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General relativistic effects on the cooling of neutron stars

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Abstract. We present a discussion of general relativistic effects on the cooling of neutron stars and show analytically that these almost cancel for the dominant neutrino processes and a very stiff equation of state (apart from a trivial redshift of the surface temperature for an observer "at infinity"). Numerical results for a "realistic" equation of state show larger general relativistic corrections. These are, however, still smaller than the uncertainties in the neutrino loss rates. Previous results of cooling curves would thus not be changed significantly by a general relativistic treatment of the thermal properties of neutron stars.

With the recent establishment of an upper limit on the effective temperature of steady emission from the Crab pulsar (Toor and Seward, 1977) and from putative neutron stars in young super-nova remnants with the use of the Imaging Proportional Counter on the Einstein Observatory (for recent summaries of these data, see Helfand et al., 1980), the subject of neutron star cooling has become of renewed interest. Cooling curves (surface temperature as a function of time) of neutron stars depend on various interesting aspects of neutron star physics (see, e.g., Straumann, 1980). In particular, a pion condensate in the central region of the neutron star would greatly enhance the neutrino emissivity (Maxwell et al., 1977).

Cooling curves have been computed by many authors (Tsuruta, 1979, and references therein; Maxwell, 1979; Glen and Sutherland, 1980; Van Riper and Lamb, 1980; Tsuruta, 1980). Most workers have neglected in the past general relativistic effects in the energy and transport equations. These are, however, potentially important (Glen and Sutherland, 1980), because they affect the temperature gradient in the core and thus the neutrino losses, which are very temperature sensitive ($\sim T^n$, $n=6.8$).

In this note we show first analytically that general relativistic effects on the cooling of neutron stars cancel (accidentally) to a large extent for the dominant neutrino processes and a very stiff equation of state. Then we present results of numerical calculations for a "realistic" equation of state. The general relativistic corrections turn out to be larger in this case, but they are still smaller than the uncertainties in the neutrino loss rates. This proves that previous results of cooling curves would not be changed significantly by a general relativistic treatment of the thermal properties of neutron stars. For this reason, we believe that the differences in the results of Van Riper and Lamb (1980) and those of Tsuruta (1979) cannot be due to general relativistic effects.

We begin by recalling the basic general relativistic equations which determine the cooling of neutron stars (see, e.g. Thorne, 1966).

The metric of a spherically symmetric neutron star is

$$g = e^{2\phi(r)}\, dr^2 - \frac{dr^2}{1-2Gm(r)/rc^2} - r^2(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2) \tag{1}$$

Here $m(r)$ is the gravitational mass enclosed in the sphere of radius r, i.e.

$$m(r) = \int_0^r \rho(r')4\pi r'^2\, dr' \tag{2}$$

where $\rho(r)$ is the total mass-energy density. The well-known equations for the pressure gradient and for $d\phi/dr$ (Tolman–Oppenheimer–Volkoff) decouple practically from the thermal equations, since the neutron star matter is essentially in its ground state (apart from a very thin envelope) shortly after formation of the neutron star. The hydrostatic structure is thus determined by the equation of state at $T=0$.

The relativistic generalization of the equation for thermal equilibrium (energy balance) is

$$\frac{d}{dr}(Le^{2\phi}) = -\frac{4\pi r^2}{(1-2Gm(r)/rc^2)^{1/2}}\, ne^{\phi} T \frac{ds}{dt} \tag{3}$$

The total luminosity, L, is the sum of the neutrino luminosity, L_ν, and the radiation/conduction luminosity L_γ. T is the local temperature and n denotes the density of baryons of specific entropy s (per baryon).

The neutrino luminosity gradient is given by

$$\frac{d}{dr}(L_\nu e^{2\phi}) = \frac{4\pi r^2}{(1-2Gm(r)/rc^2)^{1/2}}\, ne^{2\phi} q_\nu \tag{4}$$

where q_ν is the neutrino emissivity per baryon.

For thermal neutrino energies $\lesssim 1$ MeV, the neutrino mean free path is greater than the stellar radius. Hence the temperature gradient within the neutron star is governed by the radiation/conduction flux L_γ. The relativistic generalization of the equation for thermal energy transport reads

$$\frac{d}{dr}(Te^{\phi}) = -\frac{3\kappa\rho}{4acT^3}\frac{L_\gamma e^{\phi}}{4\pi r^2(1-2Gm(r)/rc^2)^{1/2}} \tag{5}$$

where κ is the total opacity: $\kappa^{-1} = \kappa_{\text{rad}}^{-1} + \kappa_{\text{cond}}^{-1}$.

The degenerate matter inside the neutron star has a very high thermal conductivity and thus equation (5) leads to $T \cdot e^{\phi} = \text{const.}$, except for a thin envelope (of a few meters thickness). The general relativistic factor e^{ϕ} is potentially important, since the dominant neutrino emissivities have a strong temperature dependence.

For the calculation of general relativistic corrections to cooling curves we integrate equations (3) and (4) over the star and neglect, for simplicity, the surface value of L_γ compared to that of L_ν. (This is valid for the first few thousand years.) From the resulting equations, one finds for the time dependence of Te^{ϕ} in the core:

$$-\frac{d}{dt}(Te^{\phi})=\frac{\int ne^{2\phi}q_\nu\,dV}{\int nc_v\,dV} \tag{6}$$

Here $dV=4\pi r^2(1-2Gm(r)/rc^2)^{-1/2}\,dr$ is the proper volume element, corresponding to the spatial part of the metric (1), and c_v is the specific heat, $T\,ds=c_v\,dT$.

Since superfluidity does not affect the cooling of neutron stars in an important way, we consider only the case of normal nucleons. Then the modified URCA-process dominates, if there exists no pion condensate. The corresponding emissivity is proportional to T^8. The emissivity due to the pion condensate β-decay has also a power dependence (T^6) of the temperature. In the following discussion, we put $\varepsilon_\nu := nq_\nu = AT^n$. The specific heat of a normal Fermi liquid is $c=B\cdot T$, where the constant B depends on the effective mass. Inserting this into equation (6) gives

$$-\frac{d}{dt}(Te^{\phi})=\frac{A}{B}(Te^{\phi})^{n-1}\frac{\int e^{-(n-2)\phi}\,dV}{\int e^{-\phi}\,dV} \tag{7}$$

In a non-relativistic approximation the time dependence of the interior temperature would be given by $-dT/dt=(A/B)T^{n-1}$. Comparing this with equation (7) at the surface of the core (before Te^{ϕ} begins to drop to the surface value on the few outermost meters), we see that the time dependence of the temperature T_i at the surface of the core is given by the same equation as in the nonrelativistic theory, but with A/B replaced by $(A/B)K$, where

$$K=e^{(n-2)\phi_s}\frac{\int e^{-(n-2)\phi}\,dV}{\int e^{-\phi}\,dV} \tag{8}$$

Here ϕ_s denotes the surface value of ϕ. Note that K can be interpreted as a "renormalization" of A, i.e. of the neutrino emissivity.

For stiff equations of state the density profiles of neutron stars are rather flat. In this case a homogeneous model ($\rho=$ const.) should provide a reasonable estimate for the magnitude of the general relativistic correction factor K. Then e^{ϕ} is given by

$$e^{\phi}=\frac{3}{2}\left(1-\frac{2GM}{c^2R}\right)^{1/2}-\frac{1}{2}\left(1-\frac{2GMr^2}{c^2R^3}\right)^{1/2} \tag{9}$$

A simple calculation shows that the leading term of $K-1$ in an expansion with respect to R_s/R (R_s: Schwarzschild radius) *vanishes for* $n=8$. For this reason it is not surprising that the numerical results for K given in Table 1 never deviate from unity by more than a few percents for the interesting values of n ($n=6, 8$). Thus we expect that the general relativistic corrections are small for a stiff equation of state.

Table 1
General relativistic correction factor K, equation (8), for various values of n and R_s/R (homogeneous model).

$n\backslash(R_s/R)$	0.05	0.1	0.2	0.3	0.4	0.5
5	0.98	0.97	0.94	0.91	0.88	0.86
6	0.99	0.98	0.96	0.94	0.94	0.95
7	1.00	0.99	0.99	0.99	1.01	1.05
8	1.00	1.00	1.02	1.03	1.08	1.17
9	1.01	1.01	1.04	1.09	1.16	1.32
10	1.01	1.02	1.07	1.12	1.25	1.48

With equation (7), the well-known expression for B, and (Friman and Maxwell, 1979)

$$\varepsilon_{\mathrm{URCA}} = (2.7\times10^{21}\ \mathrm{ergs\text{-}cm^{-3}\text{-}s^{-1}})\left(\frac{m_n^*}{m_n}\right)^3 \frac{m_p^*}{m_p}\left(\frac{\rho}{\rho_0}\right)^{2/3} T_9^8 \tag{10}$$

(ρ_0: nuclear matter density), we can easily compute the time dependence of T_i. If we take $m_p^*/m_p = 1$, $m_n^*/m_n = 0.8$ and use the relation between T_i and the surface temperature, T_s, given by Glen and Sutherland (1980), then we find the cooling curve shown in Fig. 1 (indexed by GR). This curve agrees, for the first few thousand years, completely with the one obtained in an elaborate calculation by Glen and Sutherland (1980) for a 1.25 M.-star, based on a "realistic" (stiff) equation of state (Pandharipande, Pines, and Smith, 1976).

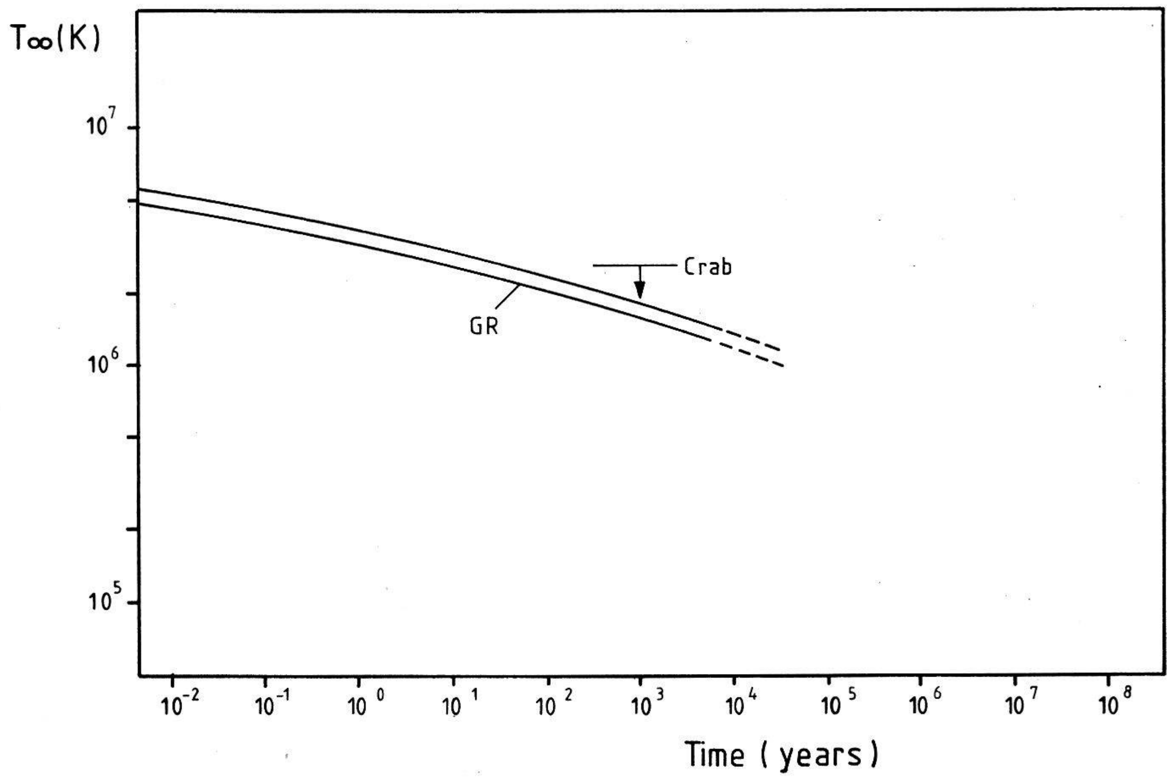

Figure 1
Observed temperature, T_∞, versus time for a homogeneous star model of normal nucleons and no pion condensate. The lower curve includes corrections due to GR. These cancel almost completely, apart from the trivial redshift of the surface temperature.

The upper curve in Fig. 1 shows the non-relativistic result. The only significant general relativistic effect is the trivial redshift of the observed temperature $T_\infty = T_s e^{\Phi_s}$ at infinity. One may expect that the general relativistic correction factor (8) is larger for more realistic neutron star models. For this reason we have computed K numerically for a representative equation of state, using computer results of Börner and Cohen (1973). For the equation of state of Bethe et al. (1970), denoted as BBS by Börner and Cohen, we have obtained the following results.

For a rather massive star with $M = 3.44 \times 10^{33}$ g and a high central density, $\log \rho_c$ $(\mathrm{g\,cm^{-3}}) = 15.5$, we obtain $K = 1.4$ for $n = 6$ and $K = 2.8$ for $n = 8$. The latter value shows that general relativistic corrections are no more very small for extreme cases (massive neutron stars with high central densities), but even then the uncertainties in the neutrino emissivities are still at least as big. In any case, K can be computed from (8) for a given neutron star model.

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