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On Optimal Financing of Pension Programs and Aaron's Rule

1 Introduction

The question of optimal financing of public pension schemes was primarily raised in a controversy between *Samuelson* [1958] and *Lerner* [1959]. In a following classic note *Aaron* [1966] observes that for *the individual* the choice between an investment fund and a pay-as-you-go system is determined by a comparison of the capital value of the actual pension at retirement with the capital value of the pension that would be received had the contributions been invested. The two capital values will be the same if the rate of growth of the labour force plus the rate of growth of average earnings equals the rate of interest. The investment fund method becomes preferable when the rate of growth of total earnings is less than the interest rate. On the other hand as long as the rate of interest is less than the rate of growth of earnings, the pay-as-you-go method is preferred. This condition is referred to as Aaron's rule. In the present note we show that Aaron's rule has no general validity. For this purpose in the following section the neo-classical model with an embedded national pension scheme is introduced. The third section deals with the question of least contributions.

2 An old-age pension in a neo-classical economy

In this section we will embody an old-age pension plan in a simple macro-economic framework. The basic neo-classical assumption is that there is continuous substitution between capital, K and labour, L. A well-behaved Cobb-Douglas production function is chosen. The neo-classical model is characterized by three equilibrium conditions in respect of full capacity, full employment and equality of saving and investment[1].

In the present study personal saving in year t comes from the public and from the pension fund, denoted by $S(t,1)$ and $S(t,2)$ respectively. Enterprises do not save and government is absent. We postulate a simple linear saving

[1] It will be clear that e.g. due to considerable unemployment rates the neo-classical model can nowadays hardly be accepted for most western countries.

function of the households. Their saving depends on disposable income and the saving propensity σ, yielding the following private saving function[2],

$$S(t,1) = \sigma[Y(t) - S(t,2)] \tag{1}$$

where $Y(t)$ is national product. The savings of the pension fund in year t are equal to the amount of contributions, $B(t)$ plus interest revenues, $R(t,2)$ minus pension payments, $U(t)$.
There is labour-augmenting technical progress at a given Harrod-neutral rate g. As a consequence the labour force can be measured in efficiency units $\bar{L}(t) = L(t)e^{gt}$. The population is given as growing at a constant rate n.
It should be observed that the pension plan under study is not necessarily entirely funded, but can be partly based on the investment fund method and partly on the pay-as-you-go method. The corresponding contribution rates as a fraction of worker's income are δ and ξ respectively and their sum is κ. The resulting pensions per retired from these two sources amount to fractions θ and η of current worker's income. Total pension income as a fraction of worker's income, γ is exogenously determined by pension regulations, forming part of labour-conditions. The neo-classical model with an embodied pension plan can be reduced to the following set of equations. Further discussion follows below.

$$Y(t) = K^{\mu}(t) \cdot \bar{L}^{(1-\mu)}(t); \qquad 0 < \mu < 1 \tag{2}$$

$$DK(t) = S(t,1) + S(t,2); \qquad DK(t) = \dot{K}(t), \quad K(0) \text{ given} \tag{3}$$

$$\bar{L}(t) = L(0)e^{(g+n)t}; \qquad L(0) \text{ given} \tag{4}$$

$$S(t,1) = \sigma[Y(t) - S(t,2)] \tag{1}$$

[2] Feldstein [1974] has argued that the introduction and growth of a pension system may depress direct saving of the households. Individuals view current contributions or anticipated pension benefits as a substitute for their own preretirement saving and thus are less motivated to save during their working years. The saving function of the households can clearly be modified in several ways, e.g. different proportions saved out of wage income and profits, out of income of the working population and income of the retired and saving out of capital gain may be considered. Clearly, due attention should be given to all the factors at work. This is not pursued here.

$$S(t,2) = B(t) - U(t) + R(t,2) \tag{5}$$

$$B(t) = \kappa \cdot W(t) \tag{6}$$

$$U(t) = \gamma \cdot W(t) \cdot [L^R(t)/L^W(t)] \tag{7}$$

$$W(t) = (1-\mu) \cdot Y(t) \tag{8}$$

$$R(t,2) = \pi \cdot R(t) \tag{9}$$

$$R(t) = \mu \cdot Y(t) \tag{10}$$

$$\pi = K(t,2)/K(t) \tag{11}$$

$$\gamma = \theta + \eta \tag{12}$$

$$\gamma = \gamma^* \tag{13}$$

$$\kappa = \delta + \xi \tag{14}$$

The equations (2), (3) and (4) are the three equilibrium conditions in respect of full capacity, the equality of saving and investment and full employment (see e.g. *Allen* [1970], *Wan* [1971]). The saving function of the households and of the pension fund are found in (1) and (5). When it can be assumed that production is based on profit maximization under perfect competition on both product and input markets, the profit rate and the wage rate at any time t are equal to the corresponding marginal products. Hence equations (8) and (10) follow, where $W(t)$ are total wages and $R(t)$ national interest revenues. Clearly their sum equals national product. As observed the pension plan considered in this paper is partly based on the capital reserve method and partly on the pay-as-you-go method. Contributions $B(t)$ amount to a fraction κ of wages (eq.(6)). Clearly, total pension payments are equal to the exogenously given ratio γ^* (eq.(13)) of wages, multiplied by the pensioner ratio, i.e. the ratio of those 65 and over, the retired (L^R) to those belonging to the active labour force, 16–64 years of age (L^W). This is shown in equation (7).
The national capital stock, like national saving, is composed of two parts, viz. the amount of capital owned by the public, $K(t,1)$ and the amount of capital owned by the pension fund, $K(t,2)$. The parameter π is introduced in equation (11). Following Van Praag & Poeth [1975] π is called the socialization ratio. Similarly the interest revenues of the pension fund are a fraction π of total interest revenues (eq.(9)).

It is a well-known characteristic of the equilibrium state of the neo-classical economy that both K and L are growing at the rate $g+n$. Since the production function used (eq.(2)) is linear and homogeneous, national product, Y, also grows at the rate $g+n$. The steady state paths follow from the model, provided that there are "right" initial values, viz.

$$K(0) = \left(\frac{s}{g+n}\right)^{\frac{1}{1-\mu}} \cdot L(0) \qquad \text{and} \qquad Y(0) = \left(\frac{s}{g+n}\right)^{\frac{\mu}{1-\mu}} \cdot L(0) \tag{15}$$

where $L(0)$ is given and s represents the fraction of total income saved[3],

$$S(t) = s \cdot Y(t) \tag{16}$$

with

$$S(t) = S(t,1) + S(t,2) \tag{17}$$

As a result

$$S(t,2) = \pi \cdot s \cdot Y(t) \tag{18}$$

It is also a well-known characteristic of the equilibrium state of the neo-classical model that the fraction of income saved affects the level at which the economy grows (the initial conditions), but that it does not affect the rate at which it grows. This circumstance gives rise to a crucial result of the neo-classical model that at a particular saving rate, viz. $s = \mu$ per capita consumption is maximized for all time. The condition $s = \mu$ is referred to as the golden rule of accumulation. Moreover $r = g + n$ holds.

[3] From the equations (1) and (11) it follows that in a stable population,

$$s = \frac{\sigma}{1-(1-\sigma)\pi}; \qquad 0 < \sigma \leq 1, \ 0 \leq \pi < 1$$

$$s = \frac{S(t,2)}{Y(t)} = \frac{S(2)}{Y}; \qquad \sigma = 0, \ \pi = 1$$

3 Least contributions

To find a formula for least contributions we return to equation (5). Substituting (6), (7), (8), (9) and (10) into (5) we get,

$$\pi \cdot \frac{s}{1-\mu} \cdot W(t) = \kappa \cdot W(t) - \gamma \cdot \frac{L^R(t)}{L^W(t)} \cdot W(t) + \frac{\mu}{1-\mu} \cdot W(t) \tag{19}$$

Rearranging terms, assuming a stable population and taking into account necessary boundary conditions yield,

$$\begin{aligned} \kappa &= \gamma \cdot \frac{L^R}{L^W} - \frac{\pi}{1-\mu} \cdot \left(\mu - \frac{\sigma}{1-(1-\sigma)\pi}\right); && \begin{array}{l} 0 < \sigma \leq 1, \\ 0 \leq \pi < 1 \end{array} \\ \kappa &= \gamma \cdot \frac{L^R}{L^W} - \frac{S(2)}{Y}; && \sigma = 0, \ \pi = 1 \end{aligned} \tag{20}$$

A stable population, i.e. a population that grows at a constant rate, is characterized by a stable age distribution. For simplicity's sake we also only consider steady state solutions of the economy[4]. All time-indices in eq. (20), therefore, can be deleted.
Three equilibrium situations can be considered, viz. the capital elasticity of output (μ) is larger than, smaller than or equal to the national saving rate (s). Or, in other words, the interest rate is larger than, smaller than or equal to the sum of the rate of technical progress and population growth. They are dealt with below.

1) $\mu > s$ or $r > g + n$. The interest rate is larger than the sum of the rate of technical progress and population growth. According to Aaron's rule for the individual participant in a pension scheme the capital reserve method is preferable. The minimal value of κ is found by maximizing the second term on the right hand side of (20),

$$\frac{\pi}{1-\mu} \cdot \left(\mu - \frac{\sigma}{1-(1-\sigma)\pi}\right); \qquad 0 < \sigma \leq 1, \ 0 \leq \pi < 1 \tag{21}$$

[4] Considering disequilibrium situations we should need to construct a model in which one of the equilibrium conditions lapses and is replaced by some error-adjustment mechanism. This matter is complicated mathematically and is not pursued here.

Differentiating (21) with respect to π and equating that to zero, we get,

$$\pi = \frac{1 - \sqrt{\sigma/\mu}}{1 - \sigma} \tag{22}$$

giving

$$\kappa = \gamma \cdot \frac{L^R}{L^W} - \frac{\mu}{(1-\mu)(1-\sigma)} \cdot \left(1 - \sqrt{\frac{\sigma}{\mu}}\right)^2 \tag{23}$$

The parameter π, representing the fraction of national capital formation originating from the pension fund gives rise to a particular value of θ, that part of pension payments that is financed by the capital reserve method. When the given pension-wage ratio γ exceeds θ, the volume of total saving and thus of pension saving is too small to guarantee incurred pension liabilities. Therefore, we introduced in the equations (12) and (14) a supplementary pension provision financed by the pay-as-you-go system with contribution rate ξ, thereby increasing pensioner's income with a fraction η of worker's income to the statutory fraction γ. As a result the financing system of the pension plan is a mixture of funding and pay-as-you-go. This contradicts Aaron's rule, which says that under the condition $r > g + n$ the capital reserve method is preferable. On the other hand, when γ is less than θ, η will be negative, which is not allowed. Only when $\gamma = \theta$ the situation described by Aaron's rule applies.

2) $\mu < s$ or $r < g + n$. The interest rate is less than the rate of technical progress and the rate of population growth. The lowest contribution level is found for $\pi = 0$. The pension plan is entirely based on the pay-as-you-go system. This result corresponds to Aaron's rule.

3) $\mu = s$ or $r = g + n$. The interest rate is equal to the rate of technical progress plus the rate of population growth. This situation is referred to as the golden rule path of an economy, where per capita consumption is maximized for all time. In a stable population the (lowest) contribution rate appears to be

$$\kappa = \gamma \cdot \frac{L^R}{L^W} \tag{24}$$

where

$$\pi = \frac{1 - \sigma/\mu}{1 - \sigma} \tag{25}$$

The contribution level is equal to that under the pay-as-you-go method. The parameter π actuarially determines θ, that part of pension payments financed by funding. Like in the first casus when γ exceeds θ, the volume of total saving and thus of pension saving required and sufficient for optimal growth is too small to guarantee incurred pension liabilities. There will be a mixed financial system therefore. On the other hand, when γ is less than θ, η is negative. This is not allowed, or, in other words, there is a minimal value of γ to arrive at the optimal growth path.

In the optimal neo-classical growth situation total contributions received are equal to the amount of pension payments. At the same time there may be or there may be not pension capital accumulation. As far as the volume of private saving, however, falls short of the amount of saving necessary for golden rule growth, the pension system will be at least partly based on funding. This result also contradicts Aaron's rule which says that under the $r = g + n$ condition the choice between pay-as-you-go and funding and any combination of them is arbitrary. The amount of pension saving, then appears to be equal to the interest revenues of the investment fund,

$$S(t, 2) = r \cdot \pi \cdot K(t) \tag{26}$$

The particular case $\sigma = 0$ implies $\pi = 1$. All necessary saving comes from the pension fund or, in other words, society consumes all its labour income and the pension fund saves and invests all its capital revenues.

4 Summary and conclusions

In this paper attention is given to Aaron's rule which says that for *the individual* the capital reserve method for financing pension schemes is preferable when the rate of growth of total earnings is less than the interest rate. It is shown, however, that often a mixed financial system is to be preferred.

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Summary

The present paper deals with Aaron's rule which says that for *the individual* the capital reserve method for financing pension schemes is preferable when the rate of growth of total earnings is less than the interest rate. It is shown that often a mixture of funding and pay-as-you-go is to be preferred.

Zusammenfassung

Die vorliegende Arbeit befasst sich mit der Regel von Aaron, welche besagt, dass für den einzelnen Versicherten einer Pensionskasse das Kapitaldeckungsverfahren vorteilhafter ist, falls die Zuwachsrate des Gesamteinkommens unter dem Zinsfuss liegt. Es wird gezeigt, dass oft eine Mischung aus Kapitaldeckungs- und Umlageverfahren vorzuziehen ist.

Résumé

Le présent article traite de la règle d'Aaron qui dit que pour l'assuré la méthode de capitalisation dans le financement des caisses de pensions est préférable lorsque le taux de croissance des revenus totaux est plus faible que le taux d'intérêt. L'auteur montre qu'il est souvent préférable de prévoir un système mixte capitalisation – répartition.