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The Time Until Ruin in Collective Risk Theory

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1. Introduction and Summary

Let $(x_1, u_1), (x_2, u_2), \ldots$ be a sequence of independent identically distributed two-dimensional random vectors with $\mu = E(x_1)$, $\sigma^2 = \text{Var}(x_1) < \infty$, $u_1 \geq 0$, $\alpha = E(u_1) > 0$, and $\beta^2 = \text{Var}(u_1) < \infty$. Let $s_n = x_1 + \ldots + x_n$, $U_n = u_1 + \ldots + u_n$, $\nu(t) = \max\{n: U_n \leq t\}$; and $\tau^* = \tau^*(b) = \inf\{t: s_{\nu(t)} \geq b\}$, where by convention $\inf \phi = +\infty$. The relation of τ^* to the time until ruin in collective risk theory is discussed by *von Bahr* (1974), who studies the asymptotic distribution of $\tau^*(b)$ as $b \to \infty$. The purpose of this note is to reproduce *von Bahr*'s result (Theorem 2) by an alternative method which seems to be simpler and provide additional insight, and to obtain some related results.

For the case $\mu \geq 0$, Section 2 gives the limiting joint distribution of $s_{\nu(\tau^*)} - b$ and τ^* properly normalized. In particular it is shown that these random variables are asymptotically independent. In Section 3 this result is combined with the well known Esscher transformation to give approximations to $P\{\tau^* \leq t\}$ in the case $\mu < 0$, under an additional assumption to the effect that the tail of the distribution of x_1^+ decreases exponentially fast. The asymptotic behavior of $E(\tau^* \mid \tau^* < \infty)$ and $\text{Var}(\tau^* \mid \tau^* < \infty)$ is also given. In Section 4 a similar result in the case of two absorbing barriers is obtained and its interpretation in queuing theory mentioned.

Let $\tau = \tau(b) = \inf\{n: n > 0, s_n \geq b\}$, so that

$$\tau^* = U_\tau, \; s_{\nu(\tau^*)} = s_\tau. \tag{1}$$

In order to understand the intuitive basis for the asymptotic independence of τ^* and $s_{\nu(\tau^*)} - b = s_\tau - b$ in the case of $\mu \geq 0$ and hence the method of proof of Theorem 1 below, consider first the special case $U_n \equiv n$, so that $\tau^* = \tau$. Assume also that x_1 is non-lattice and let $M_n = \max\limits_{1 \leq k \leq n} s_k$. Then

$$P\{\tau > n, s_{\tau(b)} - b < x\} = P\{M_n < b, s_{\tau(b)} - b < x\} \tag{2}$$

$$= \int_{(-\infty, b)} P\{s_{\tau(b)} - b < x \mid M_n < b, s_n = y\}\, P\{M_n < b, s_n \in dy\}$$

$$= \int_{(-\infty, b)} P\{s_{\tau(b-y)} - (b-y) < x\}\, P\{s_n \in dy \mid M_n < b\}\, P\{M_n < b\}.$$

It is an easy consequence of the renewal theorem (see *Feller* [1966], p. 354) that

$$\lim_{b-y\to\infty} P\{s_{\tau(b-y)}-(b-y)<x\} = \left(E(s_{\tau+})\right)^{-1} \int_{(0,\,x)} P\{s_{\tau+}>\xi\}d\xi, \tag{3}$$

where $\tau_{+} = \inf\{n: s_n>0\}$. Moreover, for large b and $n = n(b)\to\infty$ so that $P\{M_n<b\}$ remains bounded away from 0 and 1, the conditional distribution of s_n given that $M_n < b$ may be shown to concentrate on $(-\infty,\ b-f(b))$, for any $f(b)\to\infty$ more slowly than $b^{1/2}$. Hence approximately by (2) and (3) for such sequences $n = n(b)$

$$P\{\tau(b)>n,\, s_{\tau(b)}-b<x\} \cong P\{\tau(b)>n\}(Es_{\tau+})^{-1} \int_{(0,\,x)} P\{s_{\tau+}>\xi\}\, d\xi,$$

which is the desired result.

2. Joint distribution of $\tau^*(b)$ and $s_{\tau(b)} - b$ when $\mu \geqq 0$.

Theorem 1 in this section gives the joint asymptotic distribution of $\tau^*(b)$ (properly normalized) and $s_{\tau(b)}-b$ as $b\to\infty$ in the case $\mu \geq 0$. Details are given only for x_1 non-lattice and $\mu > 0$. Necessary changes in the lattice case are easy, and those for the case $\mu = 0$ are indicated at the end of this section. Lemma 1 is a standard result which forms the basis for the Doeblin-Anscombe central limit theory for sums of a random number of random variables (cf. *Rényi* [1966], p. 390).

Lemma 1. Let $y_1, y_2, \ldots,$ be independent random variables with $E(y_k) = 0$, $E(y_k^2) = 1$ $(k = 1, 2, \ldots)$. Let $m(t)$ be a positive integer valued random variable such that for some constant $c > 0$

$$p\lim_{t\to\infty} m(t)/t = c. \tag{4}$$

Then as $t\to\infty$, $t^{-\frac{1}{2}}\{\sum_1^{m(t)} y_k - \sum_1^{[ct]} y_k\} \xrightarrow{P} 0$.

Lemma 2. Assume $\mu\geqq 0$ and let Φ denote the standard normal distribution function. Then

$$\lim_{t\to\infty} P\{(s_{\nu(t)+1}-\alpha^{-1}\mu t)/t^{\frac{1}{2}}\leqq x\} = \Phi\left(\alpha^{\frac{3}{2}}x/(E(\alpha x_1-\mu u_1)^2)^{\frac{1}{2}}\right).$$

Proof. It is well known (and follows easily from the relation $U_{\nu(t)} \leqq t < U_{\nu(t)+1}$ and the strong law of large numbers) that

$$\nu(t)/t \to \alpha^{-1} \text{ a.s. } (t\to\infty). \tag{5}$$

With the notation $\tilde{\nu} = \nu+1$

$$s_{\tilde{\nu}(t)} - \alpha^{-1}\mu t = \left(s_{\tilde{\nu}(t)} - \mu\tilde{\nu}(t)\right) - \alpha^{-1}\mu\left(U_{\tilde{\nu}(t)} - \alpha\tilde{\nu}(t)\right) + \alpha^{-1}\mu\left(U_{\tilde{\nu}(t)} - t\right). \tag{6}$$

It is known from renewal theory (*Feller* [1966], p. 356) that the third term on the right hand side of (6) converges in distribution. Hence by (5) (6) and Lemma 1,

$$t^{-\frac{1}{2}}\{s_{\tilde{\nu}(t)} - \alpha^{-1}\,\mu t - (s_{[\alpha^{-1}t]} - \mu[\alpha^{-1}t]) + \alpha^{-1}\mu(U_{[\alpha^{-1}t]} - \alpha[\alpha^{-1}t])\} \xrightarrow{P} 0,$$

which together with the central limit theorem yields the lemma.

Lemma 3. Assume $\mu > 0$. If $b > b'$ and $b - b' \to \infty$ as $t\to\infty$, then

$$P\left\{\max_{0\leqq k\leqq \nu(t)} s_k > b,\ \min_{n>\nu(t)} s_n < b'\right\} \to 0\ (t\to\infty).$$

Proof. $\mathrm{P}\left\{\max_{0\leqq k\leqq \nu(t)} s_k > b,\ \min_{n>\nu(t)} s_n < b'\right\} \leqq \sum_{k=1}^{\infty} P\left\{\tau=k, \nu(t)\geqq k,\ \min_{n>k}(s_n - s_k) < b'-b\right\}$

$$\leqq \sum_{1}^{\infty} P\{\tau=k,\ \nu(t)\geqq k\}\ P\left\{\min_{n>k}(s_n - s_k) < b'-b\right\} \leqq P\left\{\min_{n\geqq 1} s_n < b'-b\right\} \to 0 \text{ as } t\to\infty.$$

Theorem 1. Assume $\mu>0$. Let $\tilde{\sigma}^2 = E(\alpha x_1 - \mu u_1)^2$ and $t = \alpha\mu^{-1}b - y\tilde{\sigma}\mu^{-\frac{3}{2}}b^{\frac{1}{2}}$. Then uniformly for $-\infty<y<\infty, 0\leqq x\leqq\infty$ $\lim_{b\to\infty} P\{\tau^*(b)>t,\ s_{\tau(b)} - b < x\} = \Phi(y)\,G(x)$, where G denotes the right side of (3).

Proof. It is easy to see that $b = \alpha^{-1}\mu t + y\tilde{\sigma}\,\alpha^{-\frac{3}{2}}t^{\frac{1}{2}} + o(t^{\frac{1}{2}})$. With the notation $M(t) = \max_{0\leq k\leq \nu(t)} s_k$, $\tilde{\nu} = \nu+1$ the probability appearing in Theorem 1 may be re-written

$$P\{M(t)<b, s_{\tau(b)}-b<x\}$$

$$= \sum_{n=0}^{\infty} \int_{(-\infty,\, b+x)} P\{s_{\tau(b)}-b<x, M(t)<b,\ U_n\leq t<U_{n+1},\ s_{n+1}\in d\xi\}$$

$$= \sum_{n=0}^{\infty} \int_{(-\infty,\, b+x)} P\{s_{\tau(b-\xi)}-(b-\xi)<x\}P\{M(t)<b, U_n \le t < U_{n+1}, s_{n+1} \in d\xi\}$$

$$= \int_{(-\infty,\, b+x)} P\{s_{\tau(b-\xi)}-(b-\xi)<x\}P\{M(t)<b, s_{\tilde{v}(t)} \in d\xi\} \tag{7}$$

$$= \int_{(-\infty,\, b+x)} P\{s_{\tau(b-\xi)}-(b-\xi)<x\}(P\{s_{\tilde{v}(t)} \in d\xi\} - P\{M(t)\ge b, s_{\tilde{v}(t)} \in d\xi\})$$

Let $\in > 0$ and $b' = \alpha^{-1}\mu t + (y - \in)\,\tilde{\sigma}\,\alpha^{-\frac{3}{2}} t^{\frac{1}{2}}$. Consider splitting the integral on the right hand side of (7) according as $-\infty < \xi < b'$ or $b' \le \xi < b+x$. Uniformly over the range $-\infty < \xi < b'$, by the renewal theorem (cf. *Feller* [1966], p. 354)

$$\lim_{b\to\infty} P\{s_{\tau(b-\xi)} - (b-\xi)<x\} = G(x). \tag{8}$$

By Lemma 3

$$\int_{(-\infty,\, b')} P\{M(t)\ge b, s_{\tilde{v}(t)} \in d\xi\} \to 0, \tag{9}$$

and by Lemma 2

$$\int_{(b',\, b+x)} P\{M(t)\ge b, s_{\tilde{v}(t)} \in d\xi\} \le \int_{(b',\, b+x)} P\{s_{\tilde{v}(t)} \in d\xi\} \to \Phi(y) - \Phi(y-\in). \tag{10}$$

The theorem follows by letting first $b \to \infty$, then $\in \to 0$ in (7) and appealing to Lemma 2, (8), (9), and (10).

Remarks. (i) It seems more natural to integrate in (7) over the values of $s_{v(t)}$, but the second equality in (7) cannot be justified if s_{n+1} is replaced by s_n.
(ii) In the case $\mu = 0$ a consequence of Donsker's invariance principle (cf. *Billingsley* [1968], pp. 68 and 146) is that for $\xi \le y$

$$\lim_{t\to\infty} P\{M(t)<y(\alpha^{-1}t)^{\frac{1}{2}}, s_{\tilde{v}(t)}<\xi(\alpha^{-1}t)^{\frac{1}{2}}\} = \Phi(\xi) - \Phi(\xi-2y),$$

which with an argument similar to the above yields

$$\lim_{b\to\infty} P\{\tau^*(b)>\alpha b^2 y, s_{\tau(b)}-b<x\} = G(x)\,\{2\Phi(y^{-\frac{1}{2}})-1\}.$$

3. The case $\mu < 0$.

Assume that $\mu < 0$ and, moreover, that $\psi(\theta) = E(\exp(\theta x_1) < \infty$ for some $\theta > 0$. By differentiation inside the expectation it is easy to see that $\psi'(0+) = \mu < 0$ and $\psi'' > 0$, and hence there exists at most one value γ necessarily positive, such that $\psi(\gamma) = 1$. Assume that such a γ exists and that

$$E\{(u_1^2+x_1^2)\exp(\gamma x_1)\} < \infty. \tag{11}$$

Let P_γ denote that probability measure under which (x_1,u_1), (x_2,u_2), ... are independent and identically distributed with

$$P_\gamma\{x_1 \in dx, u_1 \in du\} = e^{\gamma x} P\{x_1 \in dx, u_1 \in du\}. \tag{12}$$

Let α_γ, β_γ, μ_γ, σ_γ, and $\tilde{\sigma}_\gamma$ be defined with respect to P_γ analogously to α, β, μ, σ, and $\tilde{\sigma}$ in Sections 1 and 2. These quantities are finite by (11) and (12). From the convexity of ψ it follows that $\mu_\gamma > 0$, and hence Theorem 1 applies to the sequence $\{(x_k,u_k), k=1,2,\ldots\}$ under the probability P_γ. For purposes of studying the distribution under P of τ^* the relation between P and P_γ is given by

Lemma 4. Let $F_n = B((x_k,u_k), 1 \le k \le n)$. Let T be an arbitrary stopping time relative to F_n and f a non-negative random variable such that $fI_{\{T=n\}}$ is F_n-measurable for all $n = 1, 2, \ldots$ Then

$$\int_{\{T<\infty\}} f dP = \int_{\{T<\infty\}} f \exp(-\gamma s_T) dP_\gamma. \tag{13}$$

For f not necessarily non-negative, the existence of either integral in (13) implies the existence of the other and the indicated equality.

Proof. If $P^{(n)}$ $(P_\gamma^{(n)})$ denotes the restriction of $P(P_\gamma)$ to F_n, then by (12) $P^{(n)}$ and $P_\gamma^{(n)}$ are mutually absolutely continous with

$$dP^{(n)}/dP_\gamma^{(n)} = \exp(-\gamma s_n). \tag{14}$$

Obviously $fI_{\{T<\infty\}} = \sum_1^\infty fI_{(T=n)}$ and since the n^{th} term of the indicated sum is F_n-measurable, the lemma follows easily from (14).

Combining Lemma 4 with Theorem 1 gives

Theorem 2 (von Bahr). Let $t = \alpha_\gamma \mu_\gamma^{-1} b + y\, \tilde{\sigma}_\gamma \mu_\gamma^{-\frac{3}{2}} b^{\frac{1}{2}}$. Then as $b \to \infty$

$$P\{\tau^*(b) \leq t\} \sim C_\gamma e^{-\gamma b}\, \Phi(y), \tag{15}$$

where

$$C_\gamma = P\{\tau_+ = \infty\} \,/\, \gamma \mu_\gamma E_\gamma(\tau_+).$$

Proof. By (1) putting $f = I_{\{U_\tau \leq t\}}$ and $T = \tau$ in (13) yields

$$P\{\tau^*(b) \leq t\} = e^{-\gamma b} \int_{(0,\infty)} e^{-\gamma x}\, P_\gamma\{\tau^*(b) \leq t,\, s_{\tau(b)} - b \in dx\},$$

and appealing to Theorem 1 yields (15) with

$$C_\gamma = \int_{(0,\infty)} e^{-\gamma x}\; P_\gamma(s_{\tau_+} > x)dx \,/\, E_\gamma(s_{\tau_+}).$$

By Wald's lemma $E_\gamma\, s_{\tau_+} = \mu_\gamma E_\gamma\, \tau_+$, and integration by parts together with Lemma 4 gives

$$\int_{(0,\infty)} e^{-\gamma x}\, P_\gamma(s_{\tau+} > x)dx = \gamma^{-1}(1 - E_\gamma e^{-\gamma s \tau_+}) = \gamma^{-1}\, P\{\tau_+ = \infty\},$$

which completes the proof.

Remark. According to results of Spitzer (cf. *Feller* [1966], Chapter 18) the constant C_γ of Theorem 2 can be evaluated in terms of $\sum_1^\infty n^{-1} P\{s_n > 0\}$ and $\sum_1^\infty n^{-1} P_\gamma\{s_n \leq 0\}$, although such an evaluation is suitable for numerical purposes in relatively few cases.

Letting $t \to \infty$ in (13) and then applying (3) as $b \to \infty$ yields Cramér's classical result

$$P\{\tau^*(b) < \infty\} \sim C_\gamma e^{-\gamma b} \quad (b \to \infty). \tag{16}$$

Hence Theorem 2 has the consequence that for large b conditional on $\tau^*(b) < \infty$ the random variable $\tau^*(b)$ has approximately a normal distribution with mean $\alpha_\gamma \mu_\gamma^{-1} b$ and variance $\tilde{\sigma}_\gamma^2 \mu^{-3} b$.

Theorem 3. As $b \to \infty$

$$E(\tau^* \mid \tau^* < \infty) = \alpha_\gamma \mu_\gamma^{-1} b + o(b^{\frac{1}{2}}) \tag{17}$$

and

$$\operatorname{Var}(\tau^* \mid \tau^* < \infty) \sim \tilde{\sigma}_\gamma^2 \mu_\gamma^{-3} b.$$

Proof. By (1) and Lemma 4

$$\begin{aligned} b^{-1} \operatorname{Var}(\tau^* \mid \tau^* < \infty) &= E\{(U_\tau - \alpha_\gamma \mu_\gamma^{-1} b + \alpha_\gamma \mu_\gamma^{-1} b - E(\tau^* \mid \tau^* < \infty))^2 \mid \tau^* < \infty\} \\ &= (bP\{\tau^* < \infty\} e^{\gamma b})^{-1} E_\gamma\{(U_\tau - \alpha_\gamma \mu_\gamma^{-1} b)^2 \exp(-\gamma(s_\gamma - b))\} \qquad (18) \\ &\quad - b^{-1}\{E(\tau^* \mid \tau^* < \infty\} - \alpha_\gamma \mu_\gamma^{-1} b\}^2. \end{aligned}$$

By (16) and (18) it suffices to prove (17) and

$$b^{-1} E_\gamma\{(U_\tau - \alpha_\gamma \mu_\gamma^{-1} b)^2 \exp(-\gamma(s_\tau - b))\} \to C_\gamma \tilde{\sigma}_\gamma^2 \mu_\gamma^{-3}. \tag{19}$$

By Theorem 1, to prove (19), it suffices to prove the P_γ uniform integrability of $b^{-1}(U_\tau - \alpha_\gamma \mu_\gamma^{-1} b)^2$. Obviously

$$b^{-1}(U_\gamma - \alpha_\gamma \mu_\gamma^{-1} b)^2 \leq 2b^{-1}\{(U_\tau - \alpha_\gamma \tau)^2 + \alpha_\gamma (\tau - \mu_\gamma^{-1} b)^2\}. \tag{20}$$

It is well known (and follows from Theorem 1 with $u_n \equiv 1$) that under P_γ $b^{-1}(\tau - \mu_\gamma^{-1} b)^2$ converges in distribution to $\mu_\gamma^{-3} \sigma_\gamma^2 \chi_1^2$, and also $b^{-1} E_\gamma(\tau - \mu_\gamma^{-1} b)^2 \to \mu_\gamma^{-3} \sigma_\gamma^2$ (cf. *Chow, Robbins,* and *Siegmund* [1971], p. 33). Hence $b^{-1}(\tau - \mu_\gamma^{-1} b)^2$ is P_γ uniformly integrable. Similarly, it follows easily from Lemma 1 that $b^{-1}(U_\tau - \alpha_\gamma \tau)^2$ converges in distribution to $\mu_\gamma^{-1} \beta_\gamma^2 \chi_1^2$. And by Wald's lemma for squared sums $b^{-1} E_\gamma(U_\tau - \alpha_\gamma \tau)^2 = b^{-1} \beta_\gamma^2 E_\gamma(\tau)$, and since $b^{-1} E_\gamma \tau \to \mu_\gamma^{-1}$ (cf. *Chow, Robbins, Siegmund* [1971], pp. 23 and 29), it follows that $b^{-1}(U_\tau - \alpha_\gamma \tau)^2$ is P_γ uniformly integrable. Hence by (20), so is $b^{-1}(U_\tau - \alpha_\gamma \mu_\gamma^{-1} b)^2$, which proves (19). Since in particular the left hand side of (19) is finite, consideration of positive and negative parts separately together with Lemma 4 and (1) gives

$$b^{-\frac{1}{2}}(E(\tau^*\,|\,\tau^*<\infty)-\alpha_\gamma\mu_\gamma^{-1}b)$$

$$= (b^{\frac{1}{2}}e^{\gamma b}P(\tau^*<\infty))^{-1}E_\gamma\{(U_\tau-\alpha_\gamma\mu_\gamma^{-1}b)\exp(-\gamma(s_\tau-b))\}. \tag{21}$$

From (19) follows the uniform integrability of $b^{-\frac{1}{2}}(U_\tau-\alpha_\gamma\mu_\gamma^{-1}b)\exp(-\gamma(s_\tau-b))$. Hence by (16) and Theorem 1 the right hand side of (21) converges to 0, which completes the proof of the Theorem.

4. Remarks.

(i) With the same assumption as in Section 3 for $a\leq 0<b$ define

$$T^* = \inf\{t: s_{v(t)}\notin(a,b)\},\ T = \inf\{n: s_n\notin(a,b)\},$$

so that corresponding to (*i*) $U_T = T^*$. By writing

$$P\{T^*\leq t, s_T\geq b\} = P\{\tau^*\leq t\} - P\{T^*<\tau^*\leq t\}$$

$$= P\{\tau^*\leq t\} - \int_{\{T^*<t,\,s_T<a\}} P\{\tau^*\leq t\,|\,T^*, s_T\}dP, \tag{22}$$

setting $t = \alpha_\gamma\mu_\gamma^{-1}b + y\,\tilde{\sigma}_\gamma\mu_\gamma^{-\frac{3}{2}}b^{\frac{1}{2}}$ as in Theorem 2, and splitting the integral in (22) according as $T\leq t'$ or $t'<T<t$, where $t'\to\infty$ but $t' = o(b^{\frac{1}{2}})$ it is not difficult to show that as $b\to\infty$ (a fixed)

$$P\{T^*\leq t, s_T\geq b\} \sim C_\gamma e^{-\gamma b}\,\Phi(y)\,P_\gamma\{\min_{1\leq n<\infty} s_n\geq a\}. \tag{23}$$

When $|a|$ is large the last factor in (23) may be estimated using (16), although now the roles of P and P_γ have been interchanged. When $a = 0$ a formula for this probability has been obtained by Spitzer (cf. *Feller* [1966], Chapter 18). The same argument with $t = \infty$ shows that

$$P\{s_T\geq b\} \sim C_\gamma^{-\gamma b}P_\gamma\{\min_{1\leq n<\infty} s_n\geq a\}$$

and hence

$$\lim_{b\to\infty} P\{T^* \leq \alpha_\gamma \mu_\gamma^{-1} b + y\,\tilde{\sigma}_\gamma \mu_\gamma^{-\frac{3}{2}} b^{\frac{1}{2}} \,|\, s_T \geq b\} = \Phi(y).$$

In the case $U_n \equiv n$, so that $T = T^*$, with $a = 0$ this result has an interpretation in queuing theory to the effect that for large b, conditional that during a given busy period some customer must wait at least b, the number of the first customer to wait this long is approximately normally distributed with mean $\mu_\gamma^{-1} b$ and variance

$$\sigma_\gamma^2\, \mu_\gamma^{-3} b.$$

(*ii*) At least in the case $U_n \equiv n$ it is possible to extend the range of validity of Theorem 1 to include values y depending on b and tending slowly to $-\infty$ as $b \to \infty$. The limit indicated in Theorem 1 should then be replaced by asymptotic equality (i.e. the ratio of the two sides of the equality converges to 1). Similar "large deviation probabilities" may be obtained in the context of Theorem 2. It should be noted, however that this method seems to apply only to the upper tail of the distribution of τ.

(*iii*) As explained by *von Bahr* (1974), τ^* is exactly the time until ruin for a risk process in the case of positive gross premium intensity. For the case of negative gross premium intensity one can approximate the distribution of the time until ruin in terms of τ^* and $\tau^{**} = \inf\{t\colon s_{\nu(t)+1} \geq b\}$, which may also be studied by the methods of this paper. For details the reader is referred to *von Bahr*'s work.

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References

von Bahr, B. (1974): "Ruin probabilities expressed in terms of ladder height distributions," Scandinavian Actuarial Journal, 190–204.

Billingsley, P. (1968): "Convergence of Probability Measures," John Wiley and Sons, New York.

Chow, Y.S., Robbins, H., and *Siegmund, D.* (1971): "Great Expectations: The Theory of Optimal Stopping," Houghton Mifflin, Boston.

Feller, W. (1966): "An Introduction to Probability Theory and Its Applications," John Wiley and Sons, New York.

Rényi, A. (1966): "Wahrscheinlichkeitsrechnung." VEB Deutscher Verlag der Wissenschaften, Berlin.

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Zusammenfassung

Für einen Risikoprozess mit unabhängigen, identisch verteilten Schadenforderungen, die zu den Zeitpunkten eines Erneuerungsprozesses erfolgen, wird ein neuer Beweis für von Bahrs Näherungswert für die Verteilung der Ruinzeit gegeben. Einige verwandte Ergebnisse werden auch hergeleitet.

Resumé

Pour un processus de risque présentant des demandes d'indemnité indépendantes mais identiquement réparties et intervenant aux moments d'un processus de renouvellement, il a été ici apporté une nouvelle preuve de la valeur approximative de von Bahr pour la répartition du temps jusqu'à la ruine. Divers résultats apparentés ont été également déduits.

Riassunto

Per il caso di un processo stocastico con sinistri indipendenti, identicamente distribuiti, che avvengono nei momenti di un processo di rinnovo, si dà una nuova dimostrazione per l'approssimazione di von Bahr per la distribuzione del tempo fino al fallimento. Si derivano pure alcuni risultati affini.

Summary

For a risk process with independent identically distributed claims occurring at the time points of a renewal process, a new proof is given for von Bahr's approximation to the distribution of the time until ruin. Some related results are also obtained.