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1. Introduction

Credibility, by and large an invention of North American actuaries, was first used in connection with premium adjustments in workmens' compensation insurance. Credibility theory develops models for an appropriate adaptation of premium rates to a changing environment. Since the first developments in the early 1900's, the credibility approach has been used for all possible risks, with life insurance probably among the less important applications.

Nevertheless, group life insurance experience-rating plans are widely used today, mainly in North America and the United Kingdom, as well as for multinational insurance arrangements. But these experience-rating plans are not all of the prospective type, based on the credibility approach. More important are the retrospective experience-rating plans in which the excess of the billed premiums over the incurred claims, expenses, and profit margin are refunded to the policyholder. These plans in which both the insurer and the policyholder retain some risk, fall somewhere between the two extremes of the fully insured, conventional plan, and self-insurance programs or administrative services only contracts with or without a form of stop loss reinsurance cover.

Nowadays often a combination of both prospective and retrospective rating is applied to meet competitive pressures. In such cases, the billed premiums for retrospective plans are reduced, using the credibility approach, to the premium level of a prospective experience-rating plan. Such contracts may result in long-term underwriting losses to the insurer (even though there may still be overall gains from operations, partly due to such factors as the yield on contingency and claims reserves). Such losses are primarily caused by policyholders who cancel while in a deficit position.

In this paper we shall restrict ourselves to the pure prospective rating with claims projections based on a credibility formula, with premium rates afterwards guaranteed for between one and five years, without any posterior adjustment or refund. Such projections are effected by weighting the group's own prior experience and the expected experience derived either from the industry's experience or, if this seems to be justifiable, from the insurer's portfolio experience. The common formula for the projected claims level P is of the form

$$P = Z\,(\text{Actual Claims Level}) + (1-Z)\,(\text{Expected Claims Level}).$$

In the classical approach the first step is to fix the size of the group or the amount of claims experience to which full credibility (credibility factor $Z = 1$) will be granted. The second step is then to assign partial credibility weights to smaller groups or lesser amounts of claims data. In group life insurance, full credibility is usually granted for groups with 10,000 life-years or more but, due to competitive pressures, sometimes already for groups with 5,000 life-years or more. With "Life-years" we mean the number of lives in the group multiplied by the number of years of exposure of the group.
For partial credibility various formulae are in common use with credibility factors, e.g. of the form [4]

$$Z = a(L/L_0)^b, \qquad \text{for } L \leq L_0 \tag{1.1}$$

In practise, $b = ½$ seems to be a reasonable approach. L is the number of life-years of the actual case and $L_0 = 10{,}000$ or $L_0 = 25{,}000$, say.

Another common credibility factor assumes the form

$$Z = \frac{a\,L}{L+b} \tag{1.2}$$

with

$$a = \frac{L_0+b}{L_0}, \qquad \text{for } L \leq L_0 \tag{1.3}$$

These approaches, although quite practical, have the drawback of the more or less open choice for the group size for which full credibility will be attributed, even if this choice is based on a reasonable claims distribution function. Furthermore, there is the other problem of chosing a formula for partial credibility.
The Bayesian approach does not require us to fix a limit for full credibility, nor are partial credibilities a problem for these are produced automatically. Nonetheless, the problem remains that, in theory at least, the same credibility is attributed to 20 years of experience on a group of 50 lives as to 2 years of experience on a group of 500 lives. The easiest way to solve this problem is to ignore the experience that is more than 3 or 5 years old. Another approach might be to give more weight to the more recent claims data.
Credibility factors according to the Bayesian approach were developed for various well-known claims distributions. However, a statistical analysis of real claims data is needed to close the gap between theory and practical experience. In this paper, statistical data are presented for group life insurance purposes.

2. Claims Data

The results presented in this paper are based on the 3 to 5 years' claims experience of 180 groups from the United Kingdom. All together these groups represent an experience of 1,076,473 life-years, with a total of 6,618 death claims. These groups were allocated to 12 categories, according to size. Details are shown in table 1 below.

Table 1

Size of Analyzed Groups and Claims Experience

Category	Life-Years	No. of Groups	Total Life-Years	Total Claims
1	up to 500	30	9,105	72
2	501– 900	28	19,440	115
3	901– 1,300	19	20,638	112
4	1,301– 1,700	18	26,304	108
5	1,701– 2,200	17	32,830	158
6	2,201– 3,000	11	29,104	122
7	3,001– 6,000	23	93,566	385
8	6,001– 10,000	11	93,638	438
9	10,001– 20,000	14	190,699	935
10	20,001– 30,000	3	70,623	398
11	30,001– 70,000	3	125,542	647
12	70,001–150,000	3	364,984	3,128
Total		180	1,076,473	6,618

Between the groups, and even between the various categories, there are considerable differences with respect to the claims experience. Such differencies may be attributed to many different reasons, among them

- random fluctuations (of course!);
- different age structures and retirement ages; in some cases, employees who have taken early retirement are covered too;
- varying percentage of female lives in the groups;
- different industries, percentages of white and blue collar workers, occasionally hazardous occupations;
- regional variations in the United Kingdom: North and North West England, Wales, Scotland and Northern Ireland experience a heavier mortality than the rest of the United Kingdom, independent of the prevalent occupations. This may be due to the life-style and the quality of air and water. Example: For males the mortality in Scotland is about 18% higher than in England.

Indeed, it is hard to find two groups with the same "true" average mortality rate. The existence of these differences among the groups, apart from their different size, is one of the justifications for applying credibility formulae.

3. Credibility Formula

In the following we denote with

$$X_{i,j,k}, \quad \begin{array}{ll} i = I_0+1, ..., I_K; & I_0 = 0, I_K = I = 180 \\ j = 1, ..., J_i; & 3 \leq J_i \leq 5 \\ k = 1, ..., K; & K = 12 \end{array}$$

the claims ratio of the group i for the year j in the category k. I_k is the total number of groups in the category k, $K = 12$ the number of categories, and J_i the number of years of experience of the group i.

With this notation we can write the credibility formula as follows

$$P_k = (1 - Z_k)\, \bar{X}_{...} + Z_k\, \bar{X}_{..k}, \qquad k = 1, ..., K \tag{3.1}$$

with

$$\bar{X}_{i.k} = \frac{1}{J_i} \sum_{j=1}^{J_i} X_{ijk} \tag{3.2}$$

$$\bar{X}_{..k} = \frac{1}{(I_k - I_{k-1})} \sum_{i = I_{k-1}+1}^{I_k} \bar{X}_{i.k} \tag{3.3}$$

$$\bar{X}_{...} = \frac{1}{K} \sum_{k=1}^{K} \bar{X}_{..k} = \frac{1}{I} \sum_{k=1}^{K} \sum_{i = I_{k-1}+1}^{I_k} \bar{X}_{i.k} \tag{3.4}$$

It is well known that Z_k can be written in the form [1], [3]

$$Z_k = \frac{1}{1 + N_k} \tag{3.5}$$

(3.3) is the average of the group means of the category k, and at the same time the estimator for the group claims experience rate in the credibility formula.

(3.4) is the estimator for the collective claims experience rate, the expected claims rate. For the N_k in formula (3.5) we apply the following definition [1], [2]

$$N_k = \frac{e_k}{d} \tag{3.6}$$

with

$$e_k = E(\mathrm{Var}[X_{ijk}]), \quad i = I_{k-1}+1, \ldots, I_k;\ I_0 = 0,\ I_K = I \quad j = 1, \ldots, J_i;\ 3 \le J_i \le 5 \tag{3.7}$$

and

$$d = \mathrm{Var}[E(\bar{X}_{i.k})], \quad k = 1, \ldots, K \quad i = I_{k-1}+1, \ldots, I_k;\ I_0 = 0,\ I_K = I. \tag{3.8}$$

As we see from (3.7), e_k is a measure for the variance of the claims ratios within the groups of category k in time. The quantity d is the variance of the mean loss ratios within the portfolio under consideration; it measures the heterogeneity of the portfolio.
According to the above definitions (3.7) and (3.8) we are now in a position to estimate the quantities e_k and d as follows

$$\hat{e}_k = \frac{1}{(I_k - I_{k-1})} \sum_{i=I_{k-1}+1}^{I_k} \frac{1}{J_i - 1} \sum_{j=1}^{J_i} (X_{ijk} - \bar{X}_{i.k})^2 \tag{3.9}$$

and

$$\hat{d} = \frac{1}{I-1} \sum_{k=1}^{K} \sum_{i=I_{k-1}+1}^{I_k} (\bar{X}_{i.k} - X_{...})^2 \tag{3.10}$$

With formulae (3.9) and (3.10) we can estimate the quantity N_k in (3.6) according to

$$\hat{N}_k = \frac{\hat{e}_k}{\hat{d}} \qquad k = 1, \ldots, K \tag{3.11}$$

4. Numerical Results

As explained in paragraph 2, we have split our portfolio of 180 groups into 12 gategories according to size, with between 3 and 30 groups in each category. With formula (3.5) we calculated the credibility factors Z_k for each category. Table 2 shows the main results.

Table 2

Credibility Factors According to Size of Group

Category k	Average No. of Life-Years	Average No. of Claims	Z_k
1	304	2.4	0.12
2	694	4.1	0.35
3	1,086	5.9	0.48
4	1,461	6.0	0.58
5	1,931	9.3	0.62
6	2,646	11.1	0.77
7	4,068	16.7	0.81
8	8,513	39.8	0.83
9	13,621	66.8	0.88
10	23,541	132.7	0.89
11	41,847	215.7	0.96
12	121,661	1,042.7	0.99

As expected, Z_k is dependent on the number of life-years, or the average number of death claims respectively. In non-life insurance it would not be feasible to take the number of claims and the number of life-years as a basis, while this seems to be the easiest approach for life insurance, as no further calculations are needed to arrive at the credibility factor.

In table 2 the values of Z_k are attributed to specific numbers of life-years which is not very practical for actual calculations, as the credibility factors would have to be interpolated for numbers of life-years in between the values shown. Therefore we shall try to find an approximation $\hat{Z}$ of Z that is easier to handle. For this purpose we rewrite formula (3.5) in the generalized form without the index k,

$$Z = \frac{1}{1+N} \tag{4.1}$$

In (4.1) N depends on the number of life-years L. With this in mind, we shall approximate Z by a $\hat{Z}$ of the form

$$Z = \frac{L}{L+C} \tag{4.2}$$

with $C = \text{constant}$.

In order to find an appropriate value of C, we analyzed the data graphically with a logarithmic abscissa, see fig. 1. The result was that there seems to be no single value of C that is equally well suited for all numbers of life-years. Calculations were effected for three different values of C, $C_1 = 1000$, $C_2 = 1250$ and $C_3 = 1500$, resulting in three different sets of values $Z(1000)$, $Z(1250)$ and $Z(1500)$.

Table 3

Credibility Factors According to Formula (4.2)

Category	Actual Z_k	$Z(1000)$	$Z(1250)$	$Z(1500)$
1	0.12	0.23	0.20	0.17
2	0.35	0.41	0.36	0.32
3	0.48	0.52	0.46	0.42
4	0.58	0.59	0.54	0.49
5	0.62	0.66	0.61	0.56
6	0.77	0.73	0.68	0.64
7	0.81	0.80	0.76	0.73
8	0.83	0.89	0.87	0.85
9	0.88	0.93	0.92	0.90
10	0.89	0.96	0.95	0.94
11	0.96	0.98	0.97	0.97
12	0.99	0.99	0.99	0.99

It seems that $\hat{Z}(1500)$ is somewhat closer to the actual data for large groups with more than 5000 life-years, while $\hat{Z}(1000)$ seems to be the better approximation for smaller schemes. Of course it would be possible to replace the constant C by a function $C(L)$, but it is doubtful whether such an approach is justifiable based on the data available and with the objective of a practical formula in mind. The best approximation for the whole range of sizes of groups, based on the available data, is

$$\hat{Z} = \frac{L}{L+1250} \tag{4.3}$$

This function, together with the actual Z_k, is shown in fig. 1.

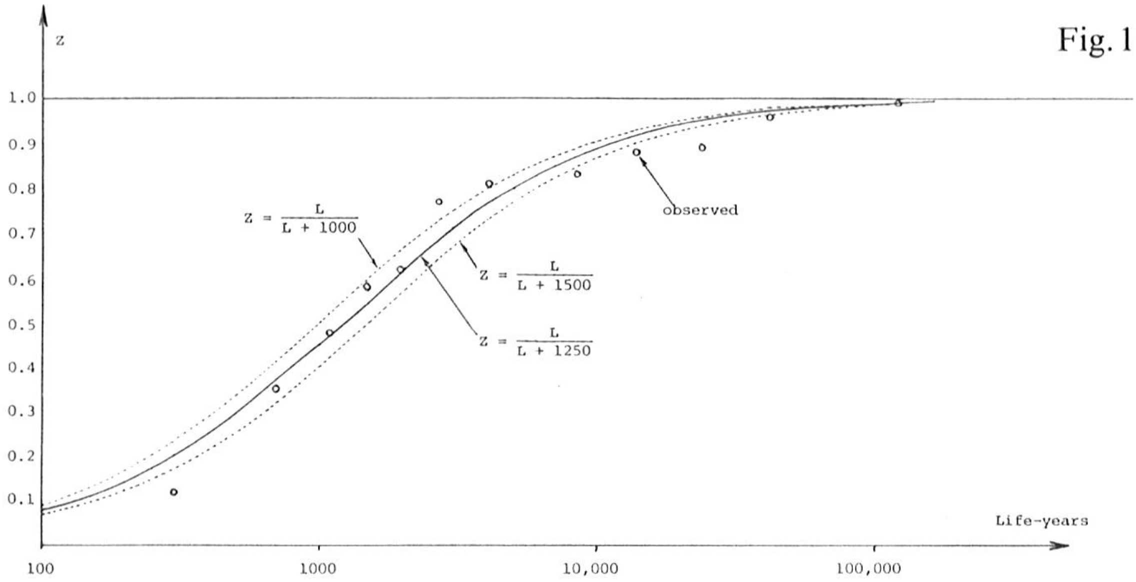

Fig. 1

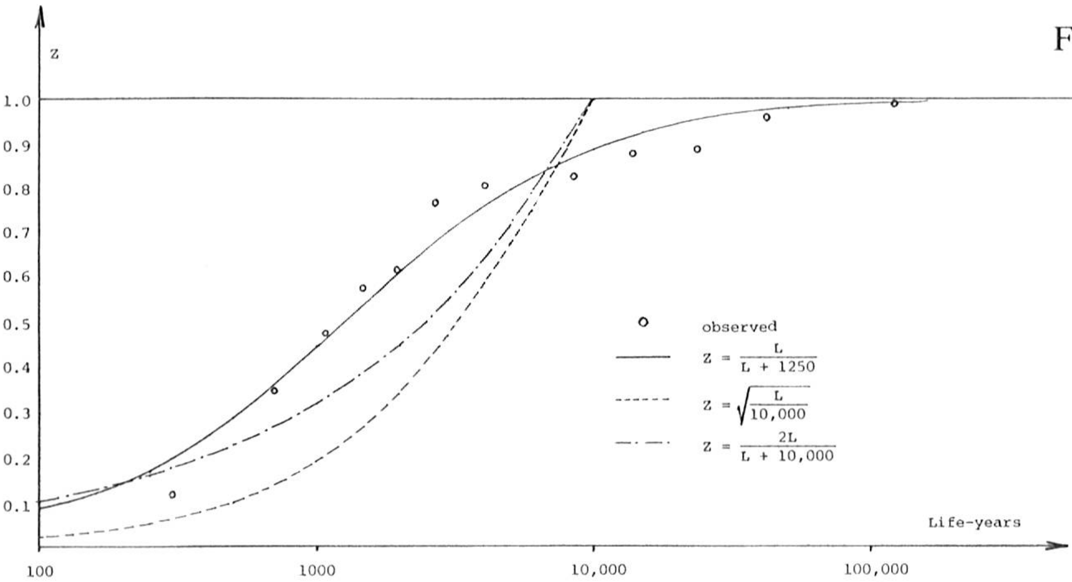

Fig. 2

5. Conclusion

We have seen that the Non-Bayesian approaches result in credibility formulae that are not fully satisfactory from the theoretical point of view. It is hard to see why there should be a limit apart from which full credibility is granted. Formula (4.3), based on a Bayesian approach, avoids this problem, and its credibility factors fit rather well into the empirical data as can be seen from fig. 2. However, more empirical work is needed in order to confirm this formula, especially for very small and for very large groups.

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Summary

The autors derive a credibility formula, based on empirical data of a group life portfolio. The results are compared with other common credibility formulae.

Zusammenfassung

Auf Grund empirischer Daten aus der Kollektiv-Lebensversicherung wird eine Formel für die Kredibilität hergeleitet. Die Resultate werden mit anderen gebräuchlichen Kredibilitätsformeln verglichen.

Résumé

Sur la base de données empiriques extraites de l'assurance vie collective, l'article établit une formule de crédibilité, puis la compare à d'autres formules déjà connues.