Zeitschrift: Mitteilungen / Vereinigung Schweizerischer Versicherungsmathematiker = Bulletin / Association des Actuaires Suisses = Bulletin / Association of Swiss Actuaries

Herausgeber: Vereinigung Schweizerischer Versicherungsmathematiker

Band: - (1981)

Heft: 2

Rubrik: Kurzmitteilungen

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. Mehr erfahren

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. En savoir plus

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. Find out more

Download PDF: 11.02.2026

ETH-Bibliothek Zürich, E-Periodica, https://www.e-periodica.ch

D. Kurzmitteilungen

BJØRN SUNDT, Oslo/Zurich[1]

Some comments to D. Zagorac: Ein Beitrag zur Intervallschätzung der Glaubwürdigkeitsparameter

(Mitteilungen 1/1981, 67–75)

1. Referring to the Fisher Lemma Zagorac states that

$$\frac{(nN-1)W}{\sigma^2+\sigma_0^2}=\sum_i\sum_j\frac{(X_{ij}-m_0)^2}{\sigma^2+\sigma_0^2}-nN\frac{(M-m_0)^2}{\sigma^2+\sigma_0^2}$$

is χ^2-distributed with $nN-1$ degrees of freedom. However, in the present case one has to be a bit careful, as for fixed j the X_{ij}'s are dependent when $\sigma_0^2 \neq 0$. By use of orthogonal transformations it can be shown that

$$Q_0=\sum_{j=1}^{N}\sum_{i=1}^{n}(X_{ij}-X_{.j})^2,$$

$$Q=n\sum_{j=1}^{N}(X_{.j}-M)^2,$$

and M are independent, and that Q_0/σ^2 and $Q/(\sigma^2+n\sigma_0^2)$ are χ^2-distributed with $N(n-1)$ resp. $N-1$ degrees of freedom.
From this we see that

$$\frac{(nN-1)W}{\sigma^2+\sigma_0^2}=\frac{Q_0+Q}{\sigma^2+\sigma_0^2}$$

is χ^2-distributed with $nN-1$ degrees of freedom if and only if $\sigma_0^2=0$.

2. Referring to the above, the confidence regions proposed in Zagorac's (1981) Sections 7.1, 7.2, and 7.4 seem questionable, and in the rest of this note I shall propose other confidence regions.

[1] The present note was submitted for publication as a «Letter to the Editor».

3. Confidence interval for m_0.
As $X_{.1}, \ldots, X_{.N}$ are independent and identically normally distributed with mean m_0,

$$T = \frac{M - m_0}{\sqrt{Q}} \sqrt{nN(N-1)}$$

is Student distributed with $N-1$ degrees of freedom. From this follows that

$$J_1 = \left[M - t_{N-1,1-\frac{\epsilon}{2}} \sqrt{\frac{Q}{nN(N-1)}}, M + t_{N-1,1-\frac{\epsilon}{2}} \sqrt{\frac{Q}{nN(N-1)}} \right],$$

with $t_{N-1,1-\frac{\epsilon}{2}}$ being the $1-\frac{\epsilon}{2}$ fractile of the Student distribution with $N-1$ degrees of freedom, is a $1-\epsilon$ confidence interval for m_0.

4. Confidence interval for σ_0^2.
Let $\chi^2_{\nu,\alpha}$ denote the α fractile in the χ^2-distribution with ν degrees of freedom. We have

$$Pr\left(\frac{Q}{\sigma^2 + n\sigma_0^2} \geqq \chi^2_{N-1,\epsilon} \right) = 1 - \epsilon,$$

that is,

$$Pr\left(\frac{Q}{n\chi^2_{N-1,\epsilon}} \geqq \frac{\sigma^2}{n} + \sigma_0^2 \right) = 1 - \epsilon.$$

Then

$$Pr\left(\frac{Q}{n\chi^2_{N-1,\epsilon}} \geqq \sigma_0^2 \right) > 1 - \epsilon,$$

and from this follows that

$$J_2 = \left(0, \frac{Q}{n\chi^2_{N-1,\epsilon}} \right]$$

is a $1-\epsilon$ confidence interval for σ_0^2.

5. Confidence interval for σ^2.
Zagorac uses that Q_0/σ^2 is χ^2-distributed with $N(n-1)$ degrees of freedom and gets as a $1-\epsilon$ confidence interval for σ^2

$$I_3 = \left[\frac{N(n-1)\,V}{q_1}, \frac{N(n-1)\,V}{q_2} \right],$$

where q_1 and q_2 are determined such that

$$Pr\left(q_2 \leqq \frac{Q_0}{\sigma^2} \leqq q_1\right) = 1 - \epsilon. \tag{1}$$

Zagorac uses $q_1 = \chi^2_{N(n-1),1-\frac{\epsilon}{2}}$ and $q_2 = \chi^2_{N(n-1),\frac{\epsilon}{2}}$.

Referring to Sverdrup (1967, Sections XIII 3.2 and XIII 4.2) I would recommend q_1 and q_2 determined by (1) and

$$\frac{\log q_1 - \log q_2}{q_1 - q_2} = \frac{1}{N(n-1)};$$

this will make the confidence interval unbiased.

6. Confidence interval for $\kappa = \sigma^2/\sigma_0^2$.
The credibility estimator of m_j is

$$\tilde{m}_j = \frac{n}{n+\kappa} X_{.j} + \frac{\kappa}{n+\kappa} m_0.$$

We see that in this formula σ^2 and σ_0^2 appear only through κ. Hence, it may be interesting to construct a confidence interval for κ.

For this purpose we use that $(1+n\kappa^{-1})^{-1} F$ with

$$F = \frac{Q}{Q_0} \frac{N(n-1)}{N-1}$$

is Fisher distributed with $N-1$ and $N(n-1)$ degrees of freedom. Let f_1 and f_2 be determined such that

$$Pr\,(f_1 \leqq (1+n\kappa^{-1})^{-1} F \leqq f_2) = 1 - \epsilon. \tag{2}$$

Then

$$Pr\left(\frac{n}{\frac{F}{f_1}-1} \leqq \kappa \leqq \frac{n}{\frac{F}{f_2}-1}\right) = 1 - \epsilon,$$

and

$$J_3 = \left[\frac{n}{\frac{F}{f_1} - 1}, \frac{n}{\frac{F}{f_2} - 1} \right]$$

is a $1 - \epsilon$ confidence interval for κ.

A simple choice of f_1 and f_2 would be the $\frac{\epsilon}{2}$ resp. $1 - \frac{\epsilon}{2}$ fractile of the Fisher distribution with $N-1$ and $N(n-1)$ degrees of freedom. However, to get the confidence interval unbiased, I suggest to find f_1 and f_2 from the equations (2) and

$$\frac{\log\left(1 + f_2 \frac{N-1}{N(n-1)}\right) - \log\left(1 + f_1 \frac{N-1}{N(n-1)}\right)}{\log f_2 - \log f_1} = \frac{N-1}{Nn-1}.$$

7. Confidence regions for (m_0, κ) and $(m_0, \sigma^2, \sigma_0^2)$.
From the Bonfferoni inequality we get

$$Pr((m_0 \in J_1) \cap (\kappa \in J_3)) \geqq 1 - 2\epsilon,$$

and thus

$$A_1 = J_1 \times J_3$$

is a $1 - 2\epsilon$ confidence region for (m_0, κ).
By using the independence of M, Q, and Q_0 we may construct a confidence region for $(m_0, \sigma^2, \sigma_0^2)$ without using inequalities like the Bonfferoni one.
Let $g_{1-\frac{\epsilon}{2}}$ denote the $1 - \frac{\epsilon}{2}$ fractile of the normal distribution $N(0, 1)$. Then, as M has the distribution $N\left(m_0, \sqrt{\frac{1}{N}\left(\frac{\sigma^2}{n} + \sigma_0^2\right)}\right)$,

$$Pr\left(M - g_{1-\frac{\epsilon}{2}} \sqrt{\frac{\sigma^2 + n\sigma_0^2}{nN}} \leqq m_0 \leqq M + g_{1-\frac{\epsilon}{2}} \sqrt{\frac{\sigma^2 + n\sigma_0^2}{nN}}\right) = 1 - \epsilon,$$

and for given $\sigma^2 + n\sigma_0^2$

$$J_4(\sigma^2 + n\sigma_0^2) = \left[M - g_{1-\frac{\epsilon}{2}} \sqrt{\frac{\sigma^2 + n\sigma_0^2}{nN}}, M + g_{1-\frac{\epsilon}{2}} \sqrt{\frac{\sigma^2 + n\sigma_0^2}{nN}} \right]$$

is a $1 - \epsilon$ confidence interval for m_0.

Using that $Q/(\sigma^2+n\sigma_0^2)$ is χ^2-distributed with $N-1$ degrees of freedom, we get, analogously to Section 5, that

$$J_5=\left[\frac{Q}{p_1},\frac{Q}{p_2}\right],$$

with p_1 and p_2 satisfying

$$Pr\left(p_2\leqq\frac{Q}{\sigma^2+n\sigma_0^2}\leqq p_1\right)=1-\epsilon$$

is a $1-\epsilon$ confidence interval for $\sigma^2+n\sigma_0^2$.

From the independence of M, Q, and Q_0 we now get

$$\begin{aligned}&Pr((m_0\in J_4(\sigma^2+n\sigma_0^2))\cap(\sigma^2+n\sigma_0^2\in J_5)\cap(\sigma^2\in I_3))=\\&Pr(m_0\in J_4(\sigma^2+n\sigma_0^2))\,Pr(\sigma^2+n\sigma_0^2\in J_5)\,Pr(\sigma^2\in I_3)=\\&(1-\epsilon)^3,\end{aligned}$$

and

$$A_2=\{(x,y,z):x\in J_4(y+nz),z+ny\in J_5,\ z\in I_3\}$$

is a $(1-\epsilon)^3$ confidence region for $(m_0,\sigma_0^2,\sigma^2)$.

Bjørn Sundt
Forschungsinstitut für
Mathematik
ETH-Zentrum
8092 Zürich

References

Sverdrup, E. (1967): *Laws and chance variations.* Vol. II. North-Holland Publishing Company. Amsterdam.

Zagorac, D. (1981): Ein Beitrag zur Intervallschätzung der Glaubwürdigkeitsparameter. *Mitteilungen der Vereinigung schweizerischer Versicherungsmathematiker*, 67–75.