and 4 above, $a_t = \ddot{a}_{[x]+t,n-t}$. Note however it is in general not true that $a_t = a_t^g$ or $a_t = a_t^e$, etc. Finally, for the net premium reserve we have:

$$V_t^g = G_t^g - P^g a_t^g$$
$$= {}_tV_x^{\text{pro}}[P_{x,n}(A_{[x],n})]$$
$$= A_{[x]+t,n-t} - \ddot{a}_{[x]+t,n-t}\, P_{x,n}(A_{[x],n})$$

The insurance G_t defined above exhibits much the same characteristics as its traditional counterpart (G_t^g) without lapse rates. For example, one can use Lemma 1 to obtain the recursion relations:

$$G_t = v_t[(1-q_t)G_{t+1} + q_t^d S_{t+1} + w_t C_{t+1}] \tag{3.11}$$
$$a_t = v_t(1-q_t)a_{t+1} + 1 \tag{3.12}$$
$$V_t = v_t(1-q_t)V_{t+1} + v_t(q_t^d S_{t+1} + w_t C_{t+1}) - P \tag{3.13}$$
$$t = 0, \dots, n-1,$$

provided one defines for convenience $G_n = a_n = V_n = 0$. From the last of these formulas follows the breakdown of P into $P = P_S + P_R + P_L$, where

$$P_{S,t} = \text{Savings premium} = v_t V_{t+1} - V_t \tag{3.14}$$

$$P_{R,t} = \text{Risk premium} = v_t q_t^d (S_{t+1} - V_{t+1}) \tag{3.15}$$

$$P_{L,t} = \text{Lapse premium} = v_t w_t (C_{t+1} - V_{t+1}), \tag{3.16}$$

$$t = 0, \dots, n-1 .$$

For the case where $t = n-1$, if $C_n > 0$,

$$\begin{aligned} P_{S,n-1} &= v_{n-1} C_n - V_{n-1} \\ P_{R,n-1} &= v_{n-1} q_{n-1}^d (S_n - C_n) \\ P_{L,n-1} &= 0 , \end{aligned}$$

and if $C_n = 0$, $P = P_{R,n-1} = v_{n-1} q_{n-1}^d S_n - V_{n-1}$, $P_{S,n-1} = P_{L,n-1} = 0$.

Now we come to the first real result of this paper:

Theorem 1:

Suppose G_t and G_t^g are defined as in 3.9 and Example 4, and assume $q_{n-1}^w = 1$. Define

$$I_t = \sum_{u=0}^{n-t-1} \frac{l_{t+u}}{l_t} w_t v_t (V_{t+1+u}^g - C_{t+1+u}) \frac{W_{t+u}}{W_t}$$

where for the purposes of this formula V_n^g is defined as C_n. Then:

a) $P^I + P = P^g$

b) $(G + I) \sim G^g$

Corollary 1:

$C_t = V_t^g$ all t if, and only if $G \sim G^g$ irrespective of the choice of the w_t.

Remark

(I) If the cash values of the insurance G are set as those of the net premium reserves for a traditional whole life (Gemischte), then from the point of view of premiums and reserves, the result is none other than the traditional whole life (Gemischte). And this occurs regardless of the choice of the lapse rates w_t. The converse of this statement is also true.

(II) From a purely theoretical point of view, one observes from the above remark that the traditional whole life (Gemischte) is completely robust against the effects of lapse rates on a portfolio. In other words, lapse rates have no effect on premium size and actuarial reserves per policy.

(III) Setting $C_t < V_t^g$ reduces the premium required per policy, however it will heighten the risk of reducing the profits generated by the portfolio. This will be looked at more closely later.

(IV) Choosing the cash values C_t so that $|V_t - C_t|$ is small, results in a net premium reserve $V(G)$ which differs only "slightly" from $V(G^g)$. Example 5 illustrates this.

Proof of Corollary:

If $C_t = V_t^g$ all t, then $I = 0$ and $G \sim G^g$. Conversely, if $G \sim G^g$ irrespective of the w_t, then $I_t = 0$ all t, irrespective of the choice of the w_t. Choose w_t such that $w_t v_t$ is non-zero all t. The result follows.

Proof of Theorem:

For $0 \leq t \leq n-1$, using 2.5,

$$\begin{aligned} v_t(1-q_t)V_{t+1} &= P - v_t q_t^d S_{t+1} - v_t w_t C_{t+1} + V_t \\ &= (P - P^g) + (P^g - v_t q_t^d S_{t+1} + V_t^g) \\ &\quad + (V_t - V_t^g) - v_t w_t C_{t+1} \\ &= (P - P^g) + (V_t - V_t^g) + v_t(1 - q_t^d)V_{t+1}^g \\ &\quad - v_t q_t^w (1 - q_t^d) V_{t+1}^g + w_t v_t (V_{t+1}^g - C_{t+1}) \\ &= (P - P^g) + (V_t - V_t^g) + v_t(1 - q_t)V_{t+1}^g \\ &\quad + w_t v_t (V_{t+1}^g - C_{t+1}) \,. \end{aligned}$$

{ Note:

- using 2.5, the expression in the second part of the second line reduces to $v_t(1-q_t^d)V_{t+1}^g$,
- the 3rd line was obtained using the relation $w_t = q_t^w(1-q_t^d)$, and
- the left hand side of the above equation is well defined because $q_{n-1} = 1$}

Thus,

$$v_t(1-q_t)D_{t+1} = (P^g - P) + D_t - v_t w_t(V_{t+1}^g - C_{t+1}),$$

where $D_t = V_t^g - V_t$. Compare this with the recursion formula for the Insurance I:

$$v_t(1-q_t)V_{t+1}^I = P^I + V_t^I - v_t w_t(V_{t+1}^g - C_{t+1}).$$

Since the difference between these two recursion relations:

$$v_t(1-q_t)X_{t+1} = (P^g - P - P^I) + X_t \qquad t = 0,\dots,n-1,$$

satisfies $X_0 = 0 = X_{n-1}$ (by definition), a contradiction results unless $P^g - P - p^I = 0$. Thus we obtain $X_t = 0$ all t, $P^g = P + P^I$ and $V_t^g = V_t^I + V_t$ all t, which is what was required to be shown.

Example 5

Using the assumptions for x, n, i_t, v_t, q_t^d, q_t^w appearing in the table below, one obtains:

$$P^g = 1240.165$$

$$P = 1061.010$$

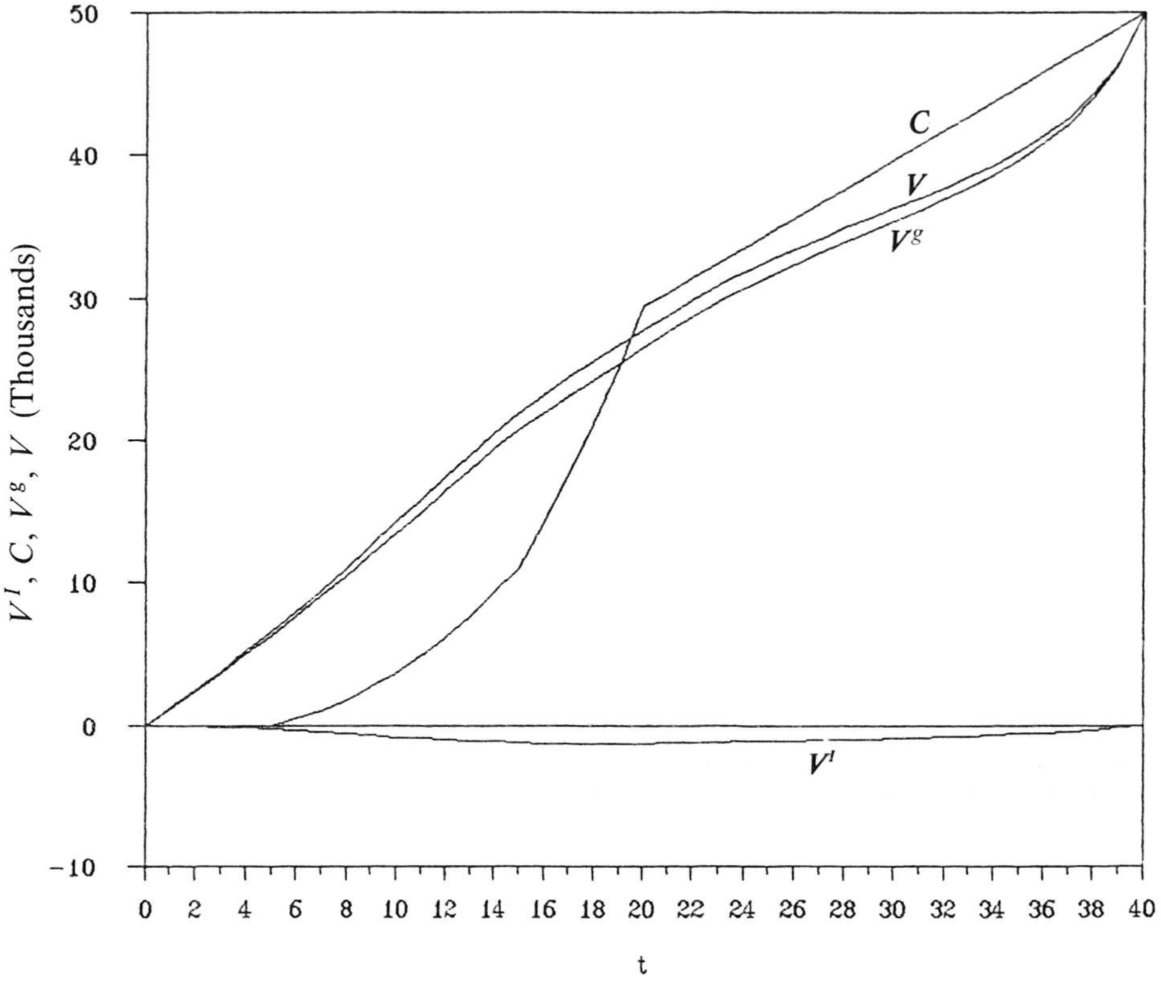
$$P^I = 179.155$$

$$V = 50000$$

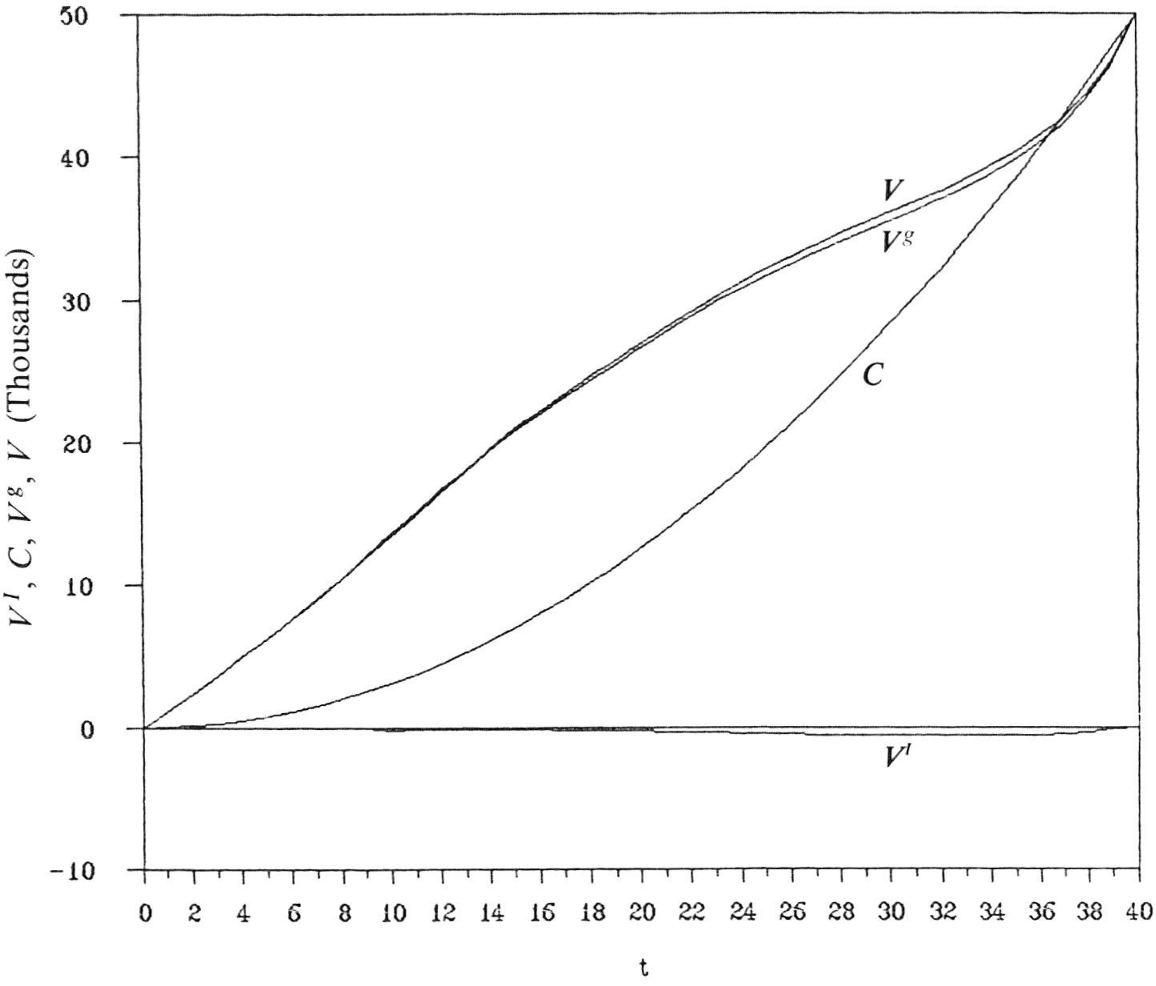
t	x	i_t	v_t	q^d_t	q^w_t	w_t	q_t
0	60	9.0%	91.7%	0.387000%	18.75%	18.68%	19.06%
1	61	8.8%	91.9%	0.488115%	9.00%	8.96%	9.44%
2	62	8.6%	92.1%	0.618003%	6.75%	6.71%	7.33%
3	63	8.4%	92.3%	0.779004%	5.25%	5.21%	5.99%
4	64	8.2%	92.4%	0.954000%	3.75%	3.71%	4.67%
5	65	8.0%	92.6%	1.134000%	3.00%	2.97%	4.10%
6	66	7.8%	92.8%	1.313001%	3.00%	2.96%	4.27%
7	67	7.6%	92.9%	1.500003%	3.00%	2.95%	4.46%
8	68	7.4%	93.1%	1.700001%	3.00%	2.95%	4.65%
9	69	7.2%	93.3%	1.929996%	3.00%	2.94%	4.87%
10	70	7.0%	93.5%	2.194002%	1.50%	1.47%	3.66%
11	71	6.8%	93.6%	2.516004%	1.50%	1.46%	3.98%
12	72	6.6%	93.8%	2.903004%	1.50%	1.46%	4.36%
13	73	6.4%	94.0%	3.326004%	1.50%	1.45%	4.78%
14	74	6.2%	94.2%	3.794004%	1.50%	1.44%	5.24%
15	75	6.0%	94.3%	4.914000%	1.50%	1.43%	6.34%
16	76	5.9%	94.4%	5.304996%	1.50%	1.42%	6.73%
17	77	5.8%	94.5%	5.744997%	1.50%	1.41%	7.16%
18	78	5.7%	94.6%	6.246000%	1.50%	1.41%	7.65%
19	79	5.6%	94.7%	6.806997%	1.50%	1.40%	8.20%
20	80	5.5%	94.8%	7.426998%	1.50%	1.39%	8.82%
21	81	5.4%	94.9%	8.112996%	1.50%	1.38%	9.49%
22	82	5.3%	95.0%	8.877996%	1.50%	1.37%	10.24%
23	83	5.2%	95.1%	9.736002%	1.50%	1.35%	11.09%
24	84	5.1%	95.1%	10.665000%	1.50%	1.34%	12.01%
25	85	5.0%	95.2%	11.644002%	1.50%	1.33%	12.97%
26	86	5.0%	95.2%	12.657996%	1.50%	1.31%	13.97%
27	87	5.0%	95.2%	13.694004%	1.50%	1.29%	14.99%
28	88	5.0%	95.2%	14.742999%	1.50%	1.28%	16.02%
29	89	5.0%	95.2%	15.800004%	1.50%	1.26%	17.06%
30	90	5.0%	95.2%	16.863003%	1.50%	1.25%	18.11%
31	91	5.0%	95.2%	17.931996%	1.50%	1.23%	19.16%
32	92	5.0%	95.2%	19.029996%	1.50%	1.21%	20.24%
33	93	5.0%	95.2%	20.208996%	1.50%	1.20%	21.41%
34	94	5.0%	95.2%	21.540996%	1.50%	1.18%	22.72%
35	95	5.0%	95.2%	23.117004%	1.50%	1.15%	24.27%
36	96	5.0%	95.2%	25.058997%	1.50%	1.12%	26.18%
37	97	5.0%	95.2%	27.505998%	1.50%	1.09%	28.59%
38	98	5.0%	95.2%	30.623004%	1.50%	1.04%	31.66%
39	99	5.0%	95.2%	34.599996%	1.00	65.40%	1.00

t	a^g_t	G^g_t	V^g_t	a_t	C_t	G_t	V_t	I_t	V^I_t
0	10.60	13150.88	0.00	6.77	0.00	7183.50	0.00	1212.96	0.00
1	10.51	14195.90	1162.78	7.77	0.00	9435.30	1189.84	1365.22	-27.06
2	10.40	15275.64	2381.97	8.14	0.00	11066.71	2434.81	1404.69	-52.84
3	10.27	16381.58	3647.18	8.36	0.00	12635.13	3763.16	1382.08	-115.98
4	10.13	17504.50	4946.92	8.49	0.00	14154.57	5148.18	1319.50	-201.26
5	9.97	18640.70	6277.31	8.50	0.00	15564.85	6546.97	1253.04	-269.66
6	9.80	19789.36	7638.49	8.45	441.00	16923.83	7963.02	1188.54	-324.52
7	9.61	20951.53	9033.30	8.38	1029.00	18340.74	9444.56	1090.89	-411.26
8	9.41	22125.73	10461.16	8.32	1764.00	19815.28	10991.54	959.55	-530.37
9	9.18	23309.29	11919.86	8.24	2646.00	21345.95	12602.28	793.98	-682.42
10	8.95	24495.32	13401.19	8.16	3675.00	22926.75	14269.14	593.93	-867.94
11	8.69	25676.33	14896.07	7.95	4851.00	24251.32	15814.05	506.68	-917.98
12	8.43	26839.60	16387.82	7.73	6174.00	25569.34	17365.11	408.01	-977.30
13	8.15	27971.53	17858.36	7.50	7644.00	26865.27	18903.51	299.22	-1045.15
14	7.87	29065.42	19299.73	7.27	9261.00	28130.91	20420.24	181.46	-1120.51
15	7.59	30112.97	20701.80	7.02	11025.00	29356.34	21904.13	56.00	-1202.33
16	7.34	30985.36	21876.49	6.82	14112.00	30386.01	23152.72	-54.86	-1276.23
17	7.10	31850.68	23050.89	6.60	17493.00	31388.84	24381.09	-146.92	-1330.20
18	6.84	32704.39	24218.81	6.39	21168.00	32353.77	25576.98	-213.89	-1358.17
19	6.59	33540.48	25371.86	6.17	25137.00	33267.13	26724.92	-248.39	-1353.07
20	6.33	34353.70	26502.85	5.94	29400.00	34114.68	27809.18	-241.63	-1306.33
21	6.07	35139.46	27605.65	5.72	30430.00	34934.64	28866.78	-236.55	-1261.13
22	5.82	35892.45	28673.25	5.50	31460.00	35721.42	29890.80	-233.03	-1217.55
23	5.57	36605.59	29696.26	5.27	32490.00	36467.62	30871.95	-230.85	-1175.69
24	5.33	37269.65	30662.41	5.06	33520.00	37163.51	31798.02	-229.63	-1135.61
25	5.09	37877.55	31563.33	4.85	34550.00	37801.44	32660.23	-228.78	-1096.89
26	4.86	38423.45	32393.58	4.64	35580.00	38374.93	33452.28	-227.49	-1058.70
27	4.64	38945.32	33187.28	4.44	36610.00	38921.67	34208.62	-225.53	-1021.34
28	4.43	39447.53	33951.08	4.25	37640.00	39445.85	34935.11	-222.38	-984.03
29	4.23	39936.21	34694.29	4.07	38670.00	39953.38	35640.10	-217.50	-945.81
30	4.02	40419.26	35428.95	3.88	39700.00	40451.95	36334.51	-210.32	-905.56
31	3.82	40906.84	36170.50	3.69	40730.00	40951.46	37032.47	-200.24	-861.97
32	3.61	41412.23	36939.12	3.50	41760.00	41464.88	37752.62	-186.68	-813.50
33	3.38	41951.14	37758.73	3.29	42790.00	42007.69	38517.25	-169.15	-758.52
34	3.13	42541.38	38656.42	3.06	43820.00	42597.46	39351.80	-147.34	-695.38
35	2.85	43204.67	39665.19	2.80	44850.00	43255.80	40287.61	-121.23	-622.42
36	2.53	43971.24	40831.04	2.49	45880.00	44013.06	41368.74	-91.20	-537.70
37	2.15	44889.04	42226.90	2.12	46910.00	44917.76	42665.60	-58.41	-438.70
38	1.66	46045.87	43986.29	1.65	47940.00	46059.26	44307.72	-25.68	-321.43
39	1.00	47619.05	46378.88	1.00	48970.00	47619.05	46558.04	0.00	-179.16
40	0.00	0.00	50000.00	0.00	50000.00	0.00	50000.00	0.00	0.00

Graph of Net Reserves



Graph of Net Reserves



4 Cost Structure

When reserves are calculated for an existing insurance product, the pricing premium, i.e. the gross premium payable by the insuree, called P_x (dependent on entry age x), is a given quantity. For a mathematical model to be of any real use, it must contain some cost components. Suppose a contract is built up of the following components.

4.1 *Basic Benefits*

$$I_t^1 = G_t . \qquad P^1 = \frac{G_0}{a_0}, \qquad V_t^1 = G_t - P^1 a_t .$$

4.2 *Administrative Costs*

Suppose the initial insured risk S_0 is positive. Let

$$x_{t+1} = \frac{\gamma(1.03)^t S_0}{v_{t+u}} \qquad t = 0, \dots, n-1,$$

then

$$I_t^2 = \sum_{u=0}^{n-t-1} \frac{l_{t+u}}{l_t} \frac{\gamma(1.03)^t S_0}{v_{t+u}} \frac{W_{t+u+1}}{W_t}$$
$$= \gamma S_0 \sum_{u=0}^{n-t-1} \frac{l_{t+u}}{l_t} (1.03)^{t+u} \frac{W_{t+u}}{W_t}$$

and $P^2 = I_0^2/a_0$, $V_t^2 = I_t^2 - P^2 a_t$ is an insurance according to the model 2.1, which covers running administration costs. These costs start at γS_0 at $t = 0$ and are inflated 3 % annually.

4.3 *Acquisition and renewal costs*

If commissions are set at $\alpha P_x (0 \leq \alpha)$ for acquisition and at $\alpha_t P_x$ at the beginning of year $[t, t+t[$, $t = 0, \dots, n-1$, for renewal, then $x_{t+1} = \alpha_t P_x / v_t$ and

$$I_t^3 = \sum_{u=0}^{n-t-1} \frac{l_{t+u}}{l_t} \frac{\alpha_{t+u} P_x}{v_{t+u}} \frac{W_{t+u+1}}{W_t} = \sum_{u=0}^{n-t-1} \frac{l_{t+u}}{l_t} \alpha_{t+u} P_x \frac{W_{t+u}}{W_t}$$

represents the insurance for the renewal commissions. Note usually $\alpha_0 = 0$. Further if we set $P^3 = (I_0^3 + \alpha P_x)/a_0$, then we obtain the modified (zillmerized) reserves

$$V_t^3 = I_t^3 - P^3 a_t$$

Using (2.3) and (2.4), we obtain

$$V_t^3 + P^3 - \alpha_t P_x = \begin{cases} v_t(1-q_t)V_{t+1}^3 & t = 0,\dots,n-1 \\ 0 & t = n-1\,. \end{cases}$$

4.4 *Tax*

Let $P^5 = \beta P_x$ so $V_t^5 = 0$. Note that the value $P = (1-\beta)P_x = P_x - P^5$ is the available premium for use in covering the three components P^1, P^2 and P^3 above. This value P will be used below.

4.5 *Deficiency Premium*

Since the value P (above) may deviate from the corresponding mathematically arrived at premium $P^a = P^1 + P^2 + P^3$, additional deficiency reserves

$$V_t^4 = -P^4 a_t$$

are required, where $P^4 = \min(P - P^a, 0)$ is the deficiency premium. From another perspective,

$$\begin{aligned} V_t &= V_t^1 + V_t^2 + V_t^3 + V_t^4 + V_t^5 \\ &= I_t^1 + I_t^2 + I_t^3 - VPa_t\,, \end{aligned}$$

where $VP = \min(P, P^a) = P^a + P^4 = P^1 + P^2 + P^3 + P^4$ is the "valuation premium". Using Formula (2.4), one obtains a recursion relation for V_t^4:

$$V_t^4 + P^4 = \begin{cases} v_t(1-q_t)V_{t+1}^4 & t = 0,\dots,n-1 \\ 0 & t = n-1\,. \end{cases}$$

Example 6 below is Example 5 extended to include the costing components above.

4.6 *Surplus Premium*

As for the deficiency premium, but it may occur that $P - P^a > 0$. In other words, the mathematically arrived at premium is less than that received from the insured. Thus we define the surplus premium

$$P^6 = \max(P - P^a, 0),$$

so,

$$P_x = P^1 + P^2 + P^3 + P^4 + P^5 + P^6$$
$$= VP + P^5 + P^6$$

Example 6

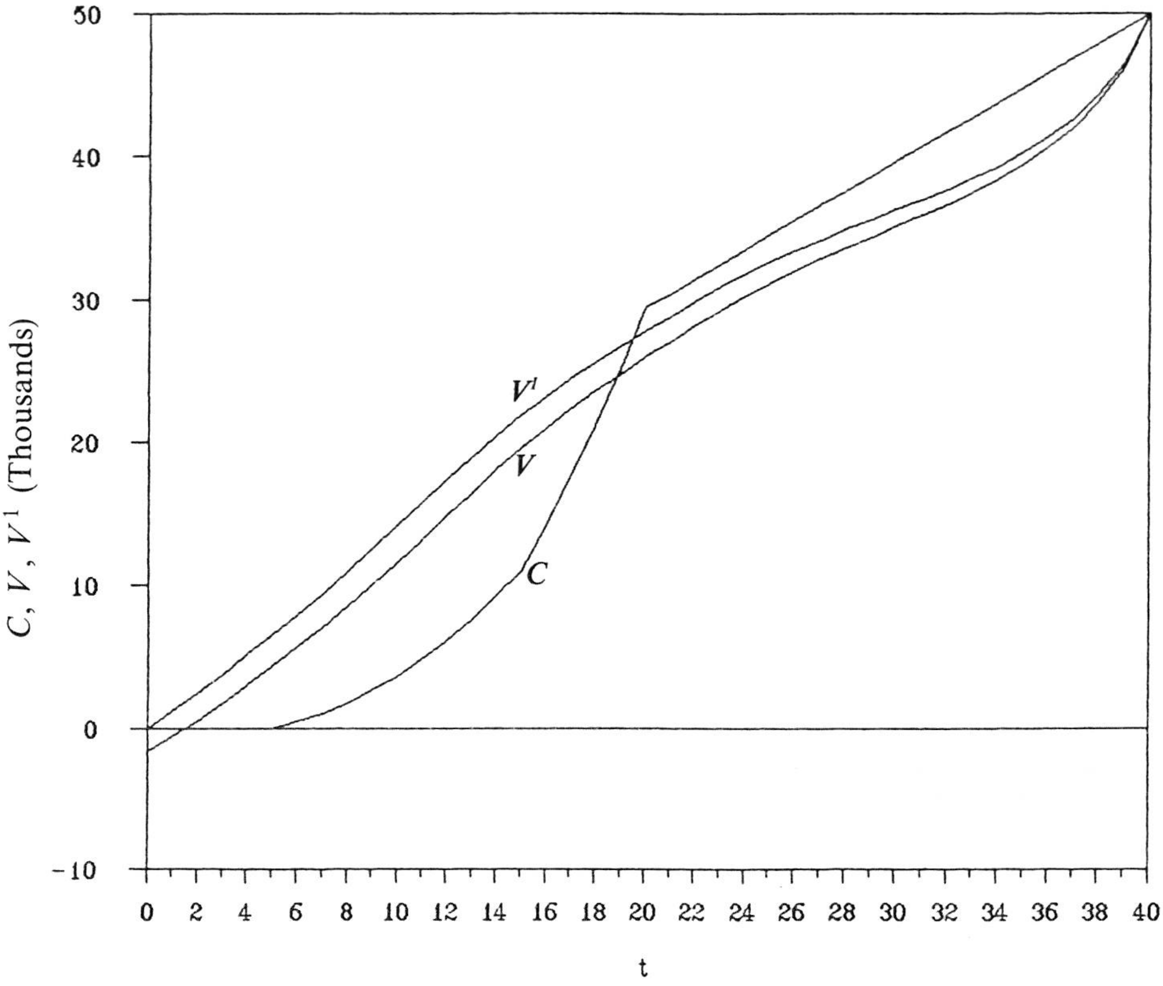
Using the same assumptions for x, n, i, v, q^d, and q^w as in Example 5 above, and the quoted figures for α, α_t, β, γ and S_t where $\alpha = 1.5\,a_0/(a_0 - 1) = 1.76$, we obtain:

P^1	=	1061.01		Basic Benefits
P^2	=	37.30	$\gamma =$ 0.06 % Inflation = 1.03	Administration Inflation
P^3	=	476.62	$\alpha =$ 1.76 $\alpha_t =$ 11.00 %	Acquisition Renewal ($t = 1, \ldots, 9$)
P^4	=	−134.33		Defic. Premium
P^5	=	29.40	$\beta =$ 2.00 %	Tax
P^6	=	0.00		Surplus Premium
P_x	=	1470.00		Gross Premium
P	=	1440.60		Gross Premium (after tax)
VP	=	1440.60		Valuation Premium
S_t	=	50 000		Sum Insured
x	=	60		Entry Age
n	=	40		Duration

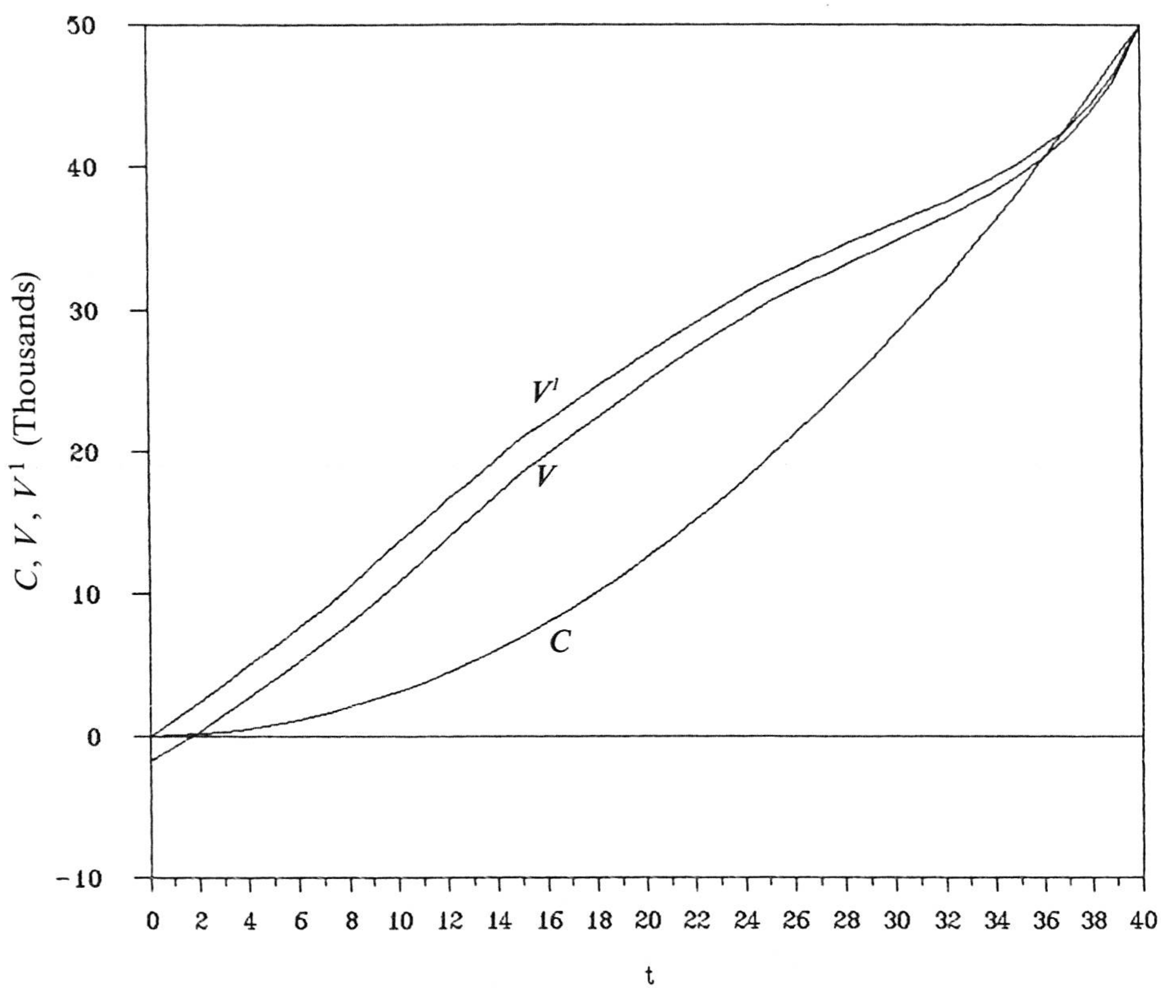
The numbers C_t and the calculated constants are given in the table below:

DUR'N	ANNUITY	BASIC BENEFITS			ADMINISTRATION COSTS			ACQUISITION & RENEWAL			DEFIC.	TOTAL	
t	a_t	C_t	I^1_t	V^1_t	$\gamma(1.03)^t S_t$	I^2_t	V^2_t	$\alpha_t P_x$	I^3_t	V^3_t	V^4_t	V_t	$\max(V_t,0)$
0	6.77	0.00	7183.50	0.00	30.000	252.55	0.00	0.00	639.78	-2587.12	909.45	-1677.67	0.00
1	7.77	0.00	9435.30	1189.84	30.900	299.71	9.83	161.70	861.62	-2842.32	1043.90	-598.76	0.00
2	8.14	0.00	11066.71	2434.81	31.827	322.97	19.50	161.70	840.93	-3036.60	1092.82	510.53	510.53
3	8.36	0.00	12635.13	3763.16	32.782	341.18	29.27	161.70	795.96	-3189.42	1123.21	1726.23	1726.23
4	8.49	0.00	14154.57	5148.18	33.765	355.60	38.96	161.70	731.33	-3314.43	1140.23	3012.94	3012.94
5	8.50	0.00	15564.85	6546.97	34.778	365.27	48.24	161.70	646.52	-3404.40	1141.69	4332.49	4332.49
6	8.45	441.00	16923.83	7963.02	35.822	372.19	57.16	161.70	545.99	-3479.29	1134.46	5675.35	5675.35
7	8.38	1029.00	18340.74	9444.56	36.896	378.80	66.04	161.70	432.76	-3563.49	1126.28	7073.39	7073.39
8	8.32	1764.00	19815.28	10991.54	38.003	385.04	74.83	161.70	305.26	-3658.45	1117.11	8525.02	8525.02
9	8.24	2646.00	21345.95	12602.28	39.143	390.89	83.49	161.70	161.70	-3766.04	1106.97	10026.71	10026.71
10	8.16	3675.00	22926.75	14269.14	40.317	396.38	92.01	0.00	0.00	-3889.08	1096.08	11568.14	11568.14
11	7.95	4851.00	24251.32	15814.05	41.527	395.47	98.84	0.00	0.00	-3790.10	1068.18	13190.98	13190.98
12	7.73	6174.00	25569.34	17365.11	42.773	393.67	105.24	0.00	0.00	-3685.42	1038.68	14823.61	14823.61
13	7.50	7644.00	26865.27	18903.51	44.056	391.11	111.20	0.00	0.00	-3576.50	1007.98	16446.19	16446.19
14	7.27	9261.00	28130.91	20420.24	45.378	387.78	116.70	0.00	0.00	-3463.71	976.19	18049.43	18049.43
15	7.02	11025.00	29356.34	21904.13	46.739	383.73	121.74	0.00	0.00	-3347.61	943.47	19621.73	19621.73
16	6.82	14112.00	30386.01	23152.72	48.141	381.39	127.10	0.00	0.00	-3249.26	915.75	20946.30	20946.30
17	6.60	17493.00	31388.84	24381.09	49.585	378.36	131.99	0.00	0.00	-3147.95	887.20	22252.34	22252.34
18	6.39	21168.00	32353.77	25576.98	51.073	374.66	136.42	0.00	0.00	-3044.20	857.96	23527.16	23527.16
19	6.17	25137.00	33267.13	26724.92	52.605	370.38	140.38	0.00	0.00	-2938.82	828.26	24754.74	24754.74
20	5.94	29400.00	34114.68	27809.18	54.183	365.56	143.88	0.00	0.00	-2832.49	798.29	25918.86	25918.86
21	5.72	30430.00	34934.64	28866.78	55.809	360.26	146.94	0.00	0.00	-2725.74	768.21	27056.18	27056.18
22	5.50	31460.00	35721.42	29890.80	57.483	354.55	149.56	0.00	0.00	-2619.17	738.17	28159.37	28159.37
23	5.27	32490.00	36467.62	30871.95	59.208	348.51	151.79	0.00	0.00	-2513.63	708.43	29218.54	29218.54
24	5.06	33520.00	37163.51	31798.02	60.984	342.31	153.68	0.00	0.00	-2410.23	679.28	30220.75	30220.75
25	4.85	34550.00	37801.44	32660.23	62.813	336.01	155.27	0.00	0.00	-2309.48	650.89	31156.90	31156.90
26	4.64	35580.00	38374.93	33452.28	64.698	329.61	156.55	0.00	0.00	-2211.30	623.22	32020.75	32020.75
27	4.44	36610.00	38921.67	34208.62	66.639	323.32	157.63	0.00	0.00	-2117.15	596.68	32845.79	32845.79
28	4.25	37640.00	39445.85	34935.11	68.638	317.03	158.45	0.00	0.00	-2026.27	571.07	33638.37	33638.37
29	4.07	38670.00	39953.38	35640.10	70.697	310.58	158.94	0.00	0.00	-1937.57	546.07	34407.54	34407.54
30	3.88	39700.00	40451.95	36334.51	72.818	303.69	158.94	0.00	0.00	-1849.60	521.28	35165.13	35165.13
31	3.69	40730.00	40951.46	37032.47	75.002	296.03	158.25	0.00	0.00	-1760.45	496.15	35926.43	35926.43
32	3.50	41760.00	41464.88	37752.62	77.252	287.09	156.58	0.00	0.00	-1667.58	469.98	36711.60	36711.60
33	3.29	42790.00	42007.69	38517.25	79.570	276.26	153.55	0.00	0.00	-1567.94	441.90	37544.76	37544.76
34	3.06	43820.00	42597.46	39351.80	81.957	262.77	148.67	0.00	0.00	-1457.98	410.91	38453.40	38453.40
35	2.80	44850.00	43255.80	40287.61	84.416	245.66	141.31	0.00	0.00	-1333.34	375.78	39471.37	39471.37
36	2.49	45880.00	44013.06	41368.74	86.948	223.57	130.61	0.00	0.00	-1187.85	334.78	40646.27	40646.27
37	2.12	46910.00	44917.76	42665.60	89.557	194.34	115.16	0.00	0.00	-1011.69	285.13	42054.20	42054.20
38	1.65	47940.00	46059.26	44307.72	92.244	154.08	92.50	0.00	0.00	-786.81	221.75	43835.17	43835.17
39	1.00	48970.00	47619.05	46558.04	95.011	95.01	57.71	0.00	0.00	-476.62	134.33	46273.46	46273.46

Graph of Modified Reserves



Graph of Modified Reserves



5 Income Statement

It is traditional to compare the valuation assumptions with those actually experienced over a period of time ("Grundlagen 2. Ordnung"). These may be obtained for a given portfolio from year to year. When they are not yet available, they may be projected into the future and adjusted as the real figures become available. These effective rates may be denoted by a dash on the appropriate symbol, e.g.:

v_t' = effective discount rate

$q_t'^d$ = effective mortality

w_t' = effective lapse rate

Once these figure are determined, one can simulate the income statement for a portfolio from year to year.

Suppose we have a portfolio of l_0 contracts (all with the same entry age x) as described in Section 4.1 to 4.6. Combining the four recursion relations for the V's above yields:

$$\begin{aligned} VP - v_t(q_t^d S_{t+1} + w_t C_{t+1}) - \alpha_t P_x - \gamma(1.03)^t S_0 \\ + V_t = \begin{cases} v_t(1-q_t)V_{t+1} & t = 0, \dots, n-1 \\ 0 & t = n-1 \end{cases} \end{aligned} \tag{5.1}$$

Thus at time point t, one obtains the breakdown

$$VP = P_S + P_R + P_L + P_A + P_K\,, \tag{5.2}$$

where

$$P_{S,t} = v_t V_{t+1} - V_t \qquad \text{(Savings)} \tag{5.3}$$

$$P_{R,t} = v_t q_t^d (S_{t+1} - V_{t+1}) \qquad \text{(Risk)} \tag{5.4}$$

$$P_{L,t} = v_t w_t (C_{t+1} - V_{t+1}) \qquad \text{(Lapses)} \tag{5.5}$$

$$P_{A,t} = \alpha_t P_x \qquad \text{(Acquisition)} \tag{5.6}$$

$$P_{K,t} = \gamma S_0 (1.03)^t \qquad \text{(Administration)} \tag{5.7}$$

and

$$P_x = VP + P^5 + P^6 \tag{5.8}$$

In analysing the portfolio for profit, the only income source consists of the premium payments and investment gains, whereas the outgoings are made up of components representing death benefits, lapse benefits, acquisition and renewal commissions, Administration costs and tax. As is traditional, each year part of the income is invested as actuarial reserves to pay for future benefits. Modified (zillmerized) reserves are used. However, no negative reserves may appear as an asset in the balance sheet, so we define the quantity $V_t' = \max(0, V_t)$, and the premium components:

$$P'_{S,t} = v_t V'_{t+1} - V'_t \tag{5.9}$$

$$P'_{R,t} = v_t q_t^d (S_{t+1} - V'_{t+1}) \tag{5.10}$$

$$P'_{L,t} = v_t w_t (C_{t+1} - V'_{t+1}) \tag{5.11}$$

In order to take the analysis further, an example is introduced.

Example 7

Again we make the same assumptions as in Example 6. The table below gives the pricing assumptions ("Grundlagen 2. Ordnung") that are used:

DUR'N	PRICING ASSUMPTIONS						
t	W'_t	i'_t	v'_t	q'^{d}_t	q'^{w}_t	w'_t	q'_t
0	100.00%	10.0%	90.9%	0.34%	15.00%	14.95%	15.29%
1	90.91%	9.8%	91.1%	0.43%	12.00%	11.95%	12.38%
2	82.80%	9.6%	91.2%	0.54%	9.00%	8.95%	9.49%
3	75.54%	9.4%	91.4%	0.67%	7.00%	6.95%	7.63%
4	69.05%	9.2%	91.6%	0.81%	5.00%	4.96%	5.77%
5	63.23%	9.0%	91.7%	0.96%	4.00%	3.96%	4.92%
6	58.01%	8.8%	91.9%	1.10%	4.00%	3.96%	5.06%
7	53.32%	8.6%	92.1%	1.24%	4.00%	3.95%	5.19%
8	49.10%	8.4%	92.3%	1.40%	4.00%	3.94%	5.34%
9	45.29%	8.2%	92.4%	1.57%	4.00%	3.94%	5.51%
10	41.86%	8.0%	92.6%	1.77%	4.00%	3.93%	5.69%
11	38.76%	7.8%	92.8%	2.00%	4.00%	3.92%	5.92%
12	35.96%	7.6%	92.9%	2.29%	4.00%	3.91%	6.20%
13	33.42%	7.4%	93.1%	2.60%	4.00%	3.90%	6.49%
14	31.11%	7.2%	93.3%	2.93%	4.00%	3.88%	6.82%
15	29.02%	7.0%	93.5%	3.76%	4.00%	3.85%	7.61%
16	27.13%	7.0%	93.5%	4.02%	4.00%	3.84%	7.86%
17	25.35%	7.0%	93.5%	4.31%	4.00%	3.83%	8.14%
18	23.69%	7.0%	93.5%	4.64%	4.00%	3.81%	8.46%
19	22.14%	7.0%	93.5%	5.01%	4.00%	3.80%	8.81%
20	20.69%	7.0%	93.5%	5.41%	4.00%	3.78%	9.19%
21	19.34%	7.0%	93.5%	5.85%	4.00%	3.77%	9.62%
22	18.07%	7.0%	93.5%	6.34%	4.00%	3.75%	10.09%
23	16.89%	7.0%	93.5%	6.88%	4.00%	3.72%	10.61%
24	15.79%	7.0%	93.5%	7.47%	4.00%	3.70%	11.17%
25	14.75%	7.0%	93.5%	8.07%	4.00%	3.68%	11.75%
26	13.79%	7.0%	93.5%	8.69%	4.00%	3.65%	12.34%
27	12.89%	7.0%	93.5%	9.30%	4.00%	3.63%	12.93%
28	12.04%	7.0%	93.5%	9.92%	4.00%	3.60%	13.52%
29	11.26%	7.0%	93.5%	10.52%	4.00%	3.58%	14.10%
30	10.52%	7.0%	93.5%	11.12%	4.00%	3.56%	14.68%
31	9.83%	7.0%	93.5%	11.71%	4.00%	3.53%	15.24%
32	9.19%	7.0%	93.5%	12.30%	4.00%	3.51%	15.81%
33	8.59%	7.0%	93.5%	12.94%	4.00%	3.48%	16.42%
34	8.03%	7.0%	93.5%	13.65%	4.00%	3.45%	17.11%
35	7.50%	7.0%	93.5%	14.51%	4.00%	3.42%	17.93%
36	7.01%	7.0%	93.5%	15.57%	4.00%	3.38%	18.95%
37	6.55%	7.0%	93.5%	16.92%	4.00%	3.32%	20.24%
38	6.12%	7.0%	93.5%	18.65%	4.00%	3.25%	21.90%
39	5.72%	7.0%	93.5%	20.86%	1.00	79.14%	1.00
40	5.35%						

Income Statement for end of year $[t, t+1[$, $t = 0, 1, \ldots, n-1$

		0=(1+6+10 14+15+16)*vt	SAVINGS 1	2	3= -8-12	4	5=1+2+3+4
t+1	l't	Premium	Savings Premium	Invs. Income	Released Reserves	Alloc. of Reserves	Interest Win
PRPG 1)							9.29%
PRESENT VALUE 2)		80561.70					7483.87
FUTURE VALUE 3)		1506482.86					139946.49
1	10.0000	14406.00	0	178	0	0	177.52
2	8.4708	12202.98	4325	466	-535	-3789	465.86
3	7.4222	10692.49	8697	1051	-1216	-7807	724.90
4	6.7179	9677.77	7670	1652	-1543	-7101	678.34
5	6.2056	8939.83	6655	2200	-1552	-6636	666.47
6	5.8473	8423.63	5825	2705	-1633	-6219	678.71
7	5.5596	8009.13	5311	3161	-1988	-5784	699.75
8	5.2785	7604.23	4825	3557	-2337	-5325	719.43
9	5.0044	7209.27	4358	3894	-2679	-4836	737.39
10	4.7371	6824.32	3882	4173	-3017	-4285	753.19
11	4.4763	6448.59	3640	4423	-3363	-3902	798.41
12	4.2214	6081.35	3105	4593	-3707	-3185	806.16
13	3.9713	5721.06	2558	4695	-4048	-2395	809.15
14	3.7251	5366.45	2051	4727	-4366	-1606	806.44
15	3.4832	5017.96	1579	4696	-4659	-818	797.99
16	3.2458	4675.91	478	4605	-5175	876	783.73
17	2.9987	4319.98	211	4535	-5245	1329	828.68
18	2.7630	3980.40	-44	4431	-5291	1769	865.24
19	2.5381	3656.42	-288	4296	-5313	2197	892.66
20	2.3235	3347.24	-516	4131	-5304	2600	910.15
21	2.1188	3052.41	-611	3938	-5271	2861	917.14
22	1.9240	2771.77	-689	3726	-5211	3088	914.94
23	1.7390	2505.19	-753	3499	-5125	3283	903.58
24	1.5636	2252.51	-809	3259	-5013	3446	883.14
25	1.3977	2013.55	-846	3008	-4863	3555	853.77
26	1.2416	1788.69	-862	2750	-4671	3598	816.19
27	1.0958	1578.55	-850	2489	-4441	3537	734.78
28	0.9606	1383.77	-816	2234	-4179	3417	656.36
29	0.8363	1204.82	-763	1988	-3891	3248	581.85
30	0.7232	1041.91	-696	1756	-3587	3039	512.03
31	0.6212	894.96	-619	1540	-3276	2803	447.48
32	0.5301	763.62	-536	1341	-2966	2550	388.54
33	0.4493	647.24	-450	1160	-2667	2293	335.38
34	0.3782	544.91	-366	998	-2388	2044	287.92
35	0.3161	455.44	-286	854	-2135	1813	245.86
36	0.2621	377.54	-209	726	-1909	1602	208.75
37	0.2151	309.86	-134	613	-1714	1411	176.11
38	0.1743	251.16	-56	514	-1547	1237	147.46
39	0.1391	200.32	34	427	-1409	1070	122.43
40	0.1086	156.44	153	352	-5430	5025	100.82
	0.0000	0.00					

1) PRPG = Present value of profit as a percentage of the present value of premium at the pricing rate of interest
2) Present Value of column at pricing rate of interest 3) Future Value of column at pricing rate of interest

RISK				LAPSE			
6	7	8	9=6+7+8	10	11	12	13=10+11+12
Risk Premium	Death Benefits	Released Reserves	Mortality Win	Lapse Premium	CV Paid	Released Reserves	Lapse Win
			8.02%				3.82%
			6463.36				3080.23
			120863.20				57599.43
1935	-1720	0	215.00	0	0	0	0.00
2046	-1819	19	245.37	-387	0	517	129.41
2214	-1998	69	284.82	-859	0	1147	287.42
2459	-2257	136	337.50	-1054	0	1407	352.97
2704	-2529	219	394.17	-999	0	1333	334.74
2939	-2804	318	453.36	-908	-102	1315	304.74
3134	-3056	432	509.59	-995	-226	1556	334.50
3284	-3282	560	561.28	-1055	-368	1778	355.19
3401	-3492	700	609.15	-1089	-522	1979	367.58
3514	-3715	860	657.95	-1100	-685	2158	372.10
3615	-3951	1042	706.02	-548	-853	2320	919.23
3736	-4231	1254	759.43	-534	-1022	2453	897.34
3868	-4547	1496	816.82	-509	-1186	2553	857.11
3959	-4838	1747	866.81	-475	-1344	2620	800.77
4015	-5110	2005	910.11	-432	-1491	2654	730.51
4634	-6106	2558	1086.01	-316	-1763	2617	537.54
4414	-6030	2684	1067.95	-203	-2014	2562	345.20
4202	-5957	2803	1048.20	-92	-2239	2488	157.33
4002	-5890	2916	1028.05	14	-2434	2397	-23.36
3809	-5818	3016	1006.37	113	-2596	2288	-194.27
3611	-5732	3102	980.33	99	-2440	2169	-171.21
3409	-5629	3170	950.26	88	-2280	2040	-151.63
3208	-5513	3221	917.19	78	-2117	1904	-135.37
3011	-5382	3253	882.05	70	-1952	1760	-122.30
2809	-5218	3251	842.50	64	-1787	1612	-111.99
2599	-5010	3209	797.67	59	-1624	1462	-103.93
2379	-4759	3126	746.47	54	-1465	1315	-96.62
2152	-4469	3006	689.84	50	-1312	1172	-89.68
1923	-4147	2854	629.17	46	-1165	1037	-82.86
1695	-3806	2677	566.06	41	-1028	910	-75.96
1474	-3454	2482	502.04	37	-900	793	-68.88
1263	-3103	2279	438.35	33	-782	687	-61.56
1065	-2764	2075	376.45	29	-674	592	-54.04
883	-2446	1881	317.67	24	-577	507	-46.40
717	-2158	1704	262.60	20	-490	431	-38.72
567	-1901	1545	211.10	16	-411	364	-31.09
428	-1674	1408	162.20	12	-341	305	-23.53
296	-1475	1293	113.78	8	-278	254	-16.00
159	-1297	1200	62.04	4	-222	209	-8.30
0	-1133	1133	0.00	0	-4297	4297	0.00

14	15	16	17	18	19	20	21
COST INCOME					COST OUTGOINGS		
Acq. Premium	Unallocated Premium	Admin. Premium	Tax Premium	Surpl. Premium	1st yr Acq Ren Comm	Admin. Costs	Tax
0	13564	327	320	0	28200	327	320
1490	5569	285	271	0	1490	285	271
1303	0	257	237	0	1303	257	237
1178	0	239	214	0	1178	239	214
1086	0	227	197	0	1086	227	197
1021	0	220	186	0	1021	220	186
969	0	215	176	0	969	215	176
918	0	210	167	0	918	210	167
869	0	204	158	0	869	204	158
821	0	199	149	0	821	199	149
0	0	193	141	0	0	193	141
0	0	187	133	0	0	187	133
0	0	181	124	0	0	181	124
0	0	175	117	0	0	175	117
0	0	168	109	0	0	168	109
0	0	161	101	0	0	161	101
0	0	153	93	0	0	153	93
0	0	145	86	0	0	145	86
0	0	137	79	0	0	137	79
0	0	129	72	0	0	129	72
0	0	121	66	0	0	121	66
0	0	113	60	0	0	113	60
0	0	105	54	0	0	105	54
0	0	97	48	0	0	97	48
0	0	90	43	0	0	90	43
0	0	82	38	0	0	82	38
0	0	74	34	0	0	74	34
0	0	67	30	0	0	67	30
0	0	60	26	0	0	60	26
0	0	54	22	0	0	54	22
0	0	47	19	0	0	47	19
0	0	42	16	0	0	42	16
0	0	36	14	0	0	36	14
0	0	32	12	0	0	32	12
0	0	27	10	0	0	27	10
0	0	23	8	0	0	23	8
0	0	20	7	0	0	20	7
0	0	16	5	0	0	16	5
0	0	13	4	0	0	13	4
0	0	11	3	0	0	11	3

22=	NET COST			26=22+	27=5+	RESERVES	
14+15-19	23=16-20	24=17-21	25=18	23+24+25	9+13+26	28=4	29
Acquis.	Admin.	Tax	Surpl. Premium	TOTAL COST	TOTAL	Change Reserve	Accuml. Reserve
				-10.79%	10.34%		
				-8694.40	8333.06		
				-162582.96	155826.16		
							0.00
-14635.76	0.00	0.00	0.00	-14635.76	-14243.24	0.00	0.00
5568.97	0.00	0.00	0.00	5568.97	6409.61	3789.32	3789.32
0.00	0.00	0.00	0.00	0.00	1297.14	7807.27	11596.59
0.00	0.00	0.00	0.00	0.00	1368.80	7100.61	18697.20
0.00	0.00	0.00	0.00	0.00	1395.38	6636.19	25333.39
0.00	0.00	0.00	0.00	0.00	1436.82	6219.14	31552.54
0.00	0.00	0.00	0.00	0.00	1543.84	5784.44	37336.97
0.00	0.00	0.00	0.00	0.00	1635.89	5325.26	42662.23
0.00	0.00	0.00	0.00	0.00	1714.12	4835.67	47497.91
0.00	0.00	0.00	0.00	0.00	1783.24	4284.78	51782.69
0.00	0.00	0.00	0.00	0.00	2423.66	3901.67	55684.36
0.00	0.00	0.00	0.00	0.00	2462.93	3184.74	58869.09
0.00	0.00	0.00	0.00	0.00	2483.07	2395.37	61264.47
0.00	0.00	0.00	0.00	0.00	2474.02	1606.09	62870.55
0.00	0.00	0.00	0.00	0.00	2438.61	817.77	63688.32
0.00	0.00	0.00	0.00	0.00	2407.28	-875.82	62812.50
0.00	0.00	0.00	0.00	0.00	2241.83	-1328.88	61483.62
0.00	0.00	0.00	0.00	0.00	2070.77	-1768.79	59714.83
0.00	0.00	0.00	0.00	0.00	1897.35	-2197.14	57517.69
0.00	0.00	0.00	0.00	0.00	1722.26	-2599.58	54918.11
0.00	0.00	0.00	0.00	0.00	1726.26	-2860.94	52057.17
0.00	0.00	0.00	0.00	0.00	1713.57	-3088.23	48968.94
0.00	0.00	0.00	0.00	0.00	1685.39	-3283.11	45685.84
0.00	0.00	0.00	0.00	0.00	1642.89	-3445.81	42240.03
0.00	0.00	0.00	0.00	0.00	1584.28	-3554.80	38685.23
0.00	0.00	0.00	0.00	0.00	1509.92	-3598.15	35087.07
0.00	0.00	0.00	0.00	0.00	1384.64	-3537.00	31550.08
0.00	0.00	0.00	0.00	0.00	1256.51	-3417.35	28132.73
0.00	0.00	0.00	0.00	0.00	1128.15	-3247.67	24885.06
0.00	0.00	0.00	0.00	0.00	1002.14	-3038.92	21846.13
0.00	0.00	0.00	0.00	0.00	880.64	-2802.62	19043.51
0.00	0.00	0.00	0.00	0.00	765.33	-2549.55	16493.96
0.00	0.00	0.00	0.00	0.00	657.79	-2292.66	14201.30
0.00	0.00	0.00	0.00	0.00	559.19	-2044.33	12156.97
0.00	0.00	0.00	0.00	0.00	469.74	-1812.75	10344.22
0.00	0.00	0.00	0.00	0.00	388.76	-1601.51	8742.71
0.00	0.00	0.00	0.00	0.00	314.78	-1410.90	7331.81
0.00	0.00	0.00	0.00	0.00	245.24	-1236.50	6095.31
0.00	0.00	0.00	0.00	0.00	176.17	-1070.35	5024.96
0.00	0.00	0.00	0.00	0.00	100.82	-5024.96	0.00

where the constants i'_t, q'^d_t and q'^w_t are as given in the table on page 113. They are calculated as follows

i'_t is as given $\times(1+$ Variation Interest)
$q'^d_t = q^d_t \times$ Mortality Factor
$\times(1+$ Mortality Improvement$)^t$,
$\times(1+$ Variation Mortality)
q'^w_t is as given times $(1 \pm$ Variation Lapses),

Variation Interest	$= 0\,\%$
Mortality Factor	$= 0.8\,/\,0.9$
Mortality Improvement	$= 1\,\%$
Variation Mortality	$= 0\,\%$
Variation Lapses	$= 0\,\%$

where the sign is "+" for the first year and "−" for the second and subsequent years. The calculation of the reserves V^1 to V^4 is identical to that in Example 6. Reference is made to the income statement above. The following is a breakdown of the various items in the statement.

5.1 *Premium Income*

Premium income can be broken down into the basic source components as follows:

Columns [1]: $l'_t P'_{S,t}/v_t$
[6]: $l'_t P'_{R,t}/v_t$
[10]: $l'_t P'_{L,t}/v_t$
[14]: $l'_t P_{A,t}/v_t$
[15]: $l'_t (VP - P'_{S,t} - P'_{R,t} - P'_{L,t} - P_{A,t} - P_{K,t})/v'_t$
[16]: $l'_t P_{K,t}/v_t$
[17]: $l'_t P^5/v_t$
[18]: $l'_t P^6/v_t$

Note:

(I) Column [0]: $l'_t VP$ is for reference only, it is not part of the income statement.

(II) Incoming item $P^6 l'_t/v'_t$ (Column [18]), when positive, is pure profit and helps offset costs. Interest gains (at the pricing rate) are allocated directly, and not as investment income.

(III) As soon as $V_t = V_t'$, item [15] equals zero because of Formula 5.2. Whilst it is positive, it is used to amortize acquisition costs.

5.2 *Investment Income* (Column [2])

In accordance with the theory, technical (valuation) interest is allocated directly to Columns [1, 6, 10]. Since Columns [14, 16, 17] are considered as income sources which are not only earned, but also spent at the beginning of the year, they are only allocated interest at the valuation rate $(1/v_t - 1)$. Similarly for the corresponding debits – Columns [19, 20, 21].
The remainder,

$$\text{Investment Income } = l_t'[(P_{S,t}' + P_{R,t}' + P_{L,t}')(i_t' - i_t) + AR_t i_t']$$

is allocated to column [2]. Here AR_t (Column [29]) is the accumulated reserve at point t after the end of year reserve allocation (Columns [4] and [28]).

5.3 *Outgoings*

Allocation of reserves	[4]:	$l_{t+1}' V_{t+1}' - l_t' V_t'$
Death Benefits	[7]:	$l_t' q_t'^d S_{t+1}$
Lapse Benefits	[11]:	$l_t' w_t' C_{t+1}$
1st year Acquisition	[19], $t = 0$:	$l_t' \alpha P_x / v_t$
Renewal Commission	[19], $t > 0$:	$l_t' \alpha_t P_x / v_t$
Administration Costs	[20]:	$l_t' P_{K,t} / v_t$
Tax	[21]:	$l_t' P^5 / v_t$

5.4 *Accumulated Reserves*

These are calculated as the accumulated reserves from the previous year plus the end of year reserve allocation. No interest gain is calculated as this appears in Column [2].

Note that

$$\begin{aligned} AR_t = {} & l'_1\, V'_1 - 0 + \\ & l'_2\, V'_2 - l'_1 V'_1 + \cdots + \\ & l'_t\, V'_t - l'_{t-1}\, V'_{t-1} \\ = {} & l'_t\, V'_t \end{aligned}$$

5.5 *Released Reserves*

Columns [8, 12] are offset against Column [3]. This is done because the death claims, a sum of the value $l'_t q'^d_t S_{t+1}$ are met in part by $l'_t q'^d_t (S_{t+1} - V'_{t+1})$, the sum covering the risk, and the rest using the reserves $[8] = l'_t q'^d_t V'_{t+1}$ (allocated to these policies) which are released. Similarly for the lapse rates. The result is

$$\text{Mortality Gain} = [9] = l'_t(q^d_t - q'^d_t)(S_{t+1} - V'_{t+1}) \quad (5.12)$$

$$\text{Lapse Gain} = [13] = l'_t(w_t - w'_t)(C_{t+1} - V'_{t+1}) \quad (5.13)$$

5.6 *Interest Gain*

$$\begin{aligned} \text{Interest Gain} & = [5] = [1+2+3+4] \\ & = l'_t\left(V'_{t+1} - \frac{V'_t}{v_t}\right) + \\ & \quad l'_t\left[(P_{S,t} + P_{R,t} + P_{L,t})(i'_t - i_t) + V'_t(i'_t - i_t) + V'_t\left(\frac{1}{v_t} - 1\right)\right] \\ & \quad - l'_t(q'^d_t + w'_t)V'_{t+1} + l'_t V'_t - l'_{t+1} V'_{t+1} \\ & = l'_t(P'_{S,t} + P'_{R,t} + P'_{L,t} + V'_t)(i'_t - i_t) \end{aligned} \quad (5.14)$$

5.7 Sensitivity Analysis

The constant PRPG = Present value of profit as a percentage of the present value of premium at the pricing rate of interest, is a useful indication of the expected success of a contract. Changing the 2[nd] order assumptions (pricing assumptions) by the amounts indicated has the indicated effect on the PRPG values. This can be used to test the contract against changes in assumptions.

PV of Premium = 80 561.70

PV	Total	Int	Mort	Lapse	Cost
Var. Interest					
2.0 %	16 581	17 200	5 061	2 639	–8 319
0.0 %	8 333	7 484	6 463	3 080	–8 694
–2.0 %	–4 077	–7 083	8 485	3 604	–9 084
Var. Mortality					
5.0 %	6 842	7 390	5 079	3 069	–8 695
0.0 %	8 333	7 484	6 463	3 080	–8 694
–5.0 %	9 853	7 581	7 874	3 091	–8 694
Var. Lapses					
–20.0 %	8 695	6 786	5 844	4 597	–8 532
0.0 %	8 333	7 484	6 463	3 080	–8 694
20.0 %	8 000	8 284	7 162	1 412	–8 857

PRPG	Total	Int	Mort	Lapse	Cost
Var. Int					
2.0 %	23.2 %	24.0 %	7.1 %	3.7 %	–11.6 %
0.0 %	10.3 %	9.3 %	8.0 %	3.8 %	–10.8 %
–2.0 %	–4.4 %	–7.7 %	9.2 %	3.9 %	–9.9 %
Var. Mor					
5.0 %	8.5 %	9.2 %	6.3 %	3.8 %	–10.8 %
0.0 %	10.3 %	9.3 %	8.0 %	3.8 %	–10.8 %
–5.0 %	12.2 %	9.4 %	9.7 %	3.8 %	–10.7 %
Var. Lap					
–20.0 %	11.2 %	8.7 %	7.5 %	5.9 %	–11.0 %
0.0 %	10.3 %	9.3 %	8.0 %	3.8 %	–10.8 %
20.0 %	9.6 %	9.9 %	8.6 %	1.7 %	–10.6 %

6 Conclusion

The main points to observe in conclusion to this paper are:

(I) The capital reserves required for an insurance which includes lapse rates as part of its premium calculation do not deviate considerably from those required for the equivalent whole life insurance (without lapse rates), even when the cash values deviate considerably from the reserves of a whole life insurance. See Example 6.

(II) Assuming one arranges matters so that in the most part the cash values C_t are less than V_t^g, then one makes a lapse profit from this contract provided $w_t' \geq w_t$ all t. This holds true in general except in the first few years where the lack of premium income due to early lapses fails to recover acquisition costs. Thus, one must be very careful in determining first and second order lapse rates.

(III) Making allowances for "mortality improvement indexation" (see Example 7, "Mortality Improvement = 1 %"), an inflation factor for the administration costs or renewal commissions (see Section 4.2) does not pose a great difficulty in the mathematics and formula apparatus.

(IV) The use of deficiency premiums and deficiency reserves as part of the calculations pose the problem of not knowing where the loss due to these reserves in the income statement is to be allocated. Should it be thought of as a discount toward the mortality, lapse, or cost premium?

(V) Having the breakdown of the gross premium into its various components representing basic benefits, risk, lapse and cost (see 5.2), allows for the possibility of administering a portfolio with this in mind. In other words, one may endeavour to keep an eye on the cashflow for an existing portfolio and determine in detail where exactly income and outgoings are to be allocated.

(VI) When setting up an account structure for analysing yearly profit (see income statement Example 7), great care has to be exercised in allocating the income, interest and cost components. One has to distinguish carefully between technical (valuation) interest yields and those that arise out of the interest spread $(i' - i)$. The former are part of the guaranteed technical income and the latter represents profits over and above those calculated.

(VII) If P^2 and/or P^5, P^6 are somewhat smaller than P^4, as is the case in Example 7, then the whole analysis makes little sense, since a considerable amount of the cost $V^4 = -P^4 a_0$ is unaccounted for.

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Summary

Unlike some industries where certain products can be marketed unchanged world-wide, in the insurance sector, cultural, legal and historical considerations mostly prevent such a happy state of affairs. Even at the pure actuarial level, the formulas used and methodologies practiced, vary from country to country. This paper is an attempt to bridge some of the differences that exist between Europe and Canada. The Canadian actuarial reader should recognize and be familiar with many of the ideas and concepts used when designing whole life or term products. He will probably find of interest how the premiums and capital reserves can be split when attempting a "gain by source" analysis as well as the effect of different assumptions on interest-, lapse- or mortality-gains (resp. losses). Whereas to the European reader, the idea of including a non-constant interest rate function, renewal commissions, administration costs that respect inflation, deficiency reserves, including lapse rate assumptions into the mathematical model, and doing a sensitivity analysis is still somewhat of a novelty in the workplace. It is certainly interesting to see that the formulas turn out to be one and the same regardless of one's point of view. They represent a generalization of the traditional 'Kontributionsformel' as found in *K. Wolff* (Springer 1970). A simplified mathematical notation is used to allow for the representation of the slightly greater complexity. Finally I would like to allude to the possibility of starting with the recursion formula (5.1) and breakdown (5.2) to obtain a parametarized model of a flexible product which under the right boundary conditions yields the traditional whole life (respectively term life, etc.). The ideal situation would be to put all product types under one umbrella distinguishing between the individual products only in terms of the parameters used and its presentation in the marketplace. This would simplify the administration of insurance products enormously.

Zusammenfassung

Anders als bei Industriezweigen, in denen gewisse Produkte weltweit unverändert auf den Markt gebracht werden können, wird ein solch idealer Zustand bei der Versicherungsindustrie durch kulturelle, rechtliche und historische Überlegungen verunmöglicht. Sogar auf rein versicherungsmathematischem Niveau variieren die angewandten Formeln und Methoden von Land zu Land. Die vorliegende Abhandlung ist ein Versuch, einige dieser zwischen Europa und Kanada existierenden Unterschiede zu überbrücken. Der kanadische Leser kennt und erkennt viele der Ideen und Konzepte, die bei der Schaffung von gemischten Lebenprodukten und Todesfallversicherungen verwendet werden. Für ihn ist es wahrscheinlich interessant zu sehen, auf welche Art Prämien und technische Reserven aufgeteilt werden können, wenn eine Gewinnanalyse gemacht wird und welche Folgen die verschiedenen Annahmen über Zins-, Storno- und Sterblichkeitsgewinne (bzw. -verluste) haben. Dagegen ist für den europäischen Leser das Einbeziehen von nicht-konstanten Zinssätzen, Inkassoprovisionen, Verwaltungskosten, welche die Inflation berücksichtigen, «Deficiency Reserves» und das Einschliessen von Stornoannahmen ins mathematische Modell sowie die Durchführung einer «Sensitivity»-Analyse in der Praxis immer noch eine Neuheit. Es ist sicher interessant zu erkennen, dass die verschiedenen Formeln eigentlich ein und dieselbe sind, ganz unabhängig, von welchem Standpunkt aus die Sache betrachtet wird. Diese Formeln sind eigentlich eine Verallgemeinerung der traditionellen Kontributionsformel, wie sie bei *K. Wolff* (Springer 1970) dargestellt wird. Eine vereinfachte mathematische Schreibweise

wurde hier verwendet, um der etwas grösseren Komplexität entgegenzukommen. Schliesslich möchte ich noch auf die Möglichkeit hinweisen, dass mit der rekursiven Formel (5.1) und Aufteilung (5.2) begonnen werden kann, um das parametrische Modell eines flexiblen Produktes zu erhalten, welches unter den richtigen Rahmenbedingungen eine gemischte Lebensversicherung (bzw. Todesfallversicherung) hervorbringt. Im Idealfall würden alle Produktetypen unter einem Dach vereint, wobei zwischen den einzelnen Produkten nur in bezug auf die verwendeten Parameter und auf die Präsentation im Markt ein Unterschied gemacht würde. Dies würde die Verwaltung von Versicherungsprodukten enorm vereinfachen.

Résumé

Contrairement à certaines industries dont les produits peuvent être vendus inchangés à travers le monde entier, dans le secteur de l'assurance des considérations culturelles, légales et historiques empêchent un état de choses aussi désirable. Même au niveau purement actuariel, les formules et les méthodologies utilisées varient d'un pays à l'autre. Le présent article tente de faire le lien entre les différences qui existent entre l'Europe et le Canada. Le lecteur Canadien de formation actuarielle devrait être familiarisé avec plusieurs des idées et des concepts utilisés pour le design de produits d'assurance vie-entière ou temporaire. Il trouvera probablement intéressant de voir comment les primes et les réserves techniques peuvent être décomposées lors d'une analyse du gain par source et l'effet de différentes hypothèses au niveau de l'intérêt, des taux de chute ou de l'amélioration (ou bien détérioration) de la mortalité. En revanche, pour le lecteur Européen, l'idée d'utiliser un taux d'intérêt non-constant, d'inclure des commissions de renouvellement, des frais d'administration qui suivent l'inflation, des «deficiency reserves», d'inclure des hypothèses de taux de chute dans le modèle mathématique, et de faire une analyse de sensibilité, constitue encore une nouveauté pour la pratique. Il est certainement intéressant de voir que les formules s'avèrent être les mêmes quel que soit le point de vue adopté. Elles représentent une généralisation de la formale de contribution traditionelle telle qu'on la retrouve dans *K. Wolff*, (Springer 1970). Un modèle mathématique simplifié est utilisé pour permettre la représentation de cette complexité légèrement accrue. Finalement, je voudrais faire référence à la possibilité de partir de la formule récursive (5.1) et de la décomposition (5.2) afin d'obtenir un modèle paramétrique pour un produit flexible lequel, sous des conditions limites adéquates, engendre l'assurance vie-entière traditionnelle (ou bien temporaire, etc.). La situation idéale serait de mettre tous les types de produits à la même enseigne et de ne faire la distinction entre les produits individuels qu'en termes de leurs paramètres et de leur présentation sur le marché. Cela simplifierait énormément l'administration des produits d'assurance.