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STATISTICAL PROPERTIES OF 2D PERIODIC LORENTZ GAS WITH INFINITE HORIZON

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Abstract. We show that in the periodic 2D Lorentz gas with infinite horizon the limit distribution of the particle displacement is Gaussian but the normalization factor is $\sqrt{t \log t}$ and not $\sqrt{t}$ as in the classical case. We find explicitly the covariance matrix of the limit distribution.

The problem we are interested in can be described in the following way. Let a point particle move freely with the unit velocity in the plane outside of a periodic set of circles of a radius $a < 1/2$ centered at the sites of a square lattice with the unit space. It is assumed that the particle reflects elastically from any circle when it reaches the circle. Denote $\vec{x}(t)$ the position of the particle at the moment t. The main problem we are interested in is: Assuming that at $t = 0$ the position of the particle in the phase space is uniformly distributed with respect to the Liouville measure of the periodic problem, what is the right normalization $N(t)$ such that there exists a (non–trivial) limit distribution of $\dfrac{\vec{x}(t) - \vec{x}(0)}{N(t)}$ as $t \to \infty$ and what is the limit distribution?

Let t_n be the time of the n–th collision of the particle with the circles. A discrete version of the problem is: What is N_n such that there exists a limit distribution of $\dfrac{\vec{x}(t_n) - \vec{x}(0)}{N_n}$ as $n \to \infty$ and what is the limit distribution? As a generalization of the problem one may think of a particle moving outside of a periodic set of non–intersecting strictly convex scatterers with the condition of elastic reflection under collisions with the scatterers.

An important feature of the original problem with the square lattice of circular scatterers is that in that case the length of free motion of the particle between subsequent collisions is unbounded. Such a configuration of scatterers is called a configuration with an infinite horizon. If the length of the particle free motion is bounded by a constant it is called a configuration with a finite horizon.

The problem on the statistical behavior of $\vec{x}(t) - \vec{x}(0)$ and the related one on the asymptotics of the velocity autocorrelation function $\langle \vec{v}(0)\vec{v}(t) \rangle$ in the periodic Lorentz gas were studied both theoretically and numerically in works of Sinai; Bunimovich, Sinai; Bunimovich; Kramli, Szasz; Casati, Comparin, Guarneri; Machta; Machta, Zwanzig; Friedman, Oono, Kubo; Friedman, Martin; Bouchad, Le Doussal; Zacherl, Geisel, Nierwetberg; Bleher and others. In particular, it was proved in [1] that if a periodic configuration of scatterers has a finite horizon then the central limit theorem holds: The limit distribution of $\dfrac{\vec{x}(t) - \vec{x}(0)}{\sqrt{t}}$ is Gaussian.

We report here the following main result (see [2]). Let us consider a periodic configuration of strictly convex non–intersecting scatterers. Assume that it has an infinite horizon. Let $\Omega_1, \ldots, \Omega_N$ be basic scatterers which generate the whole periodic configuration by means of shifts in the periods of the lattice. Let us call a corridor an open infinite strip in the plane which has no intersection with the scatterers and which is tangent from the both sides to an infinite number of scatterers. Let us enumerate $C_{k1}, \ldots, C_{kN_k}$ all the corridors which touch the scatterer Ω_k, $k = 1, \ldots, N$. Denote d_{kl} the width of the corridor C_{kl} and h_{kl} the period of the lattice in the direction of C_{kl}. Then we state that the limit in distribution,

$$\frac{\vec{x}(t_n) - \vec{x}(0)}{\sqrt{n \log n}} = \vec{\xi}$$

exists, where $\vec{\xi}$ is a Gaussian random vector variable with zero mean and the covariance matrix $(D_{lm})_{l,m=1,2}$, which can be calculated in the following way. Let $(D\vec{z}, \vec{z}) = \sum_{l,m=1,2} D_{lm} z_l z_m$. Then

$$(D\vec{z}, \vec{z}) = \frac{1}{2\sum_{k=1}^{N} |\partial \Omega_k|} \sum_{k=1}^{N} \sum_{l=1}^{N_k} h_{kl} d_{kl}^2 (\vec{\omega}_{kl}, \vec{z})^2 \tag{1}$$

where $|\partial \Omega_k|$ is the length of the boundary of the scatterer Ω_k and $\vec{\omega}_{kl}$ is the unit vector parallel to the boundary of C_{kl}. A related formula is valid for the continuous time dynamics:

$$\lim_{t \to \infty} \frac{\vec{x}(t) - \vec{x}(0)}{\sqrt{t \log t}} = \frac{\vec{\xi}}{\sqrt{\tau}}$$

where τ is the mean time of free motion. It is noteworthy that the covariance matrix D in (1) is positive if there exist two non–parallel corridors.

For the case of the square (or triangular) lattice of circular scatterers the formula (1) can be further specified. Namely, in that case D is proportional to the unit matrix, $D_{lm} = \frac{D_0}{2} \delta_{lm}$, and for small radius a we have that

$$D_0 = \frac{1}{8\pi^2 a^2}(1 + O(a^{1/4}))$$

for the square lattice and

$$D_0 = \frac{25\sqrt{3} - 36}{64\pi^2 a^2}(1 + O(a^{1/4}))$$

for the triangular lattice.

References

[1] Bunimovich, L.A., Sinai, Ya.G.: Commun. Math. Phys. **78**, 247–280 (1981)
[2] Bleher, P.M.: to appear in Journ. Statist. Phys.