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D.
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Estimation of IBNR Claims by Least Squares

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Abstract

Let c_{ij} be the amount of claims paid in development year j in respect of accident year i. A rather general run-off model is based on the assumption that the c_{ij} can be approximated by quantities $x_i\, p_j\, u_{i+j}$ where

x_i is the total amount of claims in respect of accident year i expressed in an inflation-free money unit,

p_j is the fixed proportion of the amount x_i paid in development year j, i.e. in payment year $i+j$,

u_{i+j} is the appropriate claim inflation index.

In the *classical model* it is assumed that there is no inflation, i.e. that $u_{i+j} = 1$. In this note we particularize the general model by the assumption $u_{i+j} = u^{i+j}$, where u is an unknown mean annual inflation index. We show that the practical estimation of the outstanding claims, by a method of least squares, is the same in this *exponential model* as in the classical model. Beyond the fact that it copes automatically with inflation effects, the least squares method has another advantage over the widely used chain-ladder method: practically it applies whatever be the set of cells (i,j) for which the c_{ij} are supposed to be known.

1. Observed data

An example of observed data is given in table 1. Thus, the observed claim amounts c_{ij} are supposed to be known only for certain couples (i,j). The cells in the domains I and II are empty. We want to complete the empty cells in II by estimates of the yet unobserved c_{ij}. In fact, in this particular example under consideration, we knew the values of the c_{ij} in domain I, but we decided

to forget this more than six years old information. Indeed, for any theoretical model of a practical situation, the following rather general rule seems to be valid. For short observation periods, the model may suit very well, while it generally does not so good for long observation periods. On the other side, a long observation furnishes extensive statistical data, whereas a short does not. There is no solution, except a compromise, to this dilemna.
In the sequel, no special assumption is made on the set of occupied cells, except that it must be sufficiently connected. Since this is always the case in the practical situations, we have not to make this point clearer. See *de Vylder* (to be published).

2. Classical model

2.1. Least squares rule

In the *classical model,* the known c_{ij} are approximated by quantities $x_i\, p_j$. Then values for the unknown c_{ij} are obtained at once. The unknown quantities x_i, p_j result from the solution to the following problem:

$$\text{Minimize} \sum w_{ij}\,(x_i\, p_j - c_{ij})^2, \tag{1}$$

weights equal to 1, perhaps not optimal, rather than to put unnecessary probabilistic restrictions on the model. The same remarks apply to the exponential cell (i, j), equal to 1. In this paper, we adopt a rather deterministic approach, but we believe that the whole model can be probabilized and that the right weights w_{ij} can be found as functions of the observed c_{ij}. We prefer to use weights equal to 1, perhaps not optimal, rather than to put unnecessary probabilistic restrictions on the model. The same remarks apply to the exponential model considered further.

2.2. Indetermination

If x_i, p_j is a solution of (1), then

$$x'_i = c\, x_i, \qquad p'_j = c^{-1}\, p_j, \qquad (c > 0) \tag{2}$$

is also a solution, since $x_i\, p_j = x'_i\, p'_j$. Thus, the problem (1) is indeterminate. The indetermination can be eliminated by a condition such as $\Sigma\, p_j = 1$, but this is unnecessary if one is interested only in the estimates of the unknown c_{ij}, since the products $x'_i\, p'_j$ do not depend on c. Of course, if an interpretation of the x_i, p_j themselves is wanted, then c must be fixed in one way or another.

2.3. *Iterative solution of the minimization problem*

The annulation of the partial derivatives in x_i, p_j leads to the following equations

$$x_i = \sum_j w_{ij}\, c_{ij}\, p_j \Big/ \sum_j w_{ij}\, p_j^2 \qquad (3)$$

$$p_j = \sum_i w_{ij}\, c_{ij}\, x_i \Big/ \sum_i w_{ij}\, x_i^2 \qquad (4)$$

where the summations are taken over the occupied cells only. These equations can be solved iteratively. One starts with arbitrary values of the p_j in (3) and then uses successively (4), (3), (4), ... Practically one always obtains limit solutions. The limit solutions depend on the initial values of the p_j, but different limit solutions are always connected by (2). The results obtained after four or five iterations can be considered as limit solutions from the practical point of view. The calculations can be performed on a programmable pocket calculator.

3. Exponential model

3.1. *Least squares rule*

In the *exponential model*, the c_{ij} are approximated by expressions $x_i\, p_j\, u^{i+j}$, where the unknown quantities are x_i, p_j, u. Now the rule is to

$$\text{Minimize} \quad \sum w_{ij}\, (x_i\, p_j\, u^{i+j} - c_{ij})^2. \qquad (5)$$

3.2. *Indetermination*

If x_i, p_j, u is a solution of (5), then

$$x'_i = c r^{-i}\, x_i, \quad p'_j = c^{-1}\, r^{-j}\, p_j, \quad u' = ru \qquad (6)$$

is also a solution, whatever be $c > 0$, $r > 0$.
The estimates of the unknown c_{ij} do not depend on c, r, since the product $x'_i\, p'_j\, u'^{i+j}$ does not.

3.3. *Reduction to the classical model*

From the following simple observation it results that problem (5) can be reduced to problem (1). For, if x_i, p_j is a solution of problem (1), then x_i, p_j, $u = 1$ is a solution of problem (5). Indeed, suppose a moment that it is not a solution. Then there exist x'_i, p'_j, u' such that

$$\sum w_{ij} (x'_i\, p'_j\, u'^{i+j} - c_{ij})^2 < \sum w_{ij} (x_i\, p_j - c_{ij})^2.$$

Putting $x''_i = x'_i\, u'^i$, $p''_j = p'_j\, u'^j$, we have

$$\sum w_{ij} (x''_i\, p''_j - c_{ij})^2 < \sum w_{ij} (x_i\, p_j - c_{ij})^2,$$

contradicting the fact that x_i, p_j is a solution of problem (1).

4. Practical conclusion

In order to solve problem (5), we solve problem (1), say by the iterative method. Let x'_i, p'_j be a solution of (1). Then, taking into account the indeterminations,

$$x_i = c u^{-i} x'_i, \qquad p_j = c^{-1} u^{-j} p'_j, \qquad u \tag{7}$$

is a solution of problem (5) depending on the parameters $c > 0, u > 0$. We conjecture that, under conditions always satisfied in practice, (7) is the general solution of problem (5).

5. Numerical illustration

We apply the just explained method to the data in table 1. These are real data in a sickness portfolio[1]. In table 2, we indicate the estimates of the c_{ij} corresponding to the empty domain II in table 1. In table 3, we indicate the values of the x_i, p_j under the assumptions $\sum p_j = 1, u = 1$. In table 4, we indicate the values under the assumptions $\sum p_j = 1$, $u = 1.03$.

[1] It is a pleasure to thank *M. Bardola*, Director at the VITA (Swiss), who supplied me with this and other statistical material.

Table 1. Observed data

i \ j	0	1	2	3	4	5
0						4.627
1					15.140	13.343
2				43.465	19.018	12.476
3			116.531	42.390	23.505	14.371
4		346.807	118.035	43.784	12.750	12.284
5	308.580	407.117	132.247	37.086	27.744	
6	358.211	426.329	157.415	68.219		
7	327.996	436.744	147.154			
8	377.369	561.699				
9	333.827					

Table 2. Estimated c_{ij}

i \ j	0	1	2	3	4	5
5						16.056
6					25.666	17.654
7				54.669	25.080	17.251
8			183.413	67.686	31.052	21.358
9		448.672	151.753	56.003	25.692	17.671

Table 3. Values of the x_i, p_j when $\sum p_j = 1$, $u = 1$

$p_0 =$ .323	$p_1 =$ .434	$p_2 =$ .147	$p_3 =$ .054	$p_4 =$ .025	$p_5 =$ .017
$x_0 = 270.638$	$x_1 = 664.133$	$x_2 = 790.749$	$x_3 = 796.639$	$x_4 = 798.643$	
$x_5 = 939.137$	$x_6 = 1032.577$	$x_7 = 1009.003$	$x_8 = 1249.258$	$x_9 = 1033.617$	

Table 4. Values of the x_i, p_j when $\sum p_j = 1$, $u = 1.03$

$p_0 =$ .333	$p_1 =$ .435	$p_2 =$ .143	$p_3 =$ .051	$p_4 =$ .023	$p_5 =$ .015
$x_0 = 262.305$	$x_1 = 624.937$	$x_2 = 722.407$	$x_3 = 706.591$	$x_4 = 687.736$	
$x_5 = 785.165$	$x_6 = 838.141$	$x_7 = 795.152$	$x_8 = 955.812$	$x_9 = 767.791$	

It is worthwhile to insist on the fact that the method developed in this paper does not permit to estimate the mean annual inflation index u, but that the estimates of the unknown c_{ij} do not depend on u.
Finally, note that the values of x_0 to x_4 are not very reliable, since the c_{ij} corresponding to domain I are neglected. Again, this is irrelevant, because the estimated c_{ij} corresponding to domain II do not depend on these x_i.

Bibliography

Beard R.E. (1977): Verification of outstanding claim provisions – Separation technique. ASTIN Bulletin 9.

de Vylder Fl. (to be published): Multiplicative fitting, completion and extension of matrices by least squares.

Hachemeister C.A. (CAS Symposium on Loss Reserves in Chicago 1976): Breaking down the loss reserving process.

Hachemeister C.A. and *Stanard J.N.* (to be published): IBNR claims count estimation with static lag function.

Straub E. (1972): On the calculation of IBNR reserves. Nederlandse Reassurantie Groep N.V., Amsterdam.

Straub E. and *Kramreiter* (1973): On the calculation of IBNR reserves II. Mitt. Ver. schweiz. Versicherungsmathematiker.

Taylor G.C. (1977): Separation of inflation and other effects from the distribution of non-life claim delays. ASTIN Bulletin 9.

Taylor G.C. (to the published): Runoff analysis and changing quality of business.

International Association of Consulting Actuaries

The International Association of Consulting Actuaries held its sixth conference in Toronto from June 4 through June 9, 1978. One hundred and twenty consulting actuaries attended the conference representing 13 countries.

During the three and one-half days of meetings national reports were presented and discussed for Australia, Belgium, Canada, Germany, Israel, Mexico, New Zealand, Norway, South Africa, Switzerland, West Indies, United Kingdom and United States. In addition 14 other papers on a variety of topics ranging from inflation and the pace of funding of pension plans to the general problem of devising forms of life insurance suitable for developing countries in Africa were presented.

The International Association of Consulting Actuaries was established in 1968 for the purpose of facilitating the exchange of views and information on an international basis between members on matters affecting their professional responsibilities as consulting actuaries. Meetings normally take place every two years and the next meeting is scheduled for Vienna in 1980.

At the Toronto meeting M. David R. Brown of Canada was elected Chairman of the Association, and Dr. Theo Schaetzle of Switzerland was elected Vice-Chairman.

Further information about the Association may be obtained from the Secretary/ Treasurer, Evan Innes, 600 Third Avenue, New York, New York 10016.

A full listing of the papers discussed at the Toronto Conference is attached.

Papers Presented at the Sixth Conference of the International Association of Consulting Actuaries, June 4–9, 1978

Pension Problems

Inflation and the Pace of Funding
by G. Ashley Cooper (USA)
Funding Standards for Public Pension Plans
by Edward H. Friend (USA)

Government Regulation of Pension Plans. The American Experience
by Paul H. Jackson (USA)

Pension Fund Investment

Investment Policy and Asset Mix for Pension Funds
by D. Don Ezra (Canada)
Investment Issues for the Pension Actuary
by John Graham (Australia)
Investment Advice – Problems for Consulting Actuaries
by Edward J. Jones (New Zealand)
Notes on the Relationship Between Actuary & Investment Manager
by R. David Parsons (USA)

Current Problems and Trends

Life Insurance: Contract Design under Inflationary Conditions
G. E. Barrow (UK)
Governmental and Other Influences Affecting Assumptions for Pension Plan Valuations in the USA
by Evan Innes (USA)
An Extended Role for the Consulting Actuary
by Barry King (Australia)
Professional Liability
by Frank Livsey (Canada)
Cost Comparisons in Ordinary Life Insurance
by J. Bruce MacDonald (Canada)
The Pearson Commission: Headings for Discussion
by John H. Prevett
Compensation for Loss of Earnings as a Result of Injury or Death
by Theo Schaetzle (Switzerland)

Kammer der Pensionskassen-Experten

An der letzten Generalversammlung der Kammer der Pensionskassen-Experten wurden neu in den Vorstand gewählt:

Dr. Theo Schaetzle, Präsident
Prof. Dr. Bernhard Romer
Dr. Claude Chuard

Die heute 34 Mitglieder zählende Kammer setzt sich zusammen aus unabhängigen Pensionskassen-Beratern, die alle über umfassende, praktische und theoretische Erfahrungen verfügen.

Skizze eines BVG ohne Pool

Von David Stokar, Zürich

Der Verfassungsentwurf ist gegeben und muss erfüllt werden. Im Jahre 1972 hatte die Schliessung der noch bestehenden Lücken den Vorrang gegenüber einem perfektionistischen Vollausbau, weshalb man letzteren auch heute den Betrieben überlassen und sich auf die Vorschrift von gesetzlichen Mindestanforderungen beschränken könnte. Die nachstehenden Überlegungen beziehen sich auf ein liberales Rahmengesetz, das indessen über ein blosses Beitragsprimat hinausgeht.

1. Die Expertengutachten bestätigen, dass die Verfassung sich weder für Beitrags- noch Leistungsprimat festlegt, dass innert 10–20 Jahren aber ein «gesetzlicher Mindest*schutz*» zu gewähren ist. Dies bedeutet mehr als nur Mindestbeiträge.
2. Die bisher aufgebaute Zweite Säule soll in das Obligatorium voll integriert werden. Es wäre volkswirtschaftlicher Luxus, für alle mit Null zu beginnen, wie es das BVG vorsieht.
3. Innert 5 Jahren (nicht 10!) wären für Alter, Tod und Invalidität Beiträge auf *zum Beispiel* 10% der zu versichernden Löhne stufenweise anzuheben bei voller Freizügigkeit. Innerhalb dieses Satzes könnten die Altersgutschriften zugunsten der Eintrittsgeneration sinngemäss gestaffelt werden. Während Jahr-

zehnten ist bei einem Neubeginn eine Überversicherung ausgeschlossen, weshalb ein Koordinationsabzug fakultativ sein soll. Dies wäre der Kostenklarheit förderlich.

4. Bei Erreichen des Rücktrittsalters hätte die zuständige AHV-Stelle *sämtliche* Leistungen aus Personalvorsorge festzustellen. Erreichen sie zusammen mit der einfachen AHV-Rente das *schrittweise heraufzusetzende* Mindest-Leistungsziel nicht, so werden Ausgleichsleistungen fällig. Voraussetzung ist ein lückenlos funktionierendes Freizügigkeitssystem, was indessen das Problem *jedes* Obligatoriums ist (z. B. «FZ-Pass»).
5. Die AHV-Stelle meldet den Betrag des Ausgleichsanspruchs an eine Zentrale, die den Jahresaufwand gesamtschweizerisch ermittelt. Die erforderlichen Beiträge wären zusammen mit denjenigen an die AHV/IV/EO/AlV zu entrichten, nach oben *plafoniert*.
6. Da bei Berücksichtigung sämtlicher Leistungen aus Personalvorsorge und der stufenweisen Anhebung des gesetzlichen Leistungsziels die Ausgleichszahlungen sich als sehr bescheiden erweisen würden, kommt ihnen gewissermassen der Charakter von AHV-Ergänzungsleistungen zu (könnte man auch einfach diese ausbauen?), besonders wenn man die Dritte Säule ebenfalls noch anrechnen könnte. Es würde sich somit um die Befriedigung eines echten sozialen Bedürfnisses handeln, für welches eine *gewisse landesweite Solidarität* zu rechtfertigen wäre.

Die Inanspruchnahme der leistungsfähigen AHV-Infrastruktur würde die Problematik des Pools wesentlich entschärfen und es den Betrieben ermöglichen, das Obligatorium mit dem Beitragsprimat zu erfüllen. Die Kosten könnten dadurch leicht *unter Kontrolle gehalten* werden, da man das jeweils geltende Leistungsziel unter Berücksichtigung der bestehenden wirtschaftlichen und demographischen Gegebenheiten festsetzen könnte.