Zeitschrift: Mitteilungen / Vereinigung Schweizerischer Versicherungsmathematiker = Bulletin / Association des Actuaires Suisses = Bulletin / Association of Swiss Actuaries

Herausgeber: Vereinigung Schweizerischer Versicherungsmathematiker

Band: - (1980)

Heft: 3

Artikel: Simplified bases to describe a tariff system for disability annuities

Autor: Smid, C.L.

DOI: https://doi.org/10.5169/seals-571118

C. L. Smid, Den Haag

Simplified Bases to Describe a Tariff System for Disability Annuities

1. The problem set out below has arisen in practice with disability annuities. The solution proposed deals specifically with this class of business, although it contains aspects of a more general nature. The problem may be described as follows:
We are given a set of premium rates for one terminal age of which the basis is unknown except for the rate of interest (e.g. premium rates which have been adjusted several times, losing touch with the original system of probabilities). The problem is how to construct a basis for this set of rates which will enable other rates for other ages and terms to be calculated with a sufficient degree of accuracy.
2. To fully describe a disability basis, the following probabilities are required:
(i) The probability, while not disabled, of becoming disabled or dying.
(ii) The probability, while disabled, of recovering or dying.
These probabilities underlie the following functions, from which the sets of premium rates may be calculated.

a) i_x: the probability of an x year old to become disabled within one year and still being disabled after the deferment period has elapsed;
b) $\bar{a}^{ii}_{x\overline{n}|}$: the present value of the disability annuity taking effect after the deferment period and payable continuously for n years;
c) l^{aa}_x: the number of lives alive and not disabled out of given $l^{aa}_{x_0}$ at age x_0
d) $\ddot{a}^{aa}_{x\overline{n}|}$: the present value of a premium 1 per annum payable annually in advance by the lives alive and not disabled.

Given the above functions any set of premium rates may be calculated using the formula

$$P_{x\overline{n}|}\ddot{a}^{aa}_{x\overline{n}|} = P^*_{x\overline{n}|}\ddot{a}_{x\overline{n}|} = \sum_{t=0} i_{x+t} \cdot \bar{a}^{ii}_{x+t+e+\frac{1}{2}\overline{n-t-e-\frac{1}{2}}|} \cdot v^{t+e+\frac{1}{2}} \cdot \frac{l^{aa}_{x+t}}{l^{aa}_x} \tag{1}$$

where P is the premium rate waived during disability and P^* is the premium rate not waived during disability; e is the deferment period in years.
3. P and P^* may be derived from each other using the formula $P^*(1+P) = P$ or, to be more accurate $P^*(1+P(1+\frac{1}{2}r)) = P$, where r is the rate of interest. This is explained by regarding the premium due during disability as an extra element of disability annuity P, or taking account of premiums being payable in advance and disability annuities continuously, $P(1+\frac{1}{2}r)$.

4. Furthermore, from (1) above,

$$\ddot{a}^{aa}_{x\overline{n}|} = \frac{P^*}{P}\, \ddot{a}_{x\overline{n}|}$$

so that if we use a standard mortality table to calculate $\ddot{a}_{x\overline{n}|}$, we can calculate $\ddot{a}^{aa}_{x\overline{n}|}$ and hence the non-disabled life-table l^{aa}_x may be built up using the formula

$$\frac{l^{aa}_{x+1}}{l^{aa}_x} = \frac{\ddot{a}^{aa}_{x\overline{n}|} - 1}{v \cdot \ddot{a}^{aa}_{x+1\,\overline{n-1}|}}.$$

5. Now defining $P^r_{x\overline{n}|}$ as the risk premium for age x and term n and $K_{x\overline{n}|}$ as the present value of future premiums then

$$K_{x\overline{n}|} = P_{x\overline{n}|}\ddot{a}^{aa}_{x\,\overline{n}|} = \sum_{t=0}^{n-1} P^r_{x+t\,\overline{n-t}|} \cdot v^t \cdot \frac{l^{aa}_{x+t}}{l^{aa}_x}$$

$$K_{x\overline{n}|} - v \cdot \frac{l^{aa}_{x+1}}{l^{aa}_x} \cdot K_{x+1\,\overline{n-1}|} = P^r_{x\,\overline{n}|}. \tag{2}$$

The values of $K_{x\overline{n}|}$ may now be calculated using the formula $K_{x\overline{n}|} = P_{x\overline{n}|}\ddot{a}^{aa}_{x\overline{n}|}$ and hence the values of $P^r_{x\overline{n}|}$ may be determined using formula (2).

6. We now make the assumption that $\bar{a}^{ii}_{x\overline{n}|}$ may be represented as

$$\bar{a}^{ii}_{x\overline{n}|} = \text{minimum } [A + B\bar{a}_{\overline{n}|} \quad \text{at} \quad (r+C)\%; \quad \bar{a}_{\overline{n}|} \quad \text{at} \quad (r+C)\%]$$

where A, B and C are constants independent of age and term, and r is the rate of interest.

This formula may be illustrated as follows:

Of lives who become disabled a proportion B $(0 < B < 1)$ may be regarded as long term disability cases, with $C\%$ of the corresponding disability annuities ceasing each year on account of recovery or death. The present value of these annuities is $B\bar{a}_{\overline{n}|}$ at $(r+C)\%$.

The remaining proportion $(1-B)$ may be regarded as short term disability, for which the present value of annuity payments is taken as A, on average. This would mean that

$$\bar{a}^{ii}_{x\overline{n}|} = A + B\bar{a}_{\overline{n}|} \quad \text{at} \quad (r+C)\%.$$

For small n however the value of $A + B\bar{a}_{\overline{n}|}$ may be relatively high, due to the term A, which is independent of n and for this reason the function is restricted to a maximum of $\bar{a}_{\overline{n}|}$ at $(r+C)\%$.

The assumptions made in the above representation are clearly questionable and in particular it is assumed that $\bar{a}^{ii}_{x\overline{n}|}$ is dependent only on the term n and not on the age x.

7. Now

$$P^r_{x\overline{n}|} = i_x \cdot v^{e+\frac{1}{2}} \cdot \bar{a}^{ii}_{x+e+\frac{1}{2}\,\overline{n-e-\frac{1}{2}}|}$$

and hence the i_x may be calculated from the $P^r_{x\overline{n}|}$ and $\bar{a}^{ii}_{x\overline{n}|}$ already calculated above in paragraph 5 and 6.

8. Hence given one set of net premium rates for one terminal age and deferment period all the factors necessary to calculate sets of rates for other terminal ages may be calculated for the same deferment period.

9. To proceed to a practical example a set of net premium rates for terminal age 65 and deferment period one year is set out in Table 1 below. A number of different assumptions as regards the constants A, B and C are made and set out in Table 2. Then the corresponding net premium rates for terminal ages 60 and 55 are derived using the method described above as set out in Table 3. Naturally each set of values assumed for A, B and C will result in different values of i_x and different premium rates.

In fact the net premium rates for terminal age 65 were calculated on a known rate basis, so the accurate values for the premium rates at the other terms and ages are also shown in Table 3.

Furthermore in Table 4 below are shown the equivalent net premiums for a disability annuity with premium and annuity escalating at 5% per annum compound from inception.

Table 1

Age at entry	Terminal age 65 Level net annual premium for 1000 p.a. benefit
20	58
30	81
40	106
50	111
60	47

Table 2

Basis I	$A=0$	$B=1$	$C=2\%$
II	$A=0{,}3$	$B=0{,}9$	$C=2\%$
III	$A=0$	$B=1$	$C=3\%$
IV	$A=0{,}3$	$B=0{,}85$	$C=3\%$

Table 3. Derived level net annual premium rates for 1000 p.a. benefit.

Age at entry	Terminal age 60					Terminal age 55				
	I	II	III	IV	exact	I	II	III	IV	exact
20	46	47	47	47	48	36	37	37	38	38
30	63	63	64	64	64	47	48	48	49	49
40	77	78	79	80	79	51	53	53	55	53
50	65	67	66	68	67	24	26	25	27	26
55 (53*)	33	35	34	37	35	4	5	5	5	5
58	6	6	6	7	7					

* age for terminal age 55.

Table 4. Net annual premium rates for premium and benefit escalating at 5% p.a.c.

Age at entry	Terminal age 65					Terminal age 60					Terminal age 55				
	I	II	III	IV	exact	I	II	III	IV	exact	I	II	III	IV	exact
20	120	119	118	117	122	87	86	87	86	90	61	61	62	62	64
30	137	136	135	134	138	97	97	97	98	100	66	66	67	68	69
40	151	150	150	148	152	101	102	103	103	103	63	64	65	66	65
50	137	136	136	135	137	75	76	76	78	77	26	28	27	30	28
55(53*)	105	104	105	104	105	37	38	37	40	39	5	5	5	6	5
60(58**)	52	51	51	51	51	7	7	7	7	7					

* age for terminal age 55 ** age for terminal age 60.

10. The remarkable feature of the above results is that they depend only to a small extent on the formula chosen for $\bar{a}^{ii}_{x\overline{n}|}$ and the final premium rates are a reasonably close approximation to the accurate values.

The conclusion which may be drawn from this is that the errors implicit in choosing a simplified formula for the probability of death or recovery is partly compensated by the derivation of appropriate values of the claim probability i_x.

Drs. C. L. Smid, Act. A.G.
Kortenaerkade 1
Postbus 16150
Den Haag (Niederlande)

Summary

In this article it is shown that a system of disability annuity premium rates may be adequately described by means of a simplified formula for $\bar{a}^{ii}_{x\overline{n}|}$ and errors implicit in this formula are partly offset by choosing appropriate values of the claim probability i_x. In addition a method is given for deriving the active life table, l^{aa}_x, from a set of disability premium rates.

Zusammenfassung

In dieser Arbeit wird gezeigt, wie die Prämiensätze für ein Tarifsystem von Invalidenrenten durch eine vereinfachte Formel für $\bar{a}^{ii}_{x\overline{n}|}$ ausgedrückt werden können und zudem, wie sich allfällige Approximationsfehler zumindest teilweise durch geschickte Wahl der Schadenwahrscheinlichkeit i_x auskorrigieren lassen. Ausserdem wird eine Methode beschrieben, mit der sich die Aktivitätsordnung l^{aa}_x aus einem System von Invaliditätsprämiensätzen konstruieren lässt.

Résumé

L'article montre qu'il est possible de décrire un tarif d'assurance de rente d'invalidité en utilisant une formule simplifiée pour $\bar{a}^{ii}_{x\overline{n}|}$ et d'éliminer partiellement les erreurs commises par un choix approprié de la probabilité de sinistre i_x. De plus, l'auteur donne une méthode permettant d'obtenir l'ordre des actifs l^{aa}_x à partir d'un ensemble de taux de primes.