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Helvetica Physica Acta, Vol. 62 (1989) 420–423
0018–0238/89/040420–04$1.50 + 0.20/0

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By Corrado Massa

Via Fratelli Manfredi 55, 42100 – Reggio Emilia, Italy

(20. VI. 1988, revised 24. X. 1988)

Abstract. It is shown that the temperature-dependent gravitational constant required by a variety of gauge theories implies the existence of an upper bound on the temperature of the thermal radiation. Contrary to the opinions expressed by some, a thermal radiation never attains sufficient temperature to enter the "antigravity" regime.

A variety of gauge theories used in the unification program imply a gravitational constant G that is temperature-dependent [1]

$$G = G_0(1 - \alpha G_0 T^2)^{-1} \tag{1}$$

where T is the temperature, G_0 is the zero-temperature value of G, very close to the currently observed value $6.67\,10^{-8}\,\mathrm{cm}^3\,\mathrm{g}^{-1}\,\mathrm{s}^{-2}$, and α is a constant whose numerical value depends upon the coupling constants that enter the gauge theory concerned (e.g. it may depend on the ratio of two coupling constants). It is likely that (1) $\alpha \gg k^2/\hbar c^5$ (where k is the Boltzmann constant, $\hbar$ is the reduced Planck constant, and c is the speed of light); A possible value is [1] $\alpha = 10^{40}\, k^2/\hbar c^5$ (10^{40} is the ratio of electromagnetic to gravitational coupling constants) namely $\sim 10^{-17}\,\mathrm{s}^2\,\mathrm{g}\,\mathrm{K}^{-2}\,\mathrm{cm}^{-3}$.

At the first glance, equation (1) suggests that at the critical temperature

$$T_C = (\alpha G_0)^{-1/2}$$

the gravitational constant G passes through an infinite discontinuity, and becomes negative at $T > T_C$, i.e. antigravity results (consequences of such antigravity regime are investigated in Ref 2 and in references cited therein). What follows, however, suggests that (contrary to expectations grounded on equation (1)) the critical temperature T_C is a universal upper bound on temperature, never reached by any physical process.

The stress energy tensor T^{ik} of thermal (black body) radiation is that of a perfect fluid, namely:

$$T^{ik} = (\rho + P)U^i U^k - P g^{ik} \tag{2}$$

where i and k run from 0 to 3; ρ is the rest frame energy density, and P the pressure, related to ρ by the equation of state $\rho = 3P$.

U^i is the 4-velocity of the local rest frame of the radiation, and g_{ik} is the fundamental tensor. From $\rho = 3P$ one finds:

$$T^{ik} = (4/3)\rho U^i U^k - (\rho/3)g^{ik} \tag{3}$$

Consider now a spherical box filled with radiation in a thermal distribution, and assume the distribution to be static, thus:

$$U^\beta = 0, \qquad ds^2 = e^\nu c^2\, dt^2 - e^\lambda\, dr^2 - d\Omega^2 \tag{4}$$

where $\beta = 1, 2, 3$; ds^2 is the line element of the spacetime, ν and λ are functions of r only, $d\Omega^2$ means $r^2\, d\vartheta^2 + r^2 \sin^2 \vartheta\, d\varphi^2$, and r, ϑ, φ, ct are the Schwarzschild coordinates. The normalization of 4-velocity $U^i U_i = 1$ determines $U^0 = e^{-\nu/2}$ and, by (3)(4):

$$T^{00} = \rho e^{-\nu}, \qquad T^0_0 = \rho.$$

By the standard formulas for black body radiation, ρ is given in terms of the locally measured temperature T by:

$$\rho = aT^4 \tag{5}$$

where $a = Nk^4(\hbar c)^{-3}$ is the radiation constant, N is the number of species of radiation ($N = 8N_B + 7N_F$ where N_B and N_F are respectively the number of helicity states of bosons and fermions).

The radiation is a self-gravitating fluid and therefore ρ depends on the radial coordinate r. From (5), $T = T(r)$ and, for (1), $G = G(r)$.

Assume now that no matter (and no radiation) lies outside the box, thus $T(r) = 0$ and $G(r) = G_0$ if $r > R$ where R is the radial coordinate of the box's surface. In this case, the gravitational field equation [3]

$$R^{ik} - (1/2)g^{ik}R^s_s = 8\pi G(r)T^{ik} \tag{6}$$

reads simply $R^{ik} = 0$ and the ds^2 assumes the well known form:

$$ds^2 = \gamma\, dt^2c^2 - (1/\gamma)\, dr^2 - d\Omega^2,$$
$$\gamma = 1 - 2G_0M/rc^2 \tag{7}$$

where M is the total mass of the thermal radiation field (i.e. it is the mass that governs the Keplerian motions of test particles in the distant Newtonian gravitational field). Inside the box, the (0 0) component of equation 6 reads [4]

$$(8\pi G/c^4)T^0_0 = (1/r^2) - e^{-\lambda}(1/r^2 - (1/r)\, d\lambda/dr) \tag{8}$$

which integrated gives (for $g_{11} = -e^\lambda$)

$$1/g_{11} = -1 + (8\pi/rc^4)\int_0^r G(r)T^0_0(r)r^2\, dr \tag{9}$$

where $G = G(r)$ because $r < R$.

From equation 7 and equation 9 one finds, respectively

$$\lim_{r\to R^+} 1/g_{11} = -1 + 2G_0M/Rc^2 \tag{10}$$

$$\lim_{r\to R^-} 1/g_{11} = -1 + (8\pi/Rc^4)\int_0^R G(r)T_0^0(r)r^2\,dr \tag{11}$$

Equations 10 and 11 lead (for the continuity of $g_{11}(r)$ at $r = R$) to

$$\int_0^R G(r)T_0^0(r)r^2\,dr = G_0Mc^2/4\pi. \tag{12}$$

The total mass inside the box is therefore:

$$M = (4\pi a/c^2)\int_0^R T^4(r)r^2(1-\alpha G_0T^2(r))^{-1}\,dr \tag{13}$$

because $T_0^0 = \rho = aT^4$, and $G = G_0(1-\alpha G_0T^2)^{-1}$. If, in equation 7, g_{00} is negative, then the ds^2 has the signature $(-, +, -, -)$ that is physically absurd.

If $g_{00} = 0$, then the box is a black hole and all informations about the thermal radiation field are totally lost. Therefore g_{00} is positive i.e. $M < rc^2/2G_0$, and equation 13 gives:

$$Rr_0^2T_0^4(1-\alpha G_0T_0^2)^{-1} < \beta r \tag{14}$$

where

$$0 \le r_0 \le R, \qquad T_0 = T(r_0), \quad \text{and} \quad \beta = c^4/8\pi aG_0.$$

As $r \to R^+$, one finds

$$T_0^4r_0^2 + T_0^2G_0\alpha\beta - \beta < 0, \quad \text{that gives} \quad T_0 < T_C.$$

In conclusion, T cannot reach T_C on the surface $r = r_0$. Clearly $T < T_C$ at $r > r_0$ (because a self-gravitating fluid requires $d\rho(r)/dr < 0$) and $T < \infty$ at $r < r_0$ (because $T(r)$ is continuous at $r = r_0$).

If T cannot be increased to arbitrarily large values at $r < r_0$, then there must exist, mathematically speaking, a least upper bound T_{max} to T at $r < r_0$; Since at $r \ge r_0$ the upper bound on T is T_C, it is very natural to think that $T_{max} = T_C$.

The upper bound $T < T_C$ has been obtained at the cosmological level by Starobinsky [5] and Pollock [6] which investigated the early (but nevertheless classical) universe. Starobinsky considers a homogeneous but anisotropic cosmological metric; the nonzero classical mean value $\bar{\varphi} = \langle 0|\varphi|0\rangle$ of Linde's [2] scalar field φ cannot become greater than the critical value $\bar{\varphi}_C$ whereby $G < 0$; the reason is that as $\bar{\varphi} \to \bar{\varphi}_C$ any departure from isotropy will grow without bound; $\bar{\varphi}$ depends on the temperature T of the cosmological fluid, and $\bar{\varphi} < \bar{\varphi}_C$ requires $T < T_C$.

Pollock considers a simple Friedmann cosmological model, which is isotropic and homogeneous, and finds again, with a straightforward computation, $T < T_C$

and $G > 0$ until the singularity is reached. Both authors do not investigate the very early universe (dominated by quantum gravity).

My argument, however, gives $T < T_C$ in a quite different context, i.e. at the local (non cosmological) level, and shows that the temperature of a black body radiation field confined in a box surrounded by a Schwarzschild spacetime cannot exceed T_C. I thank an anonymous referee of this Review which brought Refs. [5] and [6] to my attention.

REFERENCES

[1] P. C. W. Davies, Phys. Lett. *101*B, 399 (1981).
[2] A. D. Linde, Phys. Lett. *93*B, 394 (1980).
[3] A. K. Raychaudhuri and B. Bagchi, Phys. Lett. *124*B 168 (1983).
[4] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (Addison-Wesley, Reading, Mass., and Pergamon, London, 1971) Ch. 11, Sec. 97, eq. 6.
[5] A. Starobinsky, Sov. Astronomy Lett. *7*, 36 (1981).
[6] M. Pollock, Phys. Lett. *108*B, 386 (1982).