

Zeitschrift: Mitteilungen / Vereinigung Schweizerischer Versicherungsmathematiker
= Bulletin / Association des Actuaires Suisses = Bulletin / Association of Swiss Actuaries

Herausgeber: Vereinigung Schweizerischer Versicherungsmathematiker

Band: - (1981)

Heft: 2

Rubrik: Kurzmitteilungen

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D. Kurzmitteilungen

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Some comments to D. Zagorac: Ein Beitrag zur
Intervallschätzung der Glaubwürdigkeitsparameter

(Mitteilungen 1/1981, 67–75)

1. Referring to the Fisher Lemma Zagorac states that

$$\frac{(nN-1)W}{\sigma^2 + \sigma_0^2} = \sum_i \sum_j \frac{(X_{ij} - m_0)^2}{\sigma^2 + \sigma_0^2} - nN \frac{(M - m_0)^2}{\sigma^2 + \sigma_0^2}$$

is χ^2 -distributed with $nN - 1$ degrees of freedom. However, in the present case one has to be a bit careful, as for fixed j the X_{ij} 's are dependent when $\sigma_0^2 \neq 0$. By use of orthogonal transformations it can be shown that

$$Q_0 = \sum_{j=1}^N \sum_{i=1}^n (X_{ij} - X_{.j})^2,$$

$$Q = n \sum_{j=1}^N (X_{.j} - M)^2,$$

and M are independent, and that Q_0/σ^2 and $Q/(\sigma^2 + n\sigma_0^2)$ are χ^2 -distributed with $N(n-1)$ resp. $N-1$ degrees of freedom.

From this we see that

$$\frac{(nN-1)W}{\sigma^2 + \sigma_0^2} = \frac{Q_0 + Q}{\sigma^2 + \sigma_0^2}$$

is χ^2 -distributed with $nN - 1$ degrees of freedom if and only if $\sigma_0^2 = 0$.

2. Referring to the above, the confidence regions proposed in Zagorac's (1981) Sections 7.1, 7.2, and 7.4 seem questionable, and in the rest of this note I shall propose other confidence regions.

¹ The present note was submitted for publication as a «Letter to the Editor».

3. Confidence interval for m_0 .

As $X_{.1}, \dots, X_{.N}$ are independent and identically normally distributed with mean m_0 ,

$$T = \frac{M - m_0}{\sqrt{Q}} \sqrt{nN(N-1)}$$

is Student distributed with $N-1$ degrees of freedom. From this follows that

$$J_1 = \left[M - t_{N-1,1-\frac{\epsilon}{2}} \sqrt{\frac{Q}{nN(N-1)}}, M + t_{N-1,1-\frac{\epsilon}{2}} \sqrt{\frac{Q}{nN(N-1)}} \right],$$

with $t_{N-1,1-\frac{\epsilon}{2}}$ being the $1-\frac{\epsilon}{2}$ fractile of the Student distribution with $N-1$ degrees of freedom, is a $1-\epsilon$ confidence interval for m_0 .

4. Confidence interval for σ_0^2 .

Let $\chi_{v,\alpha}^2$ denote the α fractile in the χ^2 -distribution with v degrees of freedom. We have

$$Pr\left(\frac{Q}{\sigma^2 + n\sigma_0^2} \geq \chi_{N-1,\epsilon}^2\right) = 1 - \epsilon,$$

that is,

$$Pr\left(\frac{Q}{n\chi_{N-1,\epsilon}^2} \geq \frac{\sigma^2}{n} + \sigma_0^2\right) = 1 - \epsilon.$$

Then

$$Pr\left(\frac{Q}{n\chi_{N-1,\epsilon}^2} \geq \sigma_0^2\right) > 1 - \epsilon,$$

and from this follows that

$$J_2 = \left(0, \frac{Q}{n\chi_{N-1,\epsilon}^2} \right]$$

is a $1-\epsilon$ confidence interval for σ_0^2 .

5. Confidence interval for σ^2 .

Zagorac uses that Q_0/σ^2 is χ^2 -distributed with $N(n-1)$ degrees of freedom and gets as a $1-\epsilon$ confidence interval for σ^2

$$I_3 = \left[\frac{N(n-1)V}{q_1}, \frac{N(n-1)V}{q_2} \right],$$

where q_1 and q_2 are determined such that

$$\Pr\left(q_2 \leqq \frac{Q_0}{\sigma^2} \leqq q_1\right) = 1 - \epsilon. \quad (1)$$

Zagorac uses $q_1 = \chi_{N(n-1), 1-\frac{\epsilon}{2}}^2$ and $q_2 = \chi_{N(n-1), \frac{\epsilon}{2}}^2$.

Referring to Sverdrup (1967, Sections XIII 3.2 and XIII 4.2) I would recommend q_1 and q_2 determined by (1) and

$$\frac{\log q_1 - \log q_2}{q_1 - q_2} = \frac{1}{N(n-1)};$$

this will make the confidence interval unbiased.

6. Confidence interval for $\kappa = \sigma^2/\sigma_0^2$

The credibility estimator of m_j is

$$\tilde{m}_j = \frac{n}{n+\kappa} X_{.j} + \frac{\kappa}{n+\kappa} m_0.$$

We see that in this formula σ^2 and σ_0^2 appear only through κ . Hence, it may be interesting to construct a confidence interval for κ .

For this purpose we use that $(1+n\kappa^{-1})^{-1} F$ with

$$F = \frac{Q}{Q_0} \frac{N(n-1)}{N-1}$$

is Fisher distributed with $N-1$ and $N(n-1)$ degrees of freedom. Let f_1 and f_2 be determined such that

$$\Pr(f_1 \leqq (1+n\kappa^{-1})^{-1} F \leqq f_2) = 1 - \epsilon. \quad (2)$$

Then

$$\Pr\left(\frac{n}{F-1} \leqq \kappa \leqq \frac{n}{f_2-1}\right) = 1 - \epsilon,$$

and

$$J_3 = \left[\frac{n}{F} - 1, \frac{n}{f_2} - 1 \right]$$

is a $1 - \epsilon$ confidence interval for κ .

A simple choice of f_1 and f_2 would be the $\frac{\epsilon}{2}$ resp. $1 - \frac{\epsilon}{2}$ fractile of the Fisher distribution with $N - 1$ and $N(n - 1)$ degrees of freedom. However, to get the confidence interval unbiased, I suggest to find f_1 and f_2 from the equations (2) and

$$\frac{\log\left(1 + f_2 \frac{N-1}{N(n-1)}\right) - \log\left(1 + f_1 \frac{N-1}{N(n-1)}\right)}{\log f_2 - \log f_1} = \frac{N-1}{Nn-1}.$$

7. Confidence regions for (m_0, κ) and $(m_0, \sigma^2, \sigma_0^2)$.

From the Bonfferoni inequality we get

$$Pr((m_0 \in J_1) \cap (\kappa \in J_3)) \geq 1 - 2\epsilon,$$

and thus

$$A_1 = J_1 \times J_3$$

is a $1 - 2\epsilon$ confidence region for (m_0, κ) .

By using the independence of M , Q , and Q_0 we may construct a confidence region for $(m_0, \sigma^2, \sigma_0^2)$ without using inequalities like the Bonfferoni one.

Let $g_{1-\frac{\epsilon}{2}}$ denote the $1 - \frac{\epsilon}{2}$ fractile of the normal distribution $N(0, 1)$. Then, as M

has the distribution $N\left(m_0, \sqrt{\frac{1}{N}\left(\frac{\sigma^2}{n} + \sigma_0^2\right)}\right)$,

$$Pr\left(M - g_{1-\frac{\epsilon}{2}} \sqrt{\frac{\sigma^2 + n\sigma_0^2}{nN}} \leq m_0 \leq M + g_{1-\frac{\epsilon}{2}} \sqrt{\frac{\sigma^2 + n\sigma_0^2}{nN}}\right) = 1 - \epsilon,$$

and for given $\sigma^2 + n\sigma_0^2$

$$J_4(\sigma^2 + n\sigma_0^2) = \left[M - g_{1-\frac{\epsilon}{2}} \sqrt{\frac{\sigma^2 + n\sigma_0^2}{nN}}, M + g_{1-\frac{\epsilon}{2}} \sqrt{\frac{\sigma^2 + n\sigma_0^2}{nN}} \right]$$

is a $1 - \epsilon$ confidence interval for m_0 .

Using that $Q/(\sigma^2 + n\sigma_0^2)$ is χ^2 -distributed with $N - 1$ degrees of freedom, we get, analogously to Section 5, that

$$J_5 = \left[\frac{Q}{p_1}, \frac{Q}{p_2} \right],$$

with p_1 and p_2 satisfying

$$\Pr \left(p_2 \leq \frac{Q}{\sigma^2 + n\sigma_0^2} \leq p_1 \right) = 1 - \epsilon$$

is a $1 - \epsilon$ confidence interval for $\sigma^2 + n\sigma_0^2$.

From the independence of M , Q , and Q_0 we now get

$$\begin{aligned} \Pr((m_0 \in J_4(\sigma^2 + n\sigma_0^2)) \cap (\sigma^2 + n\sigma_0^2 \in J_5) \cap (\sigma^2 \in I_3)) &= \\ \Pr(m_0 \in J_4(\sigma^2 + n\sigma_0^2)) \Pr(\sigma^2 + n\sigma_0^2 \in J_5) \Pr(\sigma^2 \in I_3) &= \\ (1 - \epsilon)^3, \end{aligned}$$

and

$$A_2 = \{(x, y, z) : x \in J_4(y + nz), z + ny \in J_5, z \in I_3\}$$

is a $(1 - \epsilon)^3$ confidence region for $(m_0, \sigma_0^2, \sigma^2)$.

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References

- Sverdrup, E. (1967): *Laws and chance variations*. Vol. II. North-Holland Publishing Company.
Amsterdam.
- Zagorac, D. (1981): Ein Beitrag zur Intervallschätzung der Glaubwürdigkeitsparameter. *Mitteilungen der Vereinigung schweizerischer Versicherungsmathematiker*, 67–75.

