Excess claims and data trimming in the context of credibility rating procedures

Hans Bühlmann, Zurich, Alois Gisler, Winterthur,
and William S. Jewell, Berkeley*

Excess Claims and Data Trimming in the Context of Credibility Rating Procedures

1 Motivation

In Ratemaking and in Experience Rating one is often confronted with the dilemma of whether or not to fully charge very large claims to the claims load of small risk groups or of individual risks. Practitioners typically use an a posteriori argument in this situation: "If such large claims should be fully charged then the rates obtained would become 'ridiculous', hence it should not be done." The present paper aims at explaining this practical attitude from first principles.
Credibility Theory in its standard form makes the first step in the right direction. It explains to us that *all claims* should not be fully charged (but only with the constant fraction of the credibility weight). In many applications, however, it is still felt that the fraction of this charge should depend on the *size of a claim*. This leads very naturally to the idea of combining credibility procedures and data trimming.
Of course, such an idea needs to be tested. The first argument in favour of it was given by Gisler [1] who showed that in many cases the mean quadratic loss of the credibility estimator is substantially reduced if one introduces trimming of claims data. This paper goes even further. It formalizes the standard way of thinking about large claims and then shows that "optimal forecasting" of rates (using Bayes estimation techniques) and forecasting by "credibility techniques combined with data trimming" lead to almost identical results.

* The authors are greatly indebted to *R. Schnieper* who did all the numerical work on the ETH computer.

2 The Basic Model

Throughout the paper we work with the most simple model in the credibility context.

$$\underline{X} = (X_1, X_2, \ldots, X_n)$$

is the random vector representing the experience of a given risk in the years 1, 2, $\ldots, n$.

- The quality of the risk is characterized by an unknown parameter value θ, which we consider as a realisation of a random variable Θ with distribution function $U(\theta)$.
- Given the parameter value θ, $\{X_1, X_2, \ldots, X_n\}$ are i.i.d. with density function $f_\theta(x)$ [mean $\mu(\theta)$, variance $\sigma^2(\theta)$].

To these standard assumptions in credibility theory we add now some more structure regarding the distribution of the size of a claim. The main idea is introduced by the assumption that the claim sizes are drawn from two different urns (distributions). Mostly, i.e. with probability $1-\pi$, we observe an *ordinary claim* with density $p_o(x/\theta)$ [mean $\mu_o(\theta)$, variance $\sigma_o^2(\theta)$] and occasionally, i.e. with probability π, we observe an *excess claim* (catastrophic claim) with density $p_e(x/\theta)$ [mean $\mu_e(\theta)$, variance $\sigma_e^2(\theta)$].

	$p_o(x/\theta)$	$p_e(x/\theta)$
	ordinary claim amounts	excess claim amounts
occurrence	$1-\pi$	π

We have assumed that the mixing probabilities are independent of θ and from now on we shall also suppose that the density of the excess claims is independent of the risk parameter, hence formalizing the idea that large catastrophic claims have no bearing on the quality of the risk.
In mathematical shorthand all the considerations just made regarding additional structure are summed up by stating that the density $f_\theta(x)$ has the following form

$$f_\theta(x) = (1-\pi)p_o(x/\theta) + \pi p_e(x). \tag{1}$$

3 The Basic Problem

As always in the credibility context our aim is to estimate

$$\mu(\theta)$$

pure premium for
the given risk

based on the observations of

$$\underline{X}=(X_1, X_2, \ldots, X_n)$$

experience of the given risk in the
years 1, 2, ..., n.

One knows that the best estimator (with respect to quadratic loss) for this problem is

$$P[\underline{X}]=E[\mu(\theta)/X]. \tag{2}$$

Using the special structure of formula (1) we obtain

$$P[\underline{X}]=\pi\mu_e+(1-\pi)\underbrace{E[\mu_o(\theta)/X]}_{g(\underline{X})}. \tag{3}$$

If we use standard credibility techniques we estimate by

$$f[\underline{X}]=a+b\sum_{i=1}^{n} X_i \tag{4}$$

with optimal choice of a, b.
And if in addition we introduce trimming of the data we estimate by

$$f[\underline{X}]=a+b\sum_{i=1}^{n}(X_i\wedge M) \tag{5}$$

with optimal choice of a, b, M.
Using (4) we are committing the following error against optimal estimation

$$\begin{aligned}\inf_{a,b,M} E\{P[\underline{X}]-f[\underline{X}]\}^2 &= \inf_{a,b,M} E\left\{\pi\mu_e+(1-\pi)g(\underline{X})-a-b\sum_{i=1}^{n}(X_i\wedge M)\right\}^2\\ &=(1-\pi)^2\inf_{a,b,M} E\left\{\frac{\pi\mu_e-a}{1-\pi}+g(\underline{X})\right.\\ &\qquad\left.-\frac{b}{1-\pi}\sum_{i=1}^{n}(X_i\wedge M)\right\}^2\\ &=(1-\pi)^2\inf_{a',b',M} E\left\{g(\underline{X})-a'-b'\sum_{i=1}^{n}(X_i\wedge M)\right\}^2 \end{aligned} \tag{6}$$

The following two problems are therefore equivalent

A) Estimate $P[\underline{X}]$ (total premium) by $a+b\sum_{i=1}^{n}(X_i\wedge M)$ with optimal a, b, M

B) Estimate $g(\underline{X})$ (ordinary premium) by $a'+b'\sum_{i=1}^{n}(X_i\wedge M)$ with optimal a', b', M.

For the optimal choices of the parameters (denoted by $\tilde{}$) we have

$$\begin{aligned}\tilde{a}&=(1-\pi)\tilde{a}'+\pi\mu_e\\ \tilde{b}&=(1-\pi)\tilde{b}'\end{aligned}\tag{7}$$

In the following we want to illustrate that $\tilde{a}'+\tilde{b}'\sum_{i=1}^{n}(X_i\wedge\tilde{M})$ is a good approximation of $g(\underline{X})=E[\mu_o(\theta)/\underline{X}]$ (Problem B above).
We actually shall compare

$$\tilde{a}'+\tilde{b}'\sum(x_i\wedge\tilde{M})\text{ with }g(\underline{x})$$

for any observation $\underline{x}$ of $\underline{X}$.

4 The Exact Form of $g(\underline{x})$

Writing out the conditional expectation $E[\mu_o(\theta)/\underline{X}=\underline{x}]$ we obtain

$$g(\underline{x})=\frac{\int\mu_o(\theta)\left[\prod_{i=1}^{n}\{(1-\pi)p_o(x_i/\theta)+\pi p_e(x_i)\}\right]dU(\theta)}{\int\left[\prod_{i=1}^{n}\{(1-\pi)p_o(x_i/\theta)+\pi p_e(x_i)\}\right]dU(\theta)}.\tag{8}$$

Putting $I=\{1,2,\ldots,n\}$ and $S\subset I$ we rewrite

$$\prod_{i=1}^{n}\{(1-\pi)p_o(x_i/\theta)+\pi p_e(x_i)\}=\sum_{S\subset I}(1-\pi)^s\pi^{n-s}\prod_{i\in S}p_o(x_i/\theta)\prod_{i\in\bar{S}}p_e(x_i)\tag{9}$$

where the sum on the right side must be taken over all subsets

$$S\subset I\quad(\text{including }\phi\text{ and }I)\quad\text{with } s=|S|\text{ and } n=|I|.$$

We also use the abbreviations

$$p_o(x_S)=\int \prod_{i\in S} p_o(x_i/\theta)\,dU(\theta)$$

$$p_e(x_S)=\int \prod_{i\in S} p_e(x_i)\,dU(\theta)=\prod_{i\in S} p_e(x_i)$$

$$L(x_S)=\left(\frac{\pi}{1-\pi}\right)^{n-s}\frac{p_o(x_S)\,p_e(x_{\bar{S}})}{p_o(x_I)}\qquad [p_o(x_\phi)=p_e(x_\phi)=1]$$

$$E_o[\mu_o(\theta)/x_S]=\frac{\int \mu_o(\theta)\prod_{i\in S} p_o(x_i/\theta)\,dU(\theta)}{p_o(x_S)}.$$

Then introducing (9) into (8) and carrying out the integration we find for the numerator of $g(\underline{x})$

$$\sum_{S\subset I}(1-\pi)^s\pi^{n-s}\prod_{i\in\bar{S}} p_e(x_i)\int \mu_o(\theta)\prod_{i\in S} p_o(x_i/\theta)\,dU(\theta)$$

or

$$\sum_{S\subset I}(1-\pi)^s\pi^{n-s}p_o(x_S)\,p_e(x_{\bar{S}})\,E_o[\mu_o(\theta)/x_S]$$

and for the denominator of $g(\underline{x})$

$$\sum_{S\subset I}(1-\pi)^s\pi^{n-s}p_o(x_S)\,p_e(x_{\bar{S}}).$$

Dividing both numerator and denominator by $(1-\pi)^n p_o(x_I)$ we finally arrive at

$$g(\underline{x})=\frac{E_o[\mu_o(\theta)/\underline{x}]+\sum_{\substack{S\subset I\\ S\neq I}} L(x_S)\,E_o[\mu_o(\theta)/x_S]}{1+\sum_{\substack{S\subset I\\ S\neq I}} L(x_S)}. \tag{10}$$

Remarks

i) Observe that $g(\underline{x})$ is a weighted average of forecasts based on all subsamples x_S of the total sample x_I, the forecasts being calculated under the assumption that the subsample contains only claims of the ordinary type.

ii) As $\frac{\pi}{1-\pi}$ is usually rather small the weight of $L(x_S)$ is rather quickly decreasing with decreasing number of observations in x_S; for a fixed number of observations the weight $L(x_S)$ is rather big if both $p_o(x_S)$ and $p_e(x_{\bar{S}})$ are big i. e. if x_S and $x_{\bar{S}}$ are very likely to come from the ordinary and the excess urn respectively.

iii) Dividing by $p_o(x_I)$ is obviously only allowed if all the observed claims are possibly of ordinary type. The weight function $L(x_S)$ is then only positive if $p_e(x_{\bar{S}})$ is positive i. e. if the subset $x_{\bar{S}}$ is possibly of excess type. Thus the formula does what we would have done by intuition as well, it includes only predictions based on subsamples which *may be* of ordinary type. On the other hand, these predictive subsamples must contain all claims which *are surely* of ordinary type.

5 More Insight from the Single Observation Case

At this point it is worthwhile to consider the special case where the whole sample of observations contains only one observation, i. e.

$$\underline{x}=(x_1).$$

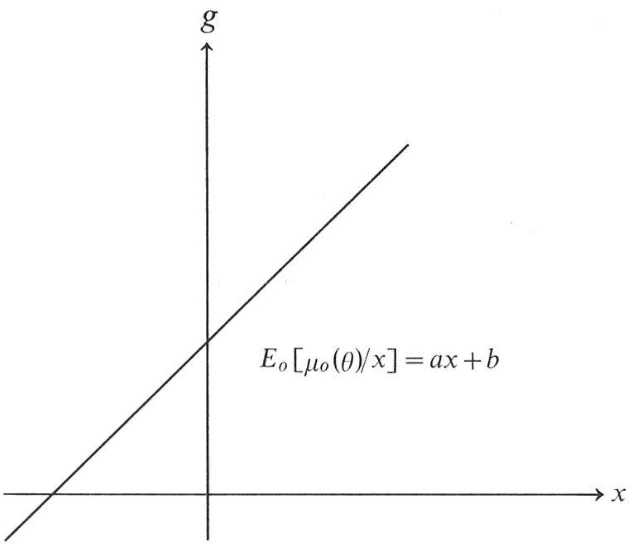
For simplicity we omit the index 1 and write x for the single observation. We have then

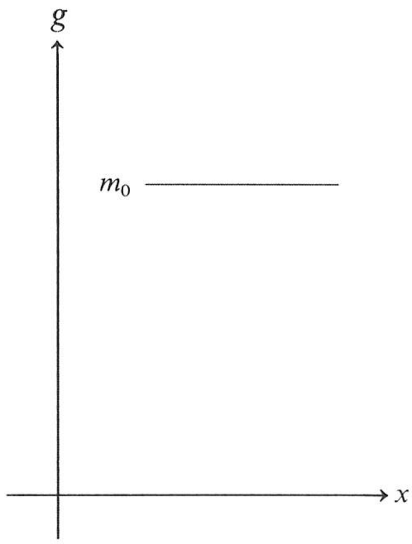
$$g(x)=\frac{E_o[\mu_o(\theta)/x]+L[x_\phi]E[\mu_o(\theta)]}{1+L[x_\phi]} \tag{11}$$

with

$$L[x_\phi]=\frac{\pi}{1-\pi}\,\frac{p_e(x)}{p_o(x)}.$$

The right hand side is a multiple of the likelihood ratio. If the latter is monotonically increasing (which is typically the case in applications) so is also the weight given to the constant estimator $E[\mu_o(\theta)]=m_o$. Assume in addition that $E_o[\mu_o(\theta)/x]$ is of linear form; then our estimator $g(x)$ is a mixture of the two cases (corresponding to the two pictures)





the weight being shifted from the estimator on the left to the estimator on the right as x increases. The resulting estimator is almost of the form $a + b \min(x, M)$. Hence credibility with trimming is almost exact! This fact will be illustrated by a numerical example in section 6. In fact our numerical example will show that this fact also carries over to higher dimensions.

6 A Numerical Example

6.1 For explicit calculations we are assuming that for *ordinary claims*

$p_o(x/\theta)$ is a normal density with mean θ
variance v

θ is normally distributed with mean m_0
variance w.

We then have

$$p_o(x_S) = \int \prod_{i \in S} p_o(x_i/\theta)\, dU(\theta)$$

which turns out to be a multidimensional normal density with mean vector

$$\begin{pmatrix} m_0 \\ m_0 \\ \cdot \\ \cdot \\ \cdot \\ m_0 \end{pmatrix} \quad \text{and covariance matrix} \quad \sum = \begin{pmatrix} w+v & w & \dots & w \\ w & w+v & \dots & \\ \cdot & \cdot & & \\ \cdot & \cdot & & \\ \cdot & \cdot & & \\ w & \dots & & w+v \end{pmatrix}$$

hence

$$p_o(x_S) = \frac{\sqrt{|A|}}{(2\pi)^{s/2}} e^{-\frac{1}{2}\sum_{\substack{i\in S\\ j\in S}} a_{ij}(x_i - m_0)(x_j - m_0)} \tag{12}$$

with $A = \sum^{-1}$.

Proof that $p_o(x_S)$ has density (12):

a) Given θ any linear combination $\sum_{i\in S} c_i X_i$ is normal with mean $\sum_{i\in S} c_i\theta$ and variance $\sum_{i\in S} c_i^2 v$. Integrating out with respect to the normal structure function of θ we obtain a normal distribution with mean $\sum_{i\in S} c_i m_0$ and variance $\left(\sum_{i\in S} c_i\right)^2 w + \sum_{i\in S} c_i^2 v$. But a sample X_S whose linear combinations are all normally distributed is multidimensional normal.

b) Let $\sum = (\sigma_{ij})_{\substack{i\in S\\ j\in S}}$

$$\begin{aligned} \sigma_{ij} = \operatorname{Cov}(X_i, X_j) &= E[\operatorname{Cov}(X_i, X_j)/\theta] + \operatorname{Cov}[E[X_i/\theta], E[X_j/\theta]] \\ &= \qquad \delta_{ij} v \qquad\qquad + \qquad w \end{aligned}$$

q.e.d.

It should be noted that

$$\det \sum = v^s + s v^{s-1} w \tag{13}$$

(subtract first row from all other rows and then develop along the first column). Also observe the explicit form of

$$\sum^{-1} = A = (a_{ij})_{\substack{i\in S\\ j\in S}},$$

namely
$$a_{ij}=\frac{1}{v}\left(\delta_{ij}-\frac{w}{v+sw}\right) \tag{14}$$

$$\left[\begin{array}{c}\text{use } (I+\alpha\beta^T)^{-1}=I-\dfrac{\alpha\beta^T}{1+\alpha^T\beta},\\ \text{where } \alpha,\ \beta \text{ are columnvectors; } I \text{ identity matrix.}\end{array}\right]$$

From elementary calculations in credibility theory we finally also know that

$$E_o[\theta/x_S]=\bar{x}_S\,\frac{sw}{v+sw}+m_0\,\frac{v}{v+sw}. \tag{15}$$

6.2 For the *excess claims* the probability law is specified by assuming that

$p_e(x)$ is a normal density with mean μ_e
variance σ_e^2.

7 Numerical Calculations of $g(\underline{x})$

For our calculations we have chosen

$$m_0=10 \qquad \mu_e=50$$
$$\left.\begin{array}{r}v=12.5\\ w=12.5\end{array}\right\}\sigma_o=5 \qquad \sigma_e=20$$
$$1-\pi=0.9 \qquad \pi=0.1$$

and we obtain

a) for $n=1$ (single observation case)

Table 1

x	$g(x)$	x	$g(x)$	x	$g(x)$	x	$g(x)$
5.	7.5091	14.	11.9850	23.	14.8946	32.	10.0370
6.	8.0068	15.	12.4755	24.	14.3712	33.	10.0151
7.	8.5049	16.	12.9602	25.	13.4968	34.	10.0059
8.	9.0033	17.	13.4348	26.	12.4521	35.	10.0022
9.	9.5017	18.	13.8919	27.	11.5065	36.	10.0008
10.	10.0000	19.	14.3177	28.	10.8265	37.	10.0003
11.	10.4979	20.	14.6876	29.	10.4153	38.	10.0001
12.	10.9951	21.	14.9586	30.	10.1952	39.	10.0000
13.	11.4910	22.	15.0602	31.	10.0870	40.	10.0000

b) for $n=2$ (two observations) $g(x_1, x_2)$

Table 2

$x_1 \backslash x_2$	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
5	6.68	7.02	7.35	7.68	8.02	8.35	8.68	9.01	9.32	9.61	9.86	10.04	10.08	9.92	9.53	8.98	8.43	8.02	7.78	7.65
6	7.02	7.35	7.68	8.01	8.35	8.68	9.01	9.33	9.65	9.94	10.21	10.41	10.50	10.42	10.12	9.62	9.07	8.62	8.32	8.17
7	7.35	7.68	8.01	8.34	8.68	9.01	9.34	9.66	9.98	10.28	10.55	10.78	10.92	10.91	10.70	10.27	9.73	9.24	8.90	8.70
8	7.68	8.01	8.34	8.68	9.01	9.34	9.67	9.99	10.31	10.62	10.90	11.15	11.33	11.38	11.26	10.92	10.42	9.90	9.51	9.26
9	8.02	8.35	8.68	9.01	9.34	9.67	10.00	10.32	10.65	10.96	11.25	11.52	11.72	11.83	11.79	11.55	11.11	10.59	10.14	9.84
10	8.35	8.68	9.01	9.34	9.67	10.00	10.33	10.66	10.98	11.30	11.60	11.88	12.11	12.27	12.30	12.15	11.79	11.30	10.81	10.44
11	8.68	9.01	9.34	9.67	10.00	10.33	10.66	10.99	11.32	11.63	11.94	12.23	12.49	12.69	12.78	12.72	12.46	12.02	11.51	11.08
12	9.01	9.33	9.66	9.99	10.32	10.66	10.99	11.32	11.65	11.97	12.29	12.59	12.86	13.09	13.24	13.26	13.09	12.73	12.23	11.76
13	9.32	9.65	9.98	10.31	10.65	10.98	11.32	11.65	11.98	12.30	12.62	12.93	13.22	13.47	13.67	13.75	13.68	13.41	12.97	12.47
14	9.61	9.94	10.28	10.62	10.96	11.30	11.63	11.97	12.30	12.63	12.96	13.27	13.57	13.85	14.07	14.22	14.23	14.06	13.69	13.20
15	9.86	10.21	10.55	10.90	11.25	11.60	11.94	12.29	12.62	12.96	13.29	13.61	13.92	14.21	14.46	14.65	14.74	14.66	14.39	13.93
16	10.04	10.41	10.78	11.15	11.52	11.88	12.23	12.59	12.93	13.27	13.61	13.94	14.26	14.56	14.83	15.06	15.20	15.21	15.03	14.65
17	10.08	10.50	10.92	11.33	11.72	12.11	12.49	12.86	13.22	13.57	13.92	14.26	14.58	14.90	15.19	15.44	15.62	15.70	15.61	15.31
18	9.92	10.42	10.91	11.38	11.83	12.27	12.69	13.09	13.47	13.85	14.21	14.56	14.90	15.22	15.52	15.79	16.01	16.13	16.12	15.90
19	9.53	10.12	10.70	11.26	11.79	12.30	12.78	13.24	13.67	14.07	14.46	14.83	15.19	15.52	15.84	16.12	16.36	16.51	16.54	16.38
20	8.98	9.62	10.27	10.92	11.55	12.15	12.72	13.26	13.75	14.22	14.65	15.06	15.44	15.79	16.12	16.42	16.66	16.83	16.87	16.73
21	8.43	9.07	9.73	10.42	11.11	11.79	12.46	13.09	13.68	14.23	14.74	15.20	15.62	16.01	16.36	16.66	16.91	17.07	17.10	16.94
22	8.02	8.62	9.24	9.90	10.59	11.30	12.02	12.73	13.41	14.06	14.66	15.21	15.70	16.13	16.51	16.83	17.07	17.21	17.20	16.97
23	7.78	8.32	8.90	9.51	10.14	10.81	11.51	12.23	12.97	13.69	14.39	15.03	15.61	16.12	16.54	16.87	17.10	17.20	17.10	16.76
24	7.65	8.17	8.70	9.26	9.84	10.44	11.08	11.76	12.47	13.20	13.93	14.65	15.31	15.90	16.38	16.73	16.94	16.97	16.76	16.28

c) for $n=5$ (five observations) $g(x_1, x_2, C_3, C_4, C_5)$
note: C_3, C_4, C_5 are chosen as "parameters" for the following tables
i) $(C_3, C_4, C_5)=(10, 10, 10)$

Table 3.1

x_1 \ x_2	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
5	8.76	8.86	8.98	9.13	9.28	9.43	9.56	9.66	9.71	9.71	9.67	9.60	9.53	9.47	9.42	9.39	9.37	9.36	9.36	9.35
6	8.86	8.94	9.06	9.20	9.35	9.50	9.62	9.71	9.76	9.77	9.73	9.67	9.60	9.54	9.49	9.46	9.44	9.43	9.43	9.43
7	8.98	9.06	9.17	9.31	9.46	9.60	9.72	9.81	9.86	9.87	9.84	9.78	9.71	9.65	9.61	9.58	9.56	9.55	9.54	9.54
8	9.13	9.20	9.31	9.44	9.58	9.72	9.84	9.94	9.99	10.00	9.97	9.92	9.85	9.80	9.75	9.72	9.70	9.69	9.69	9.69
9	9.28	9.35	9.46	9.58	9.72	9.86	9.98	10.08	10.14	10.15	10.13	10.08	10.01	9.96	9.91	9.88	9.86	9.85	9.85	9.84
10	9.43	9.50	9.60	9.72	9.86	10.00	10.12	10.22	10.29	10.31	10.28	10.24	10.17	10.12	10.07	10.04	10.02	10.01	10.00	10.00
11	9.56	9.62	9.72	9.84	9.98	10.12	10.25	10.36	10.42	10.45	10.43	10.38	10.32	10.26	10.22	10.18	10.16	10.15	10.15	10.14
12	9.66	9.71	9.81	9.94	10.08	10.22	10.36	10.46	10.53	10.56	10.55	10.50	10.44	10.38	10.33	10.30	10.28	10.27	10.26	10.26
13	9.71	9.76	9.86	9.99	10.14	10.29	10.42	10.53	10.61	10.64	10.63	10.58	10.52	10.46	10.41	10.37	10.35	10.34	10.33	10.33
14	9.71	9.77	9.87	10.00	10.15	10.31	10.45	10.56	10.64	10.67	10.66	10.61	10.55	10.48	10.43	10.40	10.37	10.36	10.35	10.35
15	9.67	9.73	9.84	9.97	10.13	10.28	10.43	10.55	10.63	10.66	10.64	10.60	10.53	10.47	10.41	10.37	10.35	10.34	10.33	10.33
16	9.60	9.67	9.78	9.92	10.08	10.24	10.38	10.50	10.58	10.61	10.60	10.55	10.48	10.41	10.36	10.32	10.29	10.28	10.27	10.27
17	9.53	9.60	9.71	9.85	10.01	10.17	10.32	10.44	10.52	10.55	10.53	10.48	10.41	10.34	10.29	10.25	10.22	10.21	10.20	10.20
18	9.47	9.54	9.65	9.80	9.96	10.12	10.26	10.38	10.46	10.48	10.47	10.41	10.34	10.27	10.22	10.18	10.16	10.14	10.14	10.13
19	9.42	9.49	9.61	9.75	9.91	10.07	10.22	10.33	10.41	10.43	10.41	10.36	10.29	10.22	10.17	10.13	10.10	10.09	10.09	10.08
20	9.39	9.46	9.58	9.72	9.88	10.04	10.18	10.30	10.37	10.40	10.37	10.32	10.25	10.18	10.13	10.09	10.07	10.06	10.05	10.05
21	9.37	9.44	9.56	9.70	9.86	10.02	10.16	10.28	10.35	10.37	10.35	10.29	10.22	10.16	10.10	10.07	10.05	10.03	10.03	10.02
22	9.36	9.43	9.55	9.69	9.85	10.01	10.15	10.27	10.34	10.36	10.34	10.28	10.21	10.14	10.09	10.06	10.03	10.02	10.02	10.01
23	9.36	9.43	9.54	9.69	9.85	10.00	10.15	10.26	10.33	10.35	10.33	10.27	10.20	10.14	10.09	10.05	10.03	10.02	10.01	10.01
24	9.35	9.43	9.54	9.69	9.84	10.00	10.14	10.26	10.33	10.35	10.33	10.27	10.20	10.13	10.08	10.05	10.02	10.01	10.01	10.00

ii) $(C_3, C_4, C_5) = (10, 10, 25)$

Table 3.2

x_1 \ x_2	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
5	8.59	8.70	8.84	9.01	9.18	9.35	9.50	9.61	9.66	9.66	9.62	9.54	9.46	9.39	9.33	9.30	9.27	9.26	9.26	9.25
6	8.70	8.80	8.93	9.09	9.26	9.43	9.57	9.67	9.73	9.73	9.69	9.62	9.54	9.47	9.41	9.38	9.36	9.35	9.34	9.34
7	8.84	8.93	9.06	9.22	9.38	9.54	9.68	9.78	9.84	9.85	9.81	9.74	9.67	9.60	9.54	9.51	9.49	9.48	9.47	9.47
8	9.01	9.09	9.22	9.37	9.53	9.68	9.82	9.93	9.99	10.00	9.97	9.90	9.83	9.76	9.71	9.68	9.66	9.65	9.64	9.64
9	9.18	9.26	9.38	9.53	9.69	9.84	9.98	10.09	10.16	10.17	10.15	10.09	10.01	9.95	9.90	9.86	9.84	9.83	9.82	9.82
10	9.35	9.43	9.54	9.68	9.84	10.00	10.14	10.26	10.33	10.35	10.33	10.27	10.20	10.13	10.08	10.05	10.02	10.01	10.01	10.00
11	9.50	9.57	9.68	9.82	9.98	10.14	10.29	10.41	10.48	10.51	10.49	10.44	10.37	10.30	10.25	10.21	10.19	10.18	10.17	10.17
12	9.61	9.67	9.78	9.93	10.09	10.26	10.41	10.53	10.61	10.64	10.63	10.57	10.51	10.44	10.38	10.34	10.32	10.31	10.30	10.30
13	9.66	9.73	9.84	9.99	10.16	10.33	10.48	10.61	10.70	10.73	10.72	10.67	10.60	10.53	10.47	10.43	10.40	10.39	10.38	10.38
14	9.66	9.73	9.85	10.00	10.17	10.35	10.51	10.64	10.73	10.77	10.75	10.70	10.63	10.56	10.50	10.46	10.43	10.42	10.41	10.41
15	9.62	9.69	9.81	9.97	10.15	10.33	10.49	10.63	10.72	10.75	10.74	10.69	10.61	10.54	10.48	10.43	10.41	10.39	10.38	10.38
16	9.54	9.62	9.74	9.90	10.09	10.27	10.44	10.57	10.67	10.70	10.69	10.63	10.55	10.47	10.41	10.37	10.34	10.33	10.32	10.31
17	9.46	9.54	9.67	9.83	10.01	10.20	10.37	10.51	10.60	10.63	10.61	10.55	10.47	10.39	10.33	10.29	10.26	10.24	10.24	10.23
18	9.39	9.47	9.60	9.76	9.95	10.13	10.30	10.44	10.53	10.56	10.54	10.47	10.39	10.32	10.25	10.21	10.18	10.17	10.16	10.16
19	9.33	9.41	9.54	9.71	9.90	10.08	10.25	10.38	10.47	10.50	10.48	10.41	10.33	10.25	10.19	10.15	10.12	10.11	10.10	10.10
20	9.30	9.38	9.51	9.68	9.86	10.05	10.21	10.34	10.43	10.46	10.43	10.37	10.29	10.21	10.15	10.11	10.08	10.07	10.06	10.05
21	9.27	9.36	9.49	9.66	9.84	10.02	10.19	10.32	10.40	10.43	10.41	10.34	10.26	10.18	10.12	10.08	10.05	10.04	10.03	10.03
22	9.26	9.35	9.48	9.65	9.83	10.01	10.18	10.31	10.39	10.42	10.39	10.33	10.24	10.17	10.11	10.07	10.04	10.03	10.02	10.02
23	9.26	9.34	9.47	9.64	9.82	10.01	10.17	10.30	10.38	10.41	10.38	10.32	10.24	10.16	10.10	10.06	10.03	10.02	10.01	10.01
24	9.25	9.34	9.47	9.64	9.82	10.00	10.17	10.30	10.38	10.41	10.38	10.31	10.23	10.16	10.10	10.05	10.03	10.02	10.01	10.01

8 Optimal Trimming

Gisler has shown [1] that for given M the optimal choice of the approximation

$$\widehat{\mu(\theta)} = a + b \sum_{i=1}^{n} (x_i \wedge M) \quad \text{to} \quad \mu(\theta) \text{ [and hence to } P[\underline{x}]]$$

can be calculated as follows

$$\tilde{b} = \frac{b_1}{(n-1)b_2 + b_3} \quad \text{where} \quad \begin{aligned} b_1 &= \text{Cov } [X_1 \wedge M, X_2] \\ b_2 &= \text{Cov } [X_1 \wedge M, X_2 \wedge M] \\ b_3 &= \text{Var } [X_1 \wedge M] \end{aligned} \tag{16}$$

$$\tilde{a} + n\tilde{b}E[X \wedge M] = E[X]. \tag{17}$$

With this optimal choice we then have

$$E[(\widehat{\mu(\theta)} - \mu(\theta))^2] = w - n\tilde{b} \cdot b_1. \tag{18}$$

Hence the trimming point M is optimal if $\tilde{b} \cdot b_1$ is maximum.
In our basic model (cf. section (2)) we find

$$b_1 = (1-\pi)^2 \text{ Cov } [\mu_o^M(\theta), \mu_o(\theta)] \quad \text{where} \quad \begin{aligned} \mu_o^M(\theta) &= E[X \wedge M/\theta, X \text{ ordinary}] \\ \mu_o(\theta) &= E[X/\theta, X \text{ ordinary}] \end{aligned} \tag{19}$$

$$b_2 = (1-\pi)^2 \text{ Var } [\mu_o^M(\theta)]$$

$$b_3 = (1-\pi)E[\sigma_o^{2\,M}(\theta)] + \pi\sigma_e^{2\,M} + (1-\pi)^2 \text{ Var } [\mu_o^M(\theta)] + \pi(1-\pi)E[(\mu_o(\theta) - \mu_e^M)^2]$$

$$\text{with } \begin{aligned} \sigma_o^{2\,M}(\theta) &= \text{Var } [X \wedge M/\theta, X \text{ ordinary}] \\ \sigma_e^{2\,M} &= \text{Var } [X \wedge M/X \text{ excess}] \\ \mu_e^M &= E[X \wedge M/X \text{ excess}]. \end{aligned}$$

Using explicitely the normal distribution as assumed both for ordinary and excess claims in section 6 we obtain from some rather tedious integrations:

Let $\Phi(.)$ denote the standardized normal distribution function and $\varphi(.)$ the standardized normal density function, then

$$b_1=(1-\pi)^2 w\Phi\left(\frac{M-m_0}{\sigma_o}\right) \qquad \sigma_o=\sqrt{v+w} \tag{20}$$

$$b_2=(1-\pi)^2 \operatorname{Cov}\,[U_1\wedge M,\ U_2\wedge M]$$

where the covariance is obtained by numerical integration.

Notation: $(U_1,\ U_2)$ is $N\left(\begin{matrix}m_0\\ m_0\end{matrix},\ \Sigma\right)$ with $\Sigma=\begin{pmatrix}v+w & w\\ w & v+w\end{pmatrix}$

$$b_3=A-B^2$$

where

$$\begin{aligned}A=(1-\pi)&\left[(m_0^2+\sigma_o^2)\ \Phi\left(\frac{M-m_0}{\sigma_o}\right)-\sigma_o(M+m_0)\,\varphi\left(\frac{M-m_0}{\sigma_o}\right)\right]\\ +\pi\ &\left[(\mu_e^2+\sigma_e^2)\ \Phi\left(\frac{M-\mu_e}{\sigma_e}\right)-\sigma_e(M+\mu_e)\ \varphi\left(\frac{M-\mu_e}{\sigma_e}\right)\right]\\ +M^2&\left[1-(1-\pi)\Phi\left(\frac{M-m_0}{\sigma_o}\right)-\pi\Phi\left(\frac{M-\mu_e}{\sigma_e}\right)\right]\end{aligned}$$

$$\begin{aligned}B=(1-\pi)&\left[(m_0-M)\ \Phi\left(\frac{M-m_0}{\sigma_o}\right)-\sigma_o\varphi\left(\frac{M-m_0}{\sigma_o}\right)\right]\\ +\pi\ &\left[(\mu_e-M)\ \Phi\left(\frac{M-\mu_e}{\sigma_e}\right)-\sigma_e\varphi\left(\frac{M-\mu_e}{\sigma_e}\right)\right]\\ +M&\end{aligned}$$

9 Numerical Calculations of $\tilde{a}+\tilde{b}\sum_{i=1}^{n}(x_i\wedge\tilde{M})$

Using the same parameter values as in section 7 we obtain the forecasts based on optimal trimming. To compare with $g(\underline{x})$ it is worthwhile to calculate also $\tilde{a}'+\tilde{b}'\sum_{i=1}^{n}(x_i\wedge\tilde{M})$ with

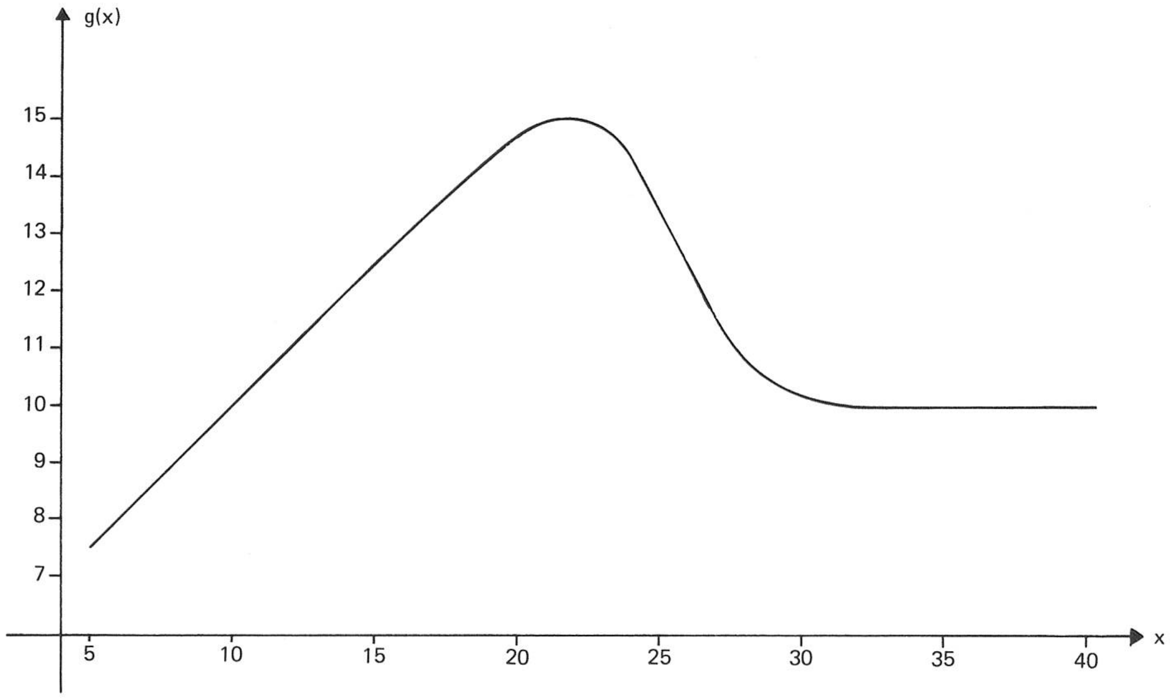
$$\tilde{a}'=\frac{\tilde{a}-\pi\mu_e}{1-\pi} \qquad \tilde{b}'=\frac{\tilde{b}}{1-\pi}.$$

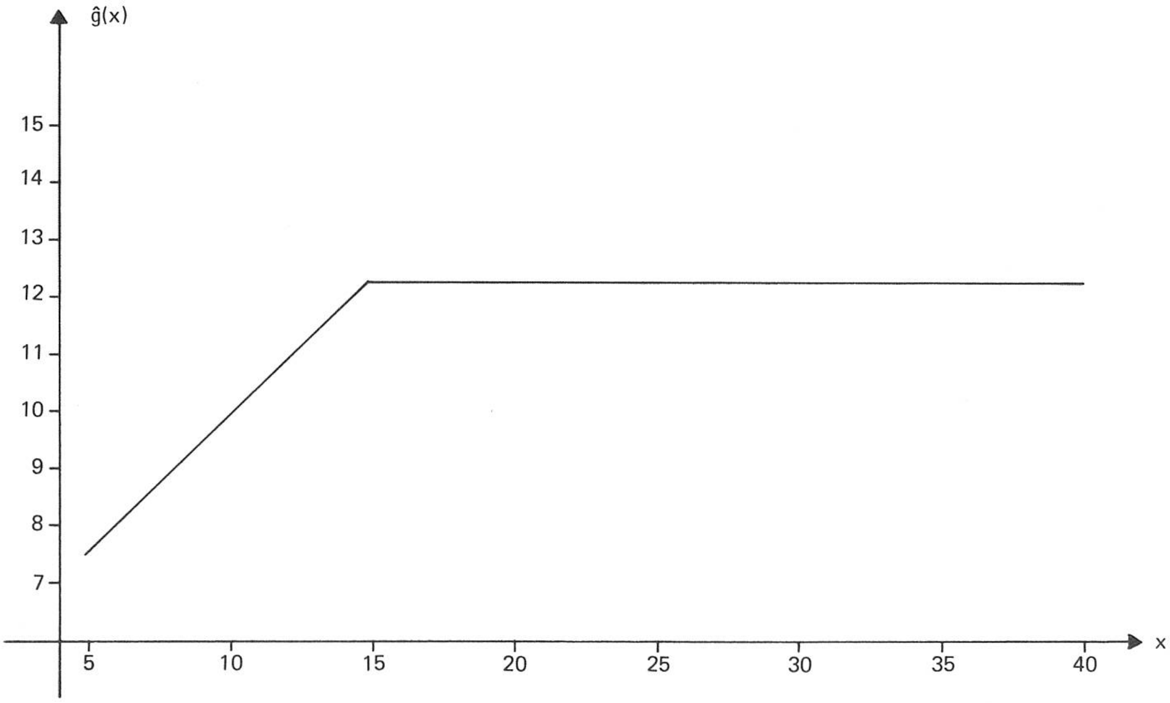
a) Results for $n=1$ (single observation case)
Truncation point $\tilde{M}=14.68$

Formula:	$\hat{P}=0.4412\,(x\wedge \tilde{M})+9.5817$	$\hat{g}=0.4902\,(x\wedge \tilde{M})+5.0908$
x		
5	11.79	7.54
6	12.23	8.03
7	12.67	8.52
8	13.11	9.01
9	13.55	9.50
10	13.99	9.99
11	14.43	10.48
12	14.88	10.97
13	15.32	11.46
14	15.76	11.95
15	16.06	12.29
16	16.06	12.29
17	16.06	12.29
18	16.06	12.29
19	16.06	12.29
20	16.06	12.29

Figure 1 shows a comparison of the true g and the approximation $\hat{g}$ by truncation. Observe the «overshoot» of the true regression function. It derives from the gradual loss of evidence that the claim is of ordinary type. For very high observations one settles for the a priori estimate $m_0=10$ which is lower than the estimate in the 15 to 25 region on the x-axis. On the other hand the approximation $\hat{g}(x)$ has the practical advantage of being monotone hence avoiding the «overshoot» region. Of course there must be a compensation for this avoidance and indeed for high x's the approximation $\hat{g}$ leads to higher values than g.

Figure 1


g(x)
15
14
13
12
11
10
9
8
7
5
10
15
20
25
30
35
40
x


ĝ(x)
15
14
13
12
11
10
9
8
7
5
10
15
20
25
30
35
40
x

b) Results for $n=2$

truncation point $\tilde{M}=19.52$

b_1*) approximation to total premium* $P[\underline{x}]$

$$\textit{formula:}\quad \hat{P}=0.2289 \sum_{i=1}^{2} (x_i \wedge \tilde{M})+9.0351$$

Table 4

$x_1 \backslash x_2$	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
5	11.32	11.55	11.78	12.01	12.24	12.47	12.70	12.93	13.16	13.38	13.61	13.84	14.07	14.30	14.53	14.65	14.65	14.65	14.65	14.65
6	11.55	11.78	12.01	12.24	12.47	12.70	12.93	13.16	13.38	13.61	13.84	14.07	14.30	14.53	14.76	14.88	14.88	14.88	14.88	14.88
7	11.78	12.01	12.24	12.47	12.70	12.93	13.16	13.38	13.61	13.84	14.07	14.30	14.53	14.76	14.99	15.11	15.11	15.11	15.11	15.11
8	12.01	12.24	12.47	12.70	12.93	13.16	13.38	13.61	13.84	14.07	14.30	14.53	14.76	14.99	15.22	15.33	15.33	15.33	15.33	15.33
9	12.24	12.47	12.70	12.93	13.16	13.38	13.61	13.84	14.07	14.30	14.53	14.76	14.99	15.22	15.44	15.56	15.56	15.56	15.56	15.56
10	12.47	12.70	12.93	13.16	13.38	13.61	13.84	14.07	14.30	14.53	14.76	14.99	15.22	15.44	15.67	15.79	15.79	15.79	15.79	15.79
11	12.70	12.93	13.16	13.38	13.61	13.84	14.07	14.30	14.53	14.76	14.99	15.22	15.44	15.67	15.90	16.02	16.02	16.02	16.02	16.02
12	12.93	13.16	13.38	13.61	13.84	14.07	14.30	14.53	14.76	14.99	15.22	15.44	15.67	15.90	16.13	16.25	16.25	16.25	16.25	16.25
13	13.16	13.38	13.61	13.84	14.07	14.30	14.53	14.76	14.99	15.22	15.44	15.67	15.90	16.13	16.36	16.48	16.48	16.48	16.48	16.48
14	13.38	13.61	13.84	14.07	14.30	14.53	14.76	14.99	15.22	15.44	15.67	15.90	16.13	16.36	16.59	16.71	16.71	16.71	16.71	16.71
15	13.61	13.84	14.07	14.30	14.53	14.76	14.99	15.22	15.44	15.67	15.90	16.13	16.36	16.59	16.82	16.94	16.94	16.94	16.94	16.94
16	13.84	14.07	14.30	14.53	14.76	14.99	15.22	15.44	15.67	15.90	16.13	16.36	16.59	16.82	17.05	17.17	17.17	17.17	17.17	17.17
17	14.07	14.30	14.53	14.76	14.99	15.22	15.44	15.67	15.90	16.13	16.36	16.59	16.82	17.05	17.28	17.39	17.39	17.39	17.39	17.39
18	14.30	14.53	14.76	14.99	15.22	15.44	15.67	15.90	16.13	16.36	16.59	16.82	17.05	17.28	17.50	17.62	17.62	17.62	17.62	17.62
19	14.53	14.76	14.99	15.22	15.44	15.67	15.90	16.13	16.36	16.59	16.82	17.05	17.28	17.50	17.73	17.85	17.85	17.85	17.85	17.85
20	14.65	14.88	15.11	15.33	15.56	15.79	16.02	16.25	16.48	16.71	16.94	17.17	17.39	17.62	17.85	17.97	17.97	17.97	17.97	17.97
21	14.65	14.88	15.11	15.33	15.56	15.79	16.02	16.25	16.48	16.71	16.94	17.17	17.39	17.62	17.85	17.97	17.97	17.97	17.97	17.97
22	14.65	14.88	15.11	15.33	15.56	15.79	16.02	16.25	16.48	16.71	16.94	17.17	17.39	17.62	17.85	17.97	17.97	17.97	17.97	17.97
23	14.65	14.88	15.11	15.33	15.56	15.79	16.02	16.25	16.48	16.71	16.94	17.17	17.39	17.62	17.85	17.97	17.97	17.97	17.97	17.97
24	14.65	14.88	15.11	15.33	15.56	15.79	16.02	16.25	16.48	16.71	16.94	17.17	17.39	17.62	17.85	17.97	17.97	17.97	17.97	17.97

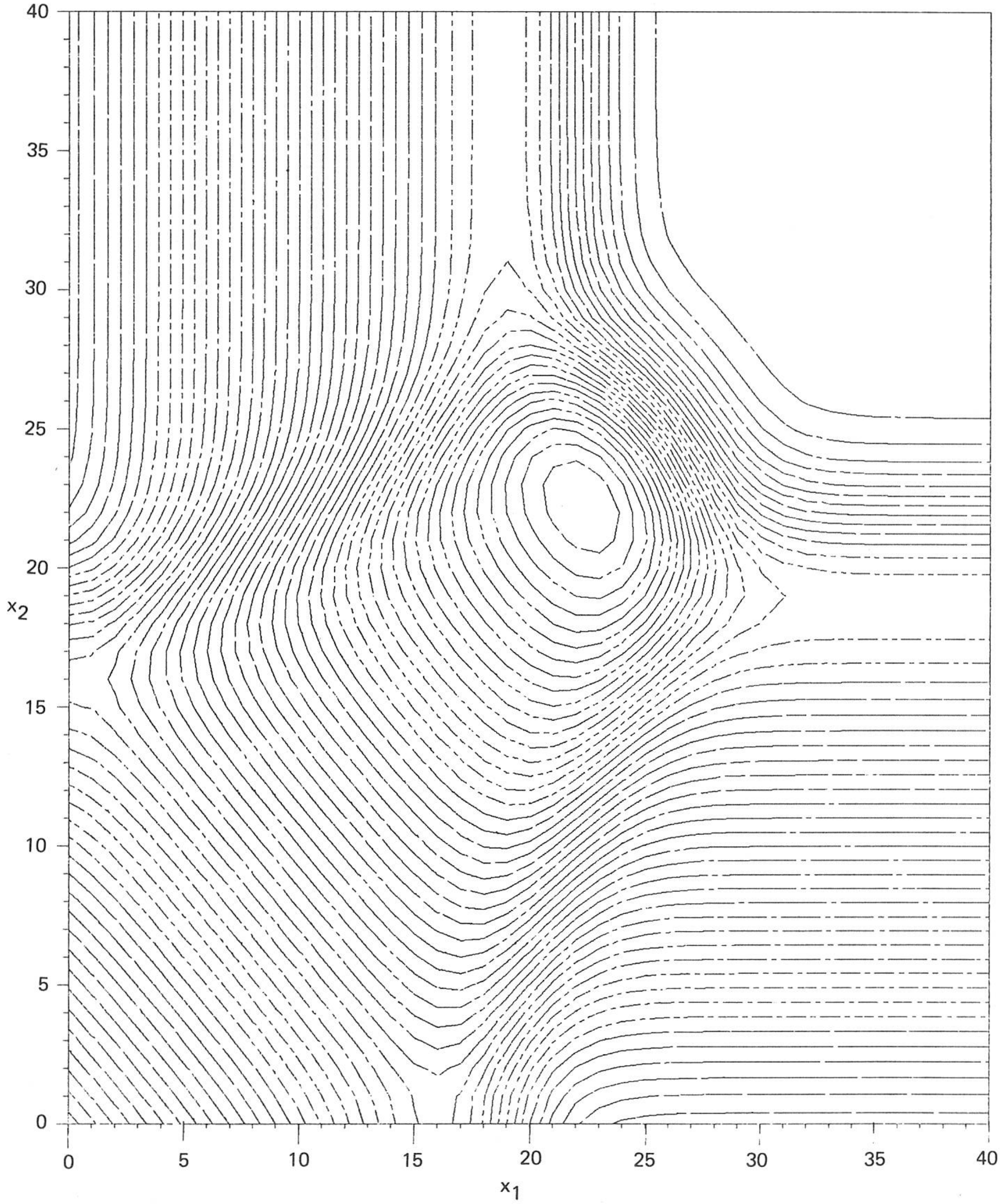
b_2) approximation to ordinary premium $g(\underline{x})$

$$\textit{formula:}\quad \hat{g}=0.2543 \sum_{i=1}^{2} (x_i \wedge \tilde{M})+4.4834$$

Table 5

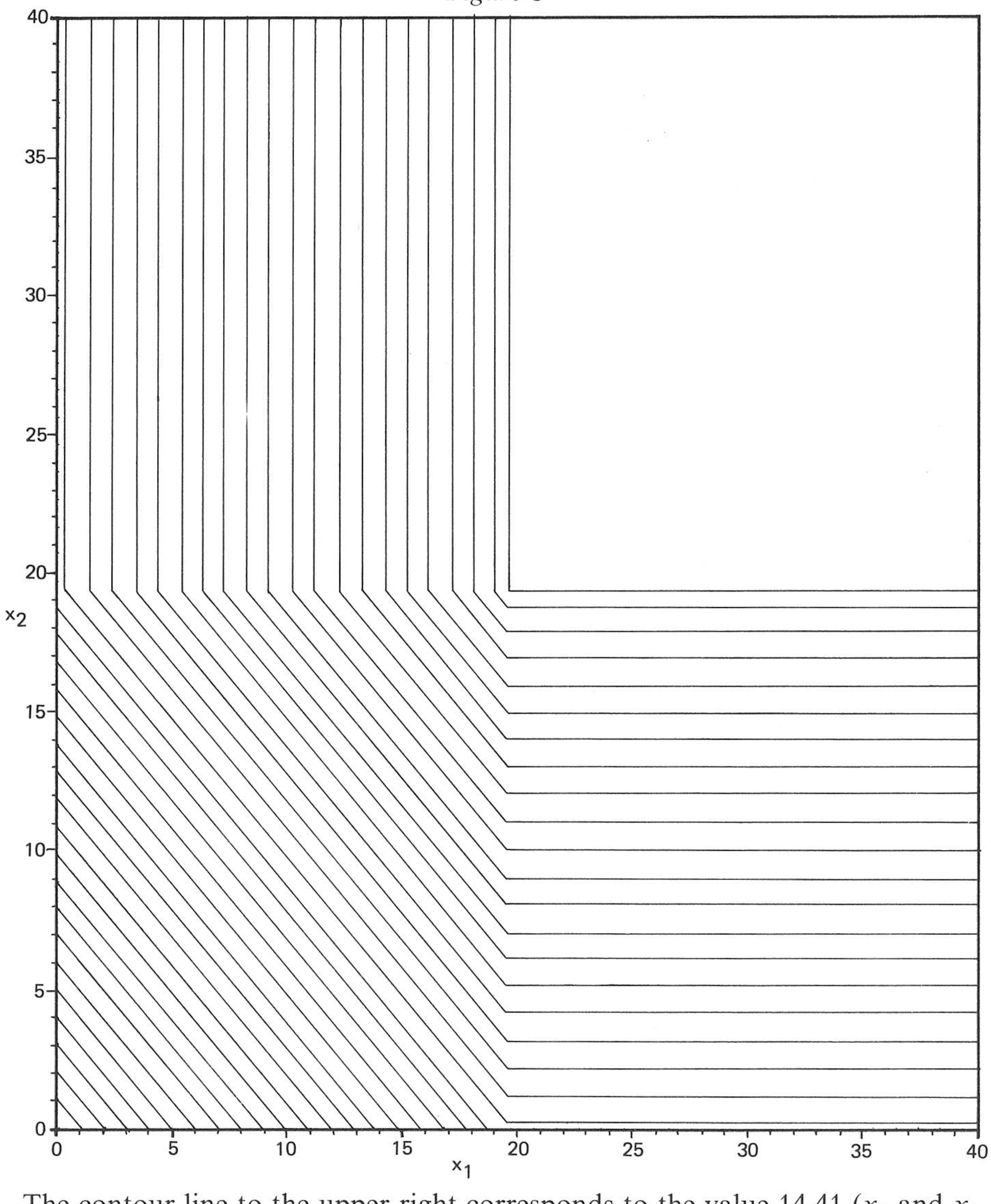
x_1 \ x_2	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
5	7.03	7.28	7.53	7.79	8.04	8.30	8.55	8.81	9.06	9.32	9.57	9.82	10.08	10.33	10.59	10.72	10.72	10.72	10.72	10.72
6	7.28	7.53	7.79	8.04	8.30	8.55	8.81	9.06	9.32	9.57	9.82	10.08	10.33	10.59	10.84	10.97	10.97	10.97	10.97	10.97
7	7.53	7.79	8.04	8.30	8.55	8.81	9.06	9.32	9.57	9.82	10.08	10.33	10.59	10.84	11.10	11.23	11.23	11.23	11.23	11.23
8	7.79	8.04	8.30	8.55	8.81	9.06	9.32	9.57	9.82	10.08	10.33	10.59	10.84	11.10	11.35	11.48	11.48	11.48	11.48	11.48
9	8.04	8.30	8.55	8.81	9.06	9.32	9.57	9.82	10.08	10.33	10.59	10.84	11.10	11.35	11.60	11.74	11.74	11.74	11.74	11.74
10	8.30	8.55	8.81	9.06	9.32	9.57	9.82	10.08	10.33	10.59	10.84	11.10	11.35	11.60	11.86	11.99	11.99	11.99	11.99	11.99
11	8.55	8.81	9.06	9.32	9.57	9.82	10.08	10.33	10.59	10.84	11.10	11.35	11.60	11.86	12.11	12.24	12.24	12.24	12.24	12.24
12	8.81	9.06	9.32	9.57	9.82	10.08	10.33	10.59	10.84	11.10	11.35	11.60	11.86	12.11	12.37	12.50	12.50	12.50	12.50	12.50
13	9.06	9.32	9.57	9.82	10.08	10.33	10.59	10.84	11.10	11.35	11.60	11.86	12.11	12.37	12.62	12.75	12.75	12.75	12.75	12.75
14	9.32	9.57	9.82	10.08	10.33	10.59	10.84	11.10	11.35	11.60	11.86	12.11	12.37	12.62	12.88	13.01	13.01	13.01	13.01	13.01
15	9.57	9.82	10.08	10.33	10.59	10.84	11.10	11.35	11.60	11.86	12.11	12.37	12.62	12.88	13.13	13.26	13.26	13.26	13.26	13.26
16	9.82	10.08	10.33	10.59	10.84	11.10	11.35	11.60	11.86	12.11	12.37	12.62	12.88	13.13	13.38	13.52	13.52	13.52	13.52	13.52
17	10.08	10.33	10.59	10.84	11.10	11.35	11.60	11.86	12.11	12.37	12.62	12.88	13.13	13.38	13.64	13.77	13.77	13.77	13.77	13.77
18	10.33	10.59	10.84	11.10	11.35	11.60	11.86	12.11	12.37	12.62	12.88	13.13	13.38	13.64	13.89	14.02	14.02	14.02	14.02	14.02
19	10.59	10.84	11.10	11.35	11.60	11.86	12.11	12.37	12.62	12.88	13.13	13.38	13.64	13.89	14.15	14.28	14.28	14.28	14.28	14.28
20	10.72	10.97	11.23	11.48	11.74	11.99	12.24	12.50	12.75	13.01	13.26	13.52	13.77	14.02	14.28	14.41	14.41	14.41	14.41	14.41
21	10.72	10.97	11.23	11.48	11.74	11.99	12.24	12.50	12.75	13.01	13.26	13.52	13.77	14.02	14.28	14.41	14.41	14.41	14.41	14.41
22	10.72	10.97	11.23	11.48	11.74	11.99	12.24	12.50	12.75	13.01	13.26	13.52	13.77	14.02	14.28	14.41	14.41	14.41	14.41	14.41
23	10.72	10.97	11.23	11.48	11.74	11.99	12.24	12.50	12.75	13.01	13.26	13.52	13.77	14.02	14.28	14.41	14.41	14.41	14.41	14.41
24	10.72	10.97	11.23	11.48	11.74	11.99	12.24	12.50	12.75	13.01	13.26	13.52	13.77	14.02	14.28	14.41	14.41	14.41	14.41	14.41

Figure 2



The equidistance of the contours equals 0.25.
The top-level contour line corresponds to the value 17.0, whereas the contour-line nearest to the point (0,0) corresponds to the value 3.5.
The contour-line to the upper right corresponds to the value 10.25.

Figure 3



The contour line to the upper right corresponds to the value 14.41 (x_1 and x_2 $\geqq$ trimming point).
The next contour line corresponds to the value 14.25.
The equidistance of the other contour lines equals 0.25.
The contour line nearest to the point (0,0) corresponds to the value 4.75.

Figures 2 and 3 allow again a comparison of g and $\hat{g}$ (now in the case of two observations x_1 and x_2). Both functions are represented by contour lines. The comparison shows similar qualitative phenomena as in the single observation case.

c) Results for $n=5$

truncation point $\tilde{M}=22.83$

formulae:

$$\hat{P}=0.1241 \sum_{i=1}^{5} (x_i \wedge \tilde{M})+7.0561 \quad \text{total premium}$$

$$\hat{g}=0.1378 \sum_{i=1}^{5} (x_i \wedge \tilde{M})+2.2845 \quad \text{ordinary premium}$$

c_1) *approximation to total premium* $P[\underline{x}] = P[x_1, x_2, \underbrace{C_3, C_4, C_5}]$

chosen as fixed parameter values

i) $(C_3, C_4, C_5) = (10, 10, 10)$

Table 6.1

$x_1 \backslash x_2$	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
5	12.02	12.14	12.27	12.39	12.52	12.64	12.76	12.89	13.01	13.14	13.26	13.39	13.51	13.63	13.76	13.88	14.01	14.13	14.23	14.23
6	12.14	12.27	12.39	12.52	12.64	12.76	12.89	13.01	13.14	13.26	13.39	13.51	13.63	13.76	13.88	14.01	14.13	14.25	14.36	14.36
7	12.27	12.39	12.52	12.64	12.76	12.89	13.01	13.14	13.26	13.39	13.51	13.63	13.76	13.88	14.01	14.13	14.25	14.38	14.48	14.48
8	12.39	12.52	12.64	12.76	12.89	13.01	13.14	13.26	13.39	13.51	13.63	13.76	13.88	14.01	14.13	14.25	14.38	14.50	14.61	14.61
9	12.52	12.64	12.76	12.89	13.01	13.14	13.26	13.39	13.51	13.63	13.76	13.88	14.01	14.13	14.25	14.38	14.50	14.63	14.73	14.73
10	12.64	12.76	12.89	13.01	13.14	13.26	13.39	13.51	13.63	13.76	13.88	14.01	14.13	14.25	14.38	14.50	14.63	14.75	14.85	14.85
11	12.76	12.89	13.01	13.14	13.26	13.39	13.51	13.63	13.76	13.88	14.01	14.13	14.25	14.38	14.50	14.63	14.75	14.87	14.98	14.98
12	12.89	13.01	13.14	13.26	13.39	13.51	13.63	13.76	13.88	14.01	14.13	14.25	14.38	14.50	14.63	14.75	14.87	15.00	15.10	15.10
13	13.01	13.14	13.26	13.39	13.51	13.63	13.76	13.88	14.01	14.13	14.25	14.38	14.50	14.63	14.75	14.87	15.00	15.12	15.23	15.23
14	13.14	13.26	13.39	13.51	13.63	13.76	13.88	14.01	14.13	14.25	14.38	14.50	14.63	14.75	14.87	15.00	15.12	15.25	15.35	15.35
15	13.26	13.39	13.51	13.63	13.76	13.88	14.01	14.13	14.25	14.38	14.50	14.63	14.75	14.87	15.00	15.12	15.25	15.37	15.47	15.47
16	13.39	13.51	13.63	13.76	13.88	14.01	14.13	14.25	14.38	14.50	14.63	14.75	14.87	15.00	15.12	15.25	15.37	15.49	15.60	15.60
17	13.51	13.63	13.76	13.88	14.01	14.13	14.25	14.38	14.50	14.63	14.75	14.87	15.00	15.12	15.25	15.37	15.49	15.62	15.72	15.72
18	13.63	13.76	13.88	14.01	14.13	14.25	14.38	14.50	14.63	14.75	14.87	15.00	15.12	15.25	15.37	15.49	15.62	15.74	15.85	15.85
19	13.76	13.88	14.01	14.13	14.25	14.38	14.50	14.63	14.75	14.87	15.00	15.12	15.25	15.37	15.49	15.62	15.74	15.87	15.97	15.97
20	13.88	14.01	14.13	14.25	14.38	14.50	14.63	14.75	14.87	15.00	15.12	15.25	15.37	15.49	15.62	15.74	15.87	15.99	16.09	16.09
21	14.01	14.13	14.25	14.38	14.50	14.63	14.75	14.87	15.00	15.12	15.25	15.37	15.49	15.62	15.74	15.87	15.99	16.12	16.22	16.22
22	14.13	14.25	14.38	14.50	14.63	14.75	14.87	15.00	15.12	15.25	15.37	15.49	15.62	15.74	15.87	15.99	16.12	16.24	16.34	16.34
23	14.23	14.36	14.48	14.61	14.73	14.85	14.98	15.10	15.23	15.35	15.47	15.60	15.72	15.85	15.97	16.09	16.22	16.34	16.45	16.45
24	14.23	14.36	14.48	14.61	14.73	14.85	14.98	15.10	15.23	15.35	15.47	15.60	15.72	15.85	15.97	16.09	16.22	16.34	16.45	16.45

ii) $(C_3, C_4, C_5) = (10, 10, 25)$

Table 6.2

$x_1 \backslash x_2$	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
5	13.61	13.74	13.86	13.98	14.11	14.23	14.36	14.48	14.61	14.73	14.85	14.98	15.10	15.23	15.35	15.47	15.60	15.72	15.83	15.83
6	13.74	13.86	13.98	14.11	14.23	14.36	14.48	14.61	14.73	14.85	14.98	15.10	15.23	15.35	15.47	15.60	15.72	15.85	15.95	15.95
7	13.86	13.98	14.11	14.23	14.36	14.48	14.61	14.73	14.85	14.98	15.10	15.23	15.35	15.47	15.60	15.72	15.85	15.97	16.07	16.07
8	13.98	14.11	14.23	14.36	14.48	14.61	14.73	14.85	14.98	15.10	15.23	15.35	15.47	15.60	15.72	15.85	15.97	16.09	16.20	16.20
9	14.11	14.23	14.36	14.48	14.61	14.73	14.85	14.98	15.10	15.23	15.35	15.47	15.60	15.72	15.85	15.97	16.09	16.22	16.32	16.32
10	14.23	14.36	14.48	14.61	14.73	14.85	14.98	15.10	15.23	15.35	15.47	15.60	15.72	15.85	15.97	16.09	16.22	16.34	16.45	16.45
11	14.36	14.48	14.61	14.73	14.85	14.98	15.10	15.23	15.35	15.47	15.60	15.72	15.85	15.97	16.09	16.22	16.34	16.47	16.57	16.57
12	14.48	14.61	14.73	14.85	14.98	15.10	15.23	15.35	15.47	15.60	15.72	15.85	15.97	16.09	16.22	16.34	16.47	16.59	16.69	16.69
13	14.61	14.73	14.85	14.98	15.10	15.23	15.35	15.47	15.60	15.72	15.85	15.97	16.09	16.22	16.34	16.47	16.59	16.71	16.82	16.82
14	14.73	14.85	14.98	15.10	15.23	15.35	15.47	15.60	15.72	15.85	15.97	16.09	16.22	16.34	16.47	16.59	16.71	16.84	16.94	16.94
15	14.85	14.98	15.10	15.23	15.35	15.47	15.60	15.72	15.85	15.97	16.09	16.22	16.34	16.47	16.59	16.71	16.84	16.96	17.07	17.07
16	14.98	15.10	15.23	15.35	15.47	15.60	15.72	15.85	15.97	16.09	16.22	16.34	16.47	16.59	16.71	16.84	16.96	17.09	17.19	17.19
17	15.10	15.23	15.35	15.47	15.60	15.72	15.85	15.97	16.09	16.22	16.34	16.47	16.59	16.71	16.84	16.96	17.09	17.21	17.31	17.31
18	15.23	15.35	15.47	15.60	15.72	15.85	15.97	16.09	16.22	16.34	16.47	16.59	16.71	16.84	16.96	17.09	17.21	17.34	17.44	17.44
19	15.35	15.47	15.60	15.72	15.85	15.97	16.09	16.22	16.34	16.47	16.59	16.71	16.84	16.96	17.09	17.21	17.34	17.46	17.56	17.56
20	15.47	15.60	15.72	15.85	15.97	16.09	16.22	16.34	16.47	16.59	16.71	16.84	16.96	17.09	17.21	17.34	17.46	17.58	17.69	17.69
21	15.60	15.72	15.85	15.97	16.09	16.22	16.34	16.47	16.59	16.71	16.84	16.96	17.09	17.21	17.34	17.46	17.58	17.71	17.81	17.81
22	15.72	15.85	15.97	16.09	16.22	16.34	16.47	16.59	16.71	16.84	16.96	17.09	17.21	17.34	17.46	17.58	17.71	17.83	17.93	17.93
23	15.83	15.95	16.07	16.20	16.32	16.45	16.57	16.69	16.82	16.94	17.07	17.19	17.31	17.44	17.56	17.69	17.81	17.93	18.04	18.04
24	15.83	15.95	16.07	16.20	16.32	16.45	16.57	16.69	16.82	16.94	17.07	17.19	17.31	17.44	17.56	17.69	17.81	17.93	18.04	18.04

c_2) *approximation to ordinary premium* $g(\underline{x})=g(x_1,x_2,\underbrace{C_1,C_2,C_3}_{\text{chosen as fixed parameters}})$

i) $(C_3,C_4,C_5)=(10,10,10)$

Table 7.1

$x_1 \backslash x_2$	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
5	7.80	7.93	8.07	8.21	8.35	8.49	8.62	8.76	8.90	9.04	9.17	9.31	9.45	9.59	9.73	9.86	10.00	10.14	10.25	10.25
6	7.93	8.07	8.21	8.35	8.49	8.62	8.76	8.90	9.04	9.17	9.31	9.45	9.59	9.73	9.86	10.00	10.14	10.28	10.39	10.39
7	8.07	8.21	8.35	8.49	8.62	8.76	8.90	9.04	9.17	9.31	9.45	9.59	9.73	9.86	10.00	10.14	10.28	10.41	10.53	10.53
8	8.21	8.35	8.49	8.62	8.76	8.90	9.04	9.17	9.31	9.45	9.59	9.73	9.86	10.00	10.14	10.28	10.41	10.55	10.67	10.67
9	8.35	8.49	8.62	8.76	8.90	9.04	9.17	9.31	9.45	9.59	9.73	9.86	10.00	10.14	10.28	10.41	10.55	10.69	10.80	10.80
10	8.49	8.62	8.76	8.90	9.04	9.17	9.31	9.45	9.59	9.73	9.86	10.00	10.14	10.28	10.41	10.55	10.69	10.83	10.94	10.94
11	8.62	8.76	8.90	9.04	9.17	9.31	9.45	9.59	9.73	9.86	10.00	10.14	10.28	10.41	10.55	10.69	10.83	10.97	11.08	11.08
12	8.76	8.90	9.04	9.17	9.31	9.45	9.59	9.73	9.86	10.00	10.14	10.28	10.41	10.55	10.69	10.83	10.97	11.10	11.22	11.22
13	8.90	9.04	9.17	9.31	9.45	9.59	9.73	9.86	10.00	10.14	10.28	10.41	10.55	10.69	10.83	10.97	11.10	11.24	11.36	11.36
14	9.04	9.17	9.31	9.45	9.59	9.73	9.86	10.00	10.14	10.28	10.41	10.55	10.69	10.83	10.97	11.10	11.24	11.38	11.49	11.49
15	9.17	9.31	9.45	9.59	9.73	9.86	10.00	10.14	10.28	10.41	10.55	10.69	10.83	10.97	11.10	11.24	11.38	11.52	11.63	11.63
16	9.31	9.45	9.59	9.73	9.86	10.00	10.14	10.28	10.41	10.55	10.69	10.83	10.97	11.10	11.24	11.38	11.52	11.65	11.77	11.77
17	9.45	9.59	9.73	9.86	10.00	10.14	10.28	10.41	10.55	10.69	10.83	10.97	11.10	11.24	11.38	11.52	11.65	11.79	11.91	11.91
18	9.59	9.73	9.86	10.00	10.14	10.28	10.41	10.55	10.69	10.83	10.97	11.10	11.24	11.38	11.52	11.65	11.79	11.93	12.04	12.04
19	9.73	9.86	10.00	10.14	10.28	10.41	10.55	10.69	10.83	10.97	11.10	11.24	11.38	11.52	11.65	11.79	11.93	12.07	12.18	12.18
20	9.86	10.00	10.14	10.28	10.41	10.55	10.69	10.83	10.97	11.10	11.24	11.38	11.52	11.65	11.79	11.93	12.07	12.21	12.32	12.32
21	10.00	10.14	10.28	10.41	10.55	10.69	10.83	10.97	11.10	11.24	11.38	11.52	11.65	11.79	11.93	12.07	12.21	12.34	12.46	12.46
22	10.14	10.28	10.41	10.55	10.69	10.83	10.97	11.10	11.24	11.38	11.52	11.65	11.79	11.93	12.07	12.21	12.34	12.48	12.60	12.60
23	10.25	10.39	10.53	10.67	10.80	10.94	11.08	11.22	11.36	11.49	11.63	11.77	11.91	12.04	12.18	12.32	12.46	12.60	12.71	12.71
24	10.25	10.39	10.53	10.67	10.80	10.94	11.08	11.22	11.36	11.49	11.63	11.77	11.91	12.04	12.18	12.32	12.46	12.60	12.71	12.71

ii) $(C_3, C_4, C_5) = (10, 10, 25)$

Table 7.2

$x_1 \backslash x_2$	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
5	9.56	9.70	9.84	9.98	10.12	10.25	10.39	10.53	10.67	10.80	10.94	11.08	11.22	11.36	11.49	11.63	11.77	11.91	12.02	12.02
6	9.70	9.84	9.98	10.12	10.25	10.39	10.53	10.67	10.80	10.94	11.08	11.22	11.36	11.49	11.63	11.77	11.91	12.04	12.16	12.16
7	9.84	9.98	10.12	10.25	10.39	10.53	10.67	10.80	10.94	11.08	11.22	11.36	11.49	11.63	11.77	11.91	12.04	12.18	12.30	12.30
8	9.98	10.12	10.25	10.39	10.53	10.67	10.80	10.94	11.08	11.22	11.36	11.49	11.63	11.77	11.91	12.04	12.18	12.32	12.43	12.43
9	10.12	10.25	10.39	10.53	10.67	10.80	10.94	11.08	11.22	11.36	11.49	11.63	11.77	11.91	12.04	12.18	12.32	12.46	12.57	12.57
10	10.25	10.39	10.53	10.67	10.80	10.94	11.08	11.22	11.36	11.49	11.63	11.77	11.91	12.04	12.18	12.32	12.46	12.60	12.71	12.71
11	10.39	10.53	10.67	10.80	10.94	11.08	11.22	11.36	11.49	11.63	11.77	11.91	12.04	12.18	12.32	12.46	12.60	12.73	12.85	12.85
12	10.53	10.67	10.80	10.94	11.08	11.22	11.36	11.49	11.63	11.77	11.91	12.04	12.18	12.32	12.46	12.60	12.73	12.87	12.99	12.99
13	10.67	10.80	10.94	11.08	11.22	11.36	11.49	11.63	11.77	11.91	12.04	12.18	12.32	12.46	12.60	12.73	12.87	13.01	13.12	13.12
14	10.80	10.94	11.08	11.22	11.36	11.49	11.63	11.77	11.91	12.04	12.18	12.32	12.46	12.60	12.73	12.87	13.01	13.15	13.26	13.26
15	10.94	11.08	11.22	11.36	11.49	11.63	11.77	11.91	12.04	12.18	12.32	12.46	12.60	12.73	12.87	13.01	13.15	13.29	13.40	13.40
16	11.08	11.22	11.36	11.49	11.63	11.77	11.91	12.04	12.18	12.32	12.46	12.60	12.73	12.87	13.01	13.15	13.29	13.42	13.54	13.54
17	11.22	11.36	11.49	11.63	11.77	11.91	12.04	12.18	12.32	12.46	12.60	12.73	12.87	13.01	13.15	13.29	13.42	13.56	13.68	13.68
18	11.36	11.49	11.63	11.77	11.91	12.04	12.18	12.32	12.46	12.60	12.73	12.87	13.01	13.15	13.29	13.42	13.56	13.70	13.81	13.81
19	11.49	11.63	11.77	11.91	12.04	12.18	12.32	12.46	12.60	12.73	12.87	13.01	13.15	13.29	13.42	13.56	13.70	13.84	13.95	13.95
20	11.63	11.77	11.91	12.04	12.18	12.32	12.46	12.60	12.73	12.87	13.01	13.15	13.29	13.42	13.56	13.70	13.84	13.97	14.09	14.09
21	11.77	11.91	12.04	12.18	12.32	12.46	12.60	12.73	12.87	13.01	13.15	13.29	13.42	13.56	13.70	13.84	13.97	14.11	14.23	14.23
22	11.91	12.04	12.18	12.32	12.46	12.60	12.73	12.87	13.01	13.15	13.29	13.42	13.56	13.70	13.84	13.97	14.11	14.25	14.36	14.36
23	12.02	12.16	12.30	12.43	12.57	12.71	12.85	12.99	13.12	13.26	13.40	13.54	13.68	13.81	13.95	14.09	14.23	14.36	14.48	14.48
24	12.02	12.16	12.30	12.43	12.57	12.71	12.85	12.99	13.12	13.26	13.40	13.54	13.68	13.81	13.95	14.09	14.23	14.36	14.48	14.48

10 Final Remarks

The Data Trimmed Credibility Formulae seem quite appropriate for Experience Rating in the presence of catastrophic (or as called in this paper *excess*) claims. With this intuitive background in our minds we have in our explicit calculations been looking at deviations from ordinary claims towards the *higher side only*. Obviously the normal distribution being symmetric one could also observe "outliers" to ordinary claims towards the lower side hence leading to a truncation at the lower end as well. But of course our assumption of normally distributed claims should only be seen as an approximation to the real world, and it is our feeling that the approximation is particularly bad at the lower tail of the distribution.
In any case truncation at the upper end of the distribution is introducing an additional parameter into the credibility formulae and we hope to have demonstrated in this paper that the labour caused by the new parameter can be worthwhile indeed.

Hans Bühlmann
Abt. Mathematik
ETH-Zentrum
8092 Zürich

Alois Gisler
Winterthur Insurances
General-Guisan-Strasse 40
8401 Winterthur

William S. Jewell
IEOR, 4173 EH
University of California
Berkely, CA 94720
USA

Bibliography

[1] *Gisler, A.:* Optimales Stutzen von Beobachtungen im Credibility-Modell (ETH Thesis 1980). See also *Gisler, A.:* Optimum Trimming of Data in the Credibility Model, BASA 1980 (3).

Appendix

For the interested reader we are attaching the explicit calculations leading to formulae (19) and (20).

A: Calculations leading to formula (19)

$$b_1 = E[\mathrm{Cov}\,[X_1 \wedge M,\ X_2/\theta]] + \mathrm{Cov}\,[E[X_1 \wedge M/\theta],\ E[X_2/\theta]]$$

$\mathrm{Cov}\,[X_1 \wedge M,\ X_2/\theta] = 0$, because X_1, X_2 are conditionally independent.

Hence

$$b_1 = \text{Cov}\ [(1-\pi)\mu_o^M(\theta) + \pi\mu_e^M,\ (1-\pi)\mu_o(\theta) + \pi\mu_e],$$

or

$$\boxed{b_1 = (1-\pi)^2\ \text{Cov}\ [\mu_o^M(\theta),\ \mu_o(\theta)]}$$

and analogously (with $X_2 \wedge M$ instead of X_2)

$$\boxed{b_2 = (1-\pi)^2\ \text{Var}\ [\mu_o^M(\theta)].}$$

Let be $Y = 1_A$ where A denotes the event $\{X \text{ is ordinary}\}$. Then

$$\begin{aligned}\text{Var}\ [X \wedge M/\theta] &= E[\text{Var}\ [X \wedge M/\theta,\ Y]/\theta] + \text{Var}\ [E[X \wedge M/\theta, Y]/\theta] \\ &= (1-\pi)\sigma_o^{2M}(\theta) + \pi\sigma_e^{2M} + \pi(1-\pi)\ (\mu_o^M(\theta) - \mu_e^M)^2.\end{aligned}$$

Hence

$$\begin{aligned}b_3 &= \text{Var}\ [X \wedge M] \\ &= E[\text{Var}\ [X \wedge M/\theta]] + \text{Var}\ [(1-\pi)\mu_o^M(\theta) + \mu_e^M], \quad \text{or}\end{aligned}$$

$$\boxed{b_3 = (1-\pi)E[\sigma_o^{2M}(\theta)] + \pi\sigma_e^{2M} + \pi(1-\pi)E[(\mu_o^M(\theta) - \mu_e^M)^2] + (1-\pi)^2\ \text{Var}\ \mu_o^M(\theta)].}$$

B: **Calculations leading to formula (20)**

i) Preparations

In the following we put $r = \sqrt{v}$, $s = \sqrt{w}$ and $\sigma_o = \sqrt{v+w} = \sqrt{r^2+s^2}$. Furthermore we denote by $\Phi(x)$ the standardized normal distribution function and by $\varphi(x)$ the standardized normal density function.
By convolution we get

$$\int_{-\infty}^{\infty} \frac{1}{s}\,\varphi\left(\frac{x-\mu}{s}\right)\Phi\left(\frac{M-x}{r}\right)dx = \Phi\left(\frac{M-\mu}{\sigma_o}\right)$$

$$\int_{-\infty}^{\infty} \frac{1}{rs}\,\varphi\left(\frac{x-\mu}{s}\right)\varphi\left(\frac{M-x}{r}\right)dx = \frac{1}{\sigma_0}\,\varphi\left(\frac{M-\mu}{\sigma_o}\right).$$

Noting that $\varphi'(x) = -x\varphi(x)$ integration by parts gives

$$\int_{-\infty}^{\infty} (x-\mu)\varphi\left(\frac{x-\mu}{s}\right)\Phi\left(\frac{M-x}{r}\right)dx = -\frac{s^2}{r}\int_{-\infty}^{\infty} \varphi\left(\frac{x-\mu}{s}\right)\varphi\left(\frac{M-x}{r}\right)dx$$

and thus

$$\int_{-\infty}^{\infty} x\varphi\left(\frac{x-\mu}{s}\right)\Phi\left(\frac{M-x}{r}\right)dx = \mu s\Phi\left(\frac{M-\mu}{\sigma_o}\right) - \frac{s^3}{\sigma_o}\,\varphi\left(\frac{M-\mu}{\sigma_o}\right).$$

Because of

$$\varphi\left(\frac{x-\mu}{s}\right)\varphi\left(\frac{M-x}{r}\right) = \varphi\left(\frac{M-\mu}{\sigma_o}\right)\varphi\left(\frac{x-\tilde{\mu}}{\tilde{\sigma}}\right)$$

where

$$\tilde{\mu} = \frac{r^2\mu + s^2 M}{r^2 + s^2} \quad \text{and} \quad \tilde{\sigma} = \frac{rs}{\sigma_o}$$

we obtain

$$\int_{-\infty}^{\infty} x\varphi\left(\frac{x-\mu}{s}\right)\varphi\left(\frac{M-x}{r}\right)dx = \varphi\left(\frac{M-\mu}{\sigma_o}\right)\cdot\tilde{\mu}\cdot\tilde{\sigma}$$

$$\int_{-\infty}^{\infty} x^2\varphi\left(\frac{x-\mu}{s}\right)\varphi\left(\frac{M-x}{r}\right)dx = \varphi\left(\frac{M-\mu}{\sigma_o}\right)\cdot\tilde{\sigma}\cdot(\tilde{\mu}^2+\tilde{\sigma}^2).$$

Integration by parts gives

$$\int_{-\infty}^{\infty} x(x-\mu)\,\varphi\left(\frac{x-\mu}{s}\right)\Phi\left(\frac{M-x}{r}\right)dx = s^2\int_{-\infty}^{\infty}\varphi\left(\frac{x-\mu}{s}\right)\Phi\left(\frac{M-x}{r}\right)dx$$

$$-\frac{s^2}{r}\int_{-\infty}^{\infty} x\varphi\left(\frac{x-\mu}{s}\right)\varphi\left(\frac{M-x}{r}\right)dx$$

and thus using the above formulae

$$\int_{-\infty}^{\infty} x^2\varphi\left(\frac{x-\mu}{s}\right)\Phi\left(\frac{M-x}{r}\right)dx = s(\mu^2+s^2)\Phi\left(\frac{M-\mu}{\sigma_o}\right)$$

$$-\left(\frac{s}{\sigma_o}\right)^3(2r^2\mu+(M+\mu)s^2)\varphi\left(\frac{M-\mu}{\sigma_o}\right).$$

ii) Actual calculations

$$\mu_o^M(\theta) = \int_{-\infty}^{M}\frac{x}{r}\,\varphi\left(\frac{x-\theta}{r}\right)dx + M\cdot Pr[X\geqq M/\theta]$$

$$= -r\varphi\left(\frac{M-\theta}{r}\right) + \theta Pr[X\leqq M/\theta] + MPr[X\geqq M/\theta]$$

$$= M + (\theta - M)\Phi\left(\frac{M-\theta}{r}\right) - r\varphi\left(\frac{M-\theta}{r}\right)$$

$$\mu_o(\theta) = \theta.$$

Applying the formulae derived in i) we get by straightforward calculations

$$E[\mu_o^M(\theta)] \quad = \int_{-\infty}^{\infty} \mu_o^M(\theta)\, \frac{1}{s}\, \varphi\left(\frac{\theta - m_0}{s}\right) d\theta$$

$$= M - (M - m_0)\, \Phi\left(\frac{M - m_0}{\sigma_o}\right) - \sigma_o \varphi\left(\frac{M - m_0}{\sigma_o}\right)$$

$$E[\theta \cdot \mu_o^M(\theta)] = M m_0 + (s^2 + m_0^2 - M m_0)\, \Phi\left(\frac{M - m_0}{\sigma_o}\right) - m_0 \sigma_o \varphi\left(\frac{M - m_0}{\sigma_o}\right).$$

Hence

$$\text{Cov}\ [\mu_o^M(\theta),\ \mu_o(\theta)] = E[\theta \cdot \mu_o^M(\theta)] - m_0 E[\mu_o^M(\theta)] = s^2 \Phi\left(\frac{M - m_0}{\sigma_o}\right)$$

and

$$\boxed{b_1 = (1 - \pi)^2 w \Phi\left(\frac{M - m_0}{\sigma_o}\right).}$$

As $\text{Cov}\ [X_1 \wedge M,\ X_2 \wedge M / X_1, X_2 \text{ ordinary}]$

$= \text{Cov}\ [U_1 \wedge M,\ U_2 \wedge M]$

$= E[\text{Cov}\ [U_1 \wedge M,\ U_2 \wedge M / \theta]] + \text{Cov}\ [E[U_1 \wedge M / \theta],\ E[U_2 \wedge M / \theta]]$

$= \text{Var}\ [\mu_o^M(\theta)]$, we conclude from 19)

$$\boxed{b_2 = (1 - \pi)^2\ \text{Cov}\ [U_1 \wedge M,\ U_2 \wedge M].}$$

To obtain a closed formula for b_3, observe

$$\int_{-\infty}^{M} x(x - \mu)\, \frac{1}{\sigma}\, \varphi(x - \mu)\, dx = -x\sigma\varphi\left(\frac{x - \mu}{\sigma}\right)\Bigg|_{-\infty}^{M} + \sigma \int_{-\infty}^{M} \varphi\left(\frac{x - \mu}{\sigma}\right) dx$$

$$= -\sigma M \varphi\left(\frac{M - \mu}{\sigma}\right) + \sigma^2 \Phi\left(\frac{M - \mu}{\sigma}\right)$$

$$\int_{-\infty}^{M} x^2\, \frac{1}{\sigma}\, \varphi(x - \mu)\, dx = -\sigma(M + \mu)\, \varphi\left(\frac{M - \mu}{\sigma}\right) + (\mu^2 + \sigma^2)\, \Phi\left(\frac{M - \mu}{\sigma}\right).$$

According to 1) the density function of X is

$$f(x) = \int (1 - \pi)\, p_o(x/\theta)\, dU(\theta) + \pi p_e(x) = (1 - \pi)\, p_o(x) + \pi p_e(x)$$

with (see 6.1 and 6.2)

$$p_o(x) = \frac{1}{\sigma_o}\,\varphi\left(\frac{x - m_0}{\sigma_o}\right)$$

$$p_e(x) = \frac{1}{\sigma_e}\,\varphi\left(\frac{x - \mu_e}{\sigma_e}\right).$$

Hence

$$\boxed{\begin{aligned} E[X \wedge M)^2] = (1-\pi)\,&\left\{(m_0^2 + \sigma_o^2)\,\Phi\left(\frac{M - m_0}{\sigma_o}\right) - \sigma_o(M + m_0)\,\varphi\left(\frac{M - m_0}{\sigma_o}\right)\right\} \\ &+ \pi\left\{(\mu_e^2 + \sigma_e^2)\,\Phi\left(\frac{M - \mu_e}{\sigma_e}\right) - \sigma_e(M + \mu_e)\,\varphi\left(\frac{M - \mu_e}{\sigma_e}\right)\right\} \\ &+ M^2\left\{1 - (1-\pi)\,\Phi\left(\frac{M - m_0}{\sigma_o}\right) - \pi\Phi\left(\frac{M - \mu_e}{\sigma_e}\right)\right\} \\ = A & \end{aligned}}$$

The same calculations as at the beginning of ii) leading to the formula for $\mu_o^M(\theta)$ are repeated to obtain $E[X \wedge M]$, of course with different parameter values. From this calculation we obtain

$$\boxed{\begin{aligned} E[X \wedge M] = M + (1-\pi)\,&\left\{(m_0 - M)\,\Phi\left(\frac{M - m_0}{\sigma_o}\right) - \sigma_o\varphi\left(\frac{M - m_0}{\sigma_o}\right)\right\} \\ &+ \pi\left\{(\mu_e - M)\,\Phi\left(\frac{M - \mu_e}{\sigma_e}\right) - \sigma_e\varphi\left(\frac{M - \mu_e}{\sigma_e}\right)\right\} \\ = B. & \end{aligned}}$$

We can now, finally, write

$$\boxed{b_3 = \mathrm{Var}\,[X \wedge M] = A - B^2.}$$

Summary

Very large claims represent a dilemma to the Ratemaker: To what extent should they be included in the claims load? We advocate that Credibility Techniques combined with Data Trimming provide the right tool for this problem. This is illustrated in the context of a model which generates both "ordinary" and "excess" (=catastrophic) claims. In this situation "optimum forecasting" (using Bayes estimation techniques) and "credibility techniques combined with data trimming" lead to almost identical results.

Zusammenfassung

Sehr grosse Schäden stellen ein besonderes Problem für die Prämienberechnung dar: Wie weit sollen sie in die Schadenlast eingeschlossen werden? Wir sind der Meinung, dass «Credibility mit Stutzen» das richtige Werkzeug für diese Situation darstellt. Dies ist illustriert im Rahmen eines Modells, welches «gewöhnliche» und «Exzess»-(Katastrophen-)Schäden produziert. In dieser Situation liefern «optimale Voraussage» (durch Bayes-Schätzungen) und «Credibility mit Stutzen» gleichwertige Resultate.

Résumé

Les sinistres de très grande importance posent un problème pour la détermination d'une prime: dans quelle mesure faut-il les prendre en considération? Pour résoudre ce problème, nous proposons la technique de la «crédibilité combinée avec la troncation». Notre point de vue est présenté à l'aide d'un modèle qui produit des sinistres «ordinaires» et des sinistres «excédents» (catastrophiques). Dans cette situation les résultats obtenus par la «prévision optimale» (selon les méthodes de Bayes) et par la technique de la «crédibilité combinée avec la troncation» sont presque identiques.