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Relation between the critical current and the order parameter in a Josephson array

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Abstract. We give a definition of the order parameter for the paracoherent–ferrocoherent phase transition in a Josephson array and we show that the amplitude of the critical current in the array is proportional to this order parameter. We then discuss some qualitative properties of a Josephson array.

1. Introduction

Systems formed by a large number of weakly coupled superconducting grains can be classified in two categories: granular superconductors, in which the grain size and the intergrain coupling vary from grain to grain, and Josephson arrays, which are regular lattices of superconducting grains. Granular superconductors have been studied quite extensively in the last ten years [1], and they show many interesting properties, but it is quite difficult, in these systems, to distinguish between effects due to disorder from possible cooperative phenomena due to the interaction of a large number of grains. On the other hand large Josephson arrays are very difficult to construct experimentally; only one- and two-dimensional systems have been constructed and experimental work is only preliminary.

Many properties of Josephson arrays promise to be quite interesting, in particular coherent behaviour in response to electromagnetic radiation (supercoherence) and phase transition properties. Ideas about a second phase transition below the intrinsic superconducting phase transition of the grain have been pioneered by J. Rosenblatt and his coworkers [2]. The intuitive picture is very simple in the limit where the grains are small compared to ξ, the coherence length. In that limit, the order parameter varies slowly over a grain size, one can attribute an order parameter $\Delta_n = |\Delta| e^{i\varphi_n}$ to each grain, and we have therefore at first sight a system analogous to a lattice of two-dimensional rods interacting through a potential

$$\mathscr{H}' = -J \sum_{n,n'} \cos(\varphi_n - \varphi_{n'})$$

Note that this would be the plane rotator model, except for the fact that there exist additional relations between currents and fields. As emphasized by Kosterlitz and Thouless [3], these distinguish a neutral superfluid from a charged superconductor, and recent theoretical progress in the two-dimensional plane rotator model cannot be applied readily to this problem.

Still one expects that at some temperature T_{cJ} smaller than T_{c0} (the transition temperature of the grains), phases will orient 'ferromagnetically' and the system will go from a 'paracoherent' to a 'ferrocoherent' state.

Descriptions of this phase transition have already been given [2, 4], and they rely on a brownian motion analysis of a single junction in the noise created by all the others. It would be satisfactory however, from the conceptual point of view, since we are dealing with a large homogeneous system, to give a description more in keeping with the standard description of phase transitions, and to define in particular a Hamiltonian and an order parameter. To understand clearly the underlying physics, we first investigate the properties of a (hypothetical) granular ferromagnet (Section 2) and we then use the strong coupling model of superconductivity to investigate Josephson arrays along the same lines (Section 3).

2. The granular ferromagnet

A granular ferromagnet is a rather academic system, consisting of N perfectly isotropic ferromagnetic grains containing each n spins, the ferromagnetic grains being coupled to each other by an exchange interaction. Both n and N are large but finite numbers.

(a) *The model*

We first write down the interaction within the grain ν as a Heisenberg exchange interaction

$$\mathscr{H}_\nu = -J\sum_{i,j} \mathbf{S}_{\nu i}\mathbf{S}_{\nu j} \tag{1}$$

The sum is over the nearest neighbours in the grain. In the following, Greek indices will refer to grains whereas Roman indices will refer to a definite position within a grain. We then couple each grain with its nearest neighbours via an exchange interaction of the form

$$\mathscr{H}_{\nu\mu} = -\frac{j}{n}\left(\sum_i \mathbf{S}_{\nu i}\right)\left(\sum_j \mathbf{S}_{\mu j}\right) \tag{2}$$

The complete Hamiltonian of the system is then

$$\mathscr{H} = -h\sum_{\nu,i} S^z_{\nu i} + \sum_\nu \mathscr{H}_\nu + \sum_{\nu\mu} \mathscr{H}_{\nu\mu} \tag{3}$$

where $h \equiv g\mu_B H_z$, and $j \ll J$.

(b) *Qualitative properties of a single grain*

Since n is large, each grain will undergo a sharp phase transition governed by the interaction J, and develop below $T_{c0} \sim J[2Z\,S(S+1)]/3k_B$ a spontaneous magnetization, which is a large magnetic moment m_0 pointing in a random direction.

It is the magnitude of this macroscopic moment which represents the order parameter of the grain; since we want this magnetic moment to be a constituent of a much larger ferromagnetic system governed by j, we must treat this moment as a dynamical variable and apply the rules of statistical mechanics to calculate its averages. In particular, the magnetization itself will be zero at zero field.

Let us cite briefly that:

(1) The susceptibility of the grain is of the order n above T_{c0}, i.e.

$$\chi \sim n\frac{C}{T - T_{c0}} \quad T \gg T_{c0} \tag{4}$$

($C = [g^2\mu_B^2 S(S+1)]/3k_B$, $T_{c0} \simeq J[2ZS(S+1)]/3k_B$, $Z =$ number of nearest neighbours).

The susceptibility will be of the order n^2 below T_{c0}, i.e.

$$\chi \sim \frac{m_0^2}{3k_B T} \quad T \ll T_{c0} \tag{5}$$

where m_0 is the order parameter of the grains which varies with T and is of the order nS.

For arbitrary fields, the magnetization m_z is of the order

$$m_z = ng\mu_B S B_s\left(\frac{g\mu_B S(h + \lambda m_z)}{kT}\right) \quad T \gg T_{c0} \tag{6}$$

and

$$m_z = M_0 B_{m0/g\mu_B}\left(\frac{m_0 H}{kT}\right) \quad T \ll T_{c0} \tag{7}$$

One notes in the last expression that for $H \gg kT/m_0$, $m_z \simeq m_0$; for a grain of 10^8 spins, f.i. $m_z \simeq m_0$ for $H > 10^{-4}$ Gauss at 1°K.

A ferromagnetic grain is thus always saturated, and it would take extremely low fields to measure the linear susceptibility (5).

(2) The transverse susceptibility for $T < T_{c0}$ is given by

$$\chi_{+-}(\mathbf{q}) = \frac{m_0}{\omega - \omega(\mathbf{q}) - h + \sum(\omega, \mathbf{q})} \tag{8}$$

for $q \neq 0$.

The point here is that the numerator of $\chi_{+-}(\mathbf{q})$ is m_0 the order parameter, not m_z (except at $\mathbf{q} = 0$). In other words this numerator does not go to zero when $h \to 0$ in a finite system, except at $\mathbf{q} = 0$. Although it is intuitively rather obvious, it goes against the usual result which one would obtain, f.i. with the equation of motion method. Since this fact is essential for our discussion of the granular superconductors, it is worth giving an explicit derivation. We do it in the simplest possible manner: starting from a block Hamiltonian [5] for the system, we show that for the limit ($\omega \to 0$, $\mathbf{q} \to 0$), $\chi_{+-} \sim m_0/h$.

The block Hamiltonian is written in the usual manner

$$\frac{\mathscr{H}(\sigma)}{T} = a_0 V + \sum \boldsymbol{\sigma}_\mathbf{k}\cdot\boldsymbol{\sigma}_\mathbf{k}\{a_2 + ck^2\} + V^{-1}\sum_{\mathbf{k},\mathbf{k}',\mathbf{k}''} a_4(\boldsymbol{\sigma}_\mathbf{k}\cdot\boldsymbol{\sigma}_{\mathbf{k}'})(\boldsymbol{\sigma}_{\mathbf{k}''}\boldsymbol{\sigma}_{-\mathbf{k}-\mathbf{k}'-\mathbf{k}''}) - V^{1/2}\boldsymbol{\sigma}_0\cdot\mathbf{h} \tag{9}$$

where $a_2 = a_2'(T - T_c)$; a_0, a_2', a_4 and c are parameters, V is the volume of the system (the grain) and

$$\boldsymbol{\sigma}_{\mathbf{k}} = V^{-1/2} \sum_i e^{-i\mathbf{k}\cdot\mathbf{R}_i} \mathbf{S}_i. \tag{10}$$

We then calculate $\langle|\sigma_{x\mathbf{k}}|^2\rangle$ in the Gaussian approximation [5], with the only difference that we do not fix the direction of $\boldsymbol{\sigma}_0$, but keep fluctuations in the angle ϑ between $\boldsymbol{\sigma}_0$ and $\mathbf{h}$. This means that in averages of the form

$$\langle f(\sigma_{i\mathbf{k}}, \sigma_{j\mathbf{k}}, \ldots)\rangle = \frac{\int \prod d\sigma_{i\mathbf{k}}\, e^{-\mathscr{H}(\sigma)/T} f(\sigma)}{\int \prod d\sigma_{i\mathbf{k}}\, e^{-\mathscr{H}(\sigma)/T}}$$

$$= \frac{\int d\sigma_0 \sigma_0^2 \sin\vartheta\, d\vartheta \prod_{k\neq 0} d\sigma_{ik}\, e^{-\mathscr{H}(\sigma)/T} f(\sigma)}{\int \prod d\sigma_{ik}\, e^{-\mathscr{H}(\sigma)/T}} \tag{11}$$

we want to make a rotationally invariant gaussian approximation. That is, we must first do the integral over ϑ exactly, then look for the minimum of the remaining probability distribution and expand around this minimum. This minimum fixes the most probable value of σ_0^2, i.e. m_0^2, the length of the effective moment. The calculation is straightforward and leads to

$$\lim_{\mathbf{k}\to 0} \langle|\sigma_{x\mathbf{k}}|^2\rangle \div \frac{m_0}{h} \tag{12}$$

in the gaussian approximation, which is the desired result.

(c) *Qualitative properties of a granular ferromagnet*

We can now infer, from the discussion above, the basic properties of a granular ferromagnet. We must now distinguish two transition temperatures

$$\left.\begin{aligned} T_{c0} &\sim \frac{2JZS(S+1)}{3k_B} \\ T_{cJ} &\sim \frac{2jZ'S(S+1)}{3k_B} \end{aligned}\right\} \tag{13}$$

The proper description of these transitions is that at T_{c0}, the system develops N effective moments, each of magnitude $m_0 \sim nS$, whereas at T_{cJ}, the system develops one effective moment of magnitude $M_0 \sim nNS$.

One sees here the relevance of a rotationally invariant formulation: if one fixes the magnetization vector in the z direction, then it will be of the order NsS below T_{c0} already, and nothing special will happen at T_{cJ}, except perhaps a small renormalization.

Again, one sees that

(1) The susceptibility will be of order nN above T_{c0}

$$\chi \simeq nN \frac{C}{T - T_{c0}} \quad T \gg T_{c0} \tag{14}$$

of the order Nn^2 for $T_{c_J} < T < T_{c0}$

$$\chi \simeq N \frac{m_0^2}{3k_B(T - T_{c_J})} \tag{15}$$

and of the order N^2n^2 for $T < T_{c_J}$

$$\chi \simeq \frac{M_0^2}{3k_B T} \tag{16}$$

where M_0 is the order parameter for the whole system, i.e. $M_0 \sim nNS$.

(2) For the transverse susceptibility, we have to distinguish between different *wavelengths*. *If* a is the spatial extension of a grain, we will have for $T_{c_J} < T < T_{c0}$

$$\chi_{+-}(\mathbf{q}) \simeq \frac{Nm_0}{\omega - \omega(\mathbf{q}) - h + \sum(\omega, \mathbf{q})} \tag{17}$$

for $q > 1/a$; and

$$\chi_{+-}(\mathbf{q}) \simeq \frac{Nm_z}{\omega - \omega(\mathbf{q}) - h + \sum(\omega, \mathbf{q})} \tag{18}$$

for $q < 1/a$, so that $\chi_{+-}(\omega, \mathbf{q}) = 0$ for $h = 0$ and $q < 1/a$. (Note also that $\omega(q) \sim Jq^2$ for $q > 1/a$, but $\omega(q) \sim jq^2$ for $q < 1/a$).

For $T < T_{c_J}$, one has

$$\chi_{+-}(\omega, \mathbf{q}) \simeq \frac{M_0}{\omega - \omega(\mathbf{q}) - h + \sum(\omega, \mathbf{q})} \tag{19}$$

for all $q \neq 0$.

So, if one considers the value of $\chi_{+-}(\omega, \mathbf{q})$ for $h = 0$ and $q < 1/a$, one sees that it goes from zero at $T > T_{c_J}$ to a finite value at $T < T_{c_J}$. The analogous property for a Josephson array will be important, but before turning to this system, we briefly comment on the specific heat.

The ratio of the specific heat discontinuity at T_{c0} and T_{c_J}

$$\frac{(\Delta C)_0}{(\Delta C)_J} \simeq n \frac{T_{c0}}{T_{c_J}} \tag{20}$$

because

$$(\Delta C)_0 \simeq \tfrac{3}{2} Nnk_B T_{c0}$$

$$(\Delta C)_J \simeq \tfrac{3}{2} Nk_B T_{c_J}$$

Similarly, the ratio of the specific heat divergent terms can be estimated at

$$\frac{C_0}{C_J} \simeq n^4 \frac{T_{c0}^2}{T_{c_J}^2} \left(\frac{j}{J}\right)^{3/2} \tag{21}$$

for equivalent temperatures: $|T_{c_J} - T| = |T_{c0} - T|$. This follows from the expression

$$C \simeq T_c^2 \left(\frac{a_c'}{c}\right)^{3/2} (T - T_c)^{-1/2}$$

valid in the gaussian approximation for $T > T_c$, and the estimates $c \sim J/n$ at T_{c0} and $c \sim j/N$ at T_{c_J}.

So one sees that in a granular ferromagnet, the specific heat anomalies at T_{c_J} are very small compared to the same anomalies at T_{c0}.

3. The Josephson array

A Josephson array can be represented by a Hamiltonian very similar to (3), and most of our qualitative conclusions about granular ferromagnets can thus be taken over simply for Josephson arrays.

(a) *The model*

The BCS Hamiltonian can be written as [6, 7]

$$\mathscr{H} = -2\sum \varepsilon_k \sigma_{\mathbf{k}}^z - V \sum_{\mathbf{k}\mathbf{k}'} \sigma_{\mathbf{k}}^- \sigma_{\mathbf{k}'}^+ \tag{22}$$

if one excludes simply occupied k states. The sum over k's runs over the n states contained in a shell of width $\hbar\omega_{\mathrm{D}}$ (the Debye frequency) around the Fermi surface, and $\sigma_{\mathbf{k}}^x, \ldots$ are Pauli matrices. If one makes the approximation $\varepsilon_k \sim \varepsilon_f$ one gets the so-called strong coupling model of superconductivity:

$$\mathscr{H} = -2\varepsilon_f \sum \sigma_{\mathbf{k}}^z - V \sum \sigma_{\mathbf{k}}^- \sigma_{\mathbf{k}'}^+ \tag{23}$$

In this model the superconducting phase transition has a very simple interpretation: at some temperature T_{c0}, the system develops a macroscopic magnetic moment in the x–y plane, whose size is proportional to the gap, and whose direction is random. Indeed the ground state of Hamiltonian (23) is the state with maximum S and $S_z = 0$ (if one chooses properly ε_f), whereas the BCS ground state is not the true ground state but a coherent superposition of all states with maximum S and all values of S_z.

The Hamiltonian (22) is readily extended to a Josephson array by writing

$$\mathscr{H} = -2\sum_{\nu\mathbf{k}} \varepsilon_f \sigma_{\nu\mathbf{k}}^z - V \sum_{\nu\mathbf{k}\mathbf{k}'} \sigma_{\nu\mathbf{k}}^- \sigma_{\nu\mathbf{k}'}^+ - v \sum_{\nu\nu'} \left\{ \sum_{\mathbf{k}\mathbf{k}'} \sigma_{\nu\mathbf{k}}^+ \sigma_{\nu'\mathbf{k}}^- + h.c. \right\} \tag{23}$$

Since $\sigma_{\nu\mathbf{k}}^+(\sigma_{\nu\mathbf{k}})$ destroys (creates) a Cooper pair in the grain ν, the last term is a two-electron tunnelling Hamiltonian. The sum runs over all (ν, ν') nearest neighbours.

Formally and physically (24) is very similar to (3), and the transition at $T_{c_J} < T_{c0}$ is marked by the formation of a large macroscopic magnetic moment in the x–y plane. This moment is not directly observable however, and we now analyse the behaviour of the resistivity at T_{c_J}.

(b) *The Josephson conductivity*

To study the resistivity we consider the response of the system to an electromagnetic field. We first define a Fourier transform for the granular system in the following way. If $\mathbf{R}_\nu$ denotes the coordinates of the νth grain (f.i. the centre of the grain), we define

$$f(\mathbf{k}) = N^{-1/2} \sum_{\nu} e^{+i\mathbf{k}\cdot\mathbf{R}_\nu} f(\mathbf{R}_\nu)$$

$$f(\mathbf{R}_\nu) = N^{-1/2} \sum_{\mathbf{k}} e^{-i\mathbf{k}\mathbf{R}_\nu} f(\mathbf{k}) \tag{24}$$

Then the interaction with an electromagnetic field can be defined as

$$\mathscr{H}' = -\sum_{\nu\nu'} \{p_\nu S_\nu^+ S_{\nu'}^- + p_\nu^* S_\nu^- S'\} \tag{25}$$

where the sum over ν' is over the nearest neighbours of $\mathbf{R}_\nu$ and we assume that e_ν varies slowly with ν. Or in Fourier transform

$$\mathscr{H}' = N^{-1/2} \sum_{\mathbf{k}\mathbf{k}''} S_{\mathbf{k}}^+ S_{-\mathbf{k}-\mathbf{k}''}^- p_{\mathbf{k}''} g(\mathbf{k}'') + \text{h.c.} \tag{26}$$

where

$$g(\mathbf{k}) = \sum_{\mathbf{R}_\delta} e^{-i\mathbf{k}\mathbf{R}_\delta}$$

$\mathbf{R}_\delta$ being the vectors leading from a given site to its nearest neighbours. We now consider the variation of the charge on the νth grain: $\dot{S}_\nu^z$. If the current varies slowly over a distance a (the intergrain spacing), one has

$$\dot{S}_{\mathbf{k}}^z = -ia^3 \mathbf{k}\cdot\mathbf{j}_{\mathbf{k}} \tag{27}$$

where $\mathbf{j}_{\mathbf{k}}$ is the Fourier transform of the current. So, for simplicity, we just consider the response of $\dot{S}_{\mathbf{k}}^z$ to the external field. Since

$$\begin{aligned} \dot{S}_\nu^z &= i[\mathscr{H}, S_\nu^+] \\ &= -i\nu \sum_{\nu'} (S_\nu^+ S_{\nu'}^- - \text{h.c.}) \end{aligned} \tag{28}$$

the amplitude of supercurrent will be proportional to the amplitude of response functions of the type

$$\chi_{+-}(\mathbf{k}, t) = -i\vartheta(t) \sum_{\mathbf{k}'\mathbf{k}''} g(\mathbf{k}-\mathbf{k}') g(\mathbf{k}+\mathbf{k}'') \langle [S_{\mathbf{k}'}^+ S_{\mathbf{k}'-\mathbf{k}'}^- \cdot S_{\mathbf{k}''}^+ S_{\mathbf{k}+\mathbf{k}''}^-] \rangle \tag{29}$$

This response function is very similar to the transverse susceptibility.

One can apply standard many body techniques to evaluate it [8] and one gets expressions of the form

$$\chi_{+-}(\mathbf{k}\omega) = \frac{\alpha M_1}{\omega - \omega(\mathbf{k}) + \sum(\omega\mathbf{k})} \tag{30}$$

and we have taken into account, to calculate the numerator, of arguments similar to the one developed for granular ferromagnets: M_1 is here the amplitude of the order parameter of the whole system, and goes to zero at T_{cJ}, and α is a constant.

To understand the meaning of equation (30) we consider a single junction in an electromagnetic field. The supercurrent flowing through the junction will be

$$J_s = J_0 \sin\left\{\frac{2e}{\hbar\omega} p \sin(\omega t)\right\}$$

where p is the amplitude of the electromagnetic field. In the linear approximation

$$\left.\begin{aligned} J_s &= J_0 p \frac{2e}{\hbar\omega} \sin(\omega t) \\ &\equiv \chi(\omega) p \sin(\omega t) \end{aligned}\right\} \tag{31}$$

using the definition $\chi(\omega)$ corresponding to (30). We see therefore that indeed

$$\chi(\omega) = \frac{2eJ_0}{\hbar\omega} \tag{32}$$

and that the amplitude of $\chi(\omega)$ is proportional to J_0.

If one applies for a single junction problem the formalism used here, one finds the effects of fluctuations described by Boltzman factors, so that the amplitude of J_s goes to zero for

$$kT \gg vn^2$$

This will be discussed in detail elsewhere [8].

(c) *Qualitative properties of a Josephson array*

From the discussion above, we can now summarize some properties of a Josephson array.

(i) If the condition

$$Z'vn^2 < k_B T_{c0}$$

is satisfied, i.e. if, for the constitutive junctions of the array

$$\hbar J_0 = 2evn^2 < 2ek_B T_{c0}/Z' \tag{33}$$

there will appear a second phase transition at a temperature $T_{cJ} < T_{c0}$.

To evaluate condition (33), we set

$$J_0 \simeq \frac{\pi}{2} \frac{\Delta}{eR}$$

so that one must have

$$R \geqslant \frac{\hbar Z'}{(2e)^2} \frac{\pi\Delta}{k_B T_{c0}} \simeq Z' \, 10^4 \text{ Ohms.}$$

Meaningful arrays have to be constructed with very weak links.

(ii) Below T_{cJ}, the critical Josephson current will rise from zero to a finite value following an order parameter law, i.e., in the molecular field approximation

$$I_c = I_{c0}\left(1 - \frac{T}{T_{cJ}}\right)^{1/2} \tag{34}$$

This law is compatible with known experimental results [9]. Above T_{c_J}, I_c will be zero, i.e. the array will be resistive.

(iii) The specific heat anomalies at T_{c_J} will be smaller than the corresponding anomalies at T_{c0} by a factor $1/n$ at least (equations (20) and (21)).

(iv) Below T_{c_J}, the array will behave essentially as a superconductor, except for possible supercoherence properties, which we will not discuss here. Thus one has to determine the parameters in the Ginzburg–Landau theory.

$$f = f_{n0} + \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \gamma\left|\left(\frac{\hbar}{i}\nabla - \frac{e^*}{c}\mathbf{A}\right)\psi\right|^2 \tag{35}$$

We will now focus on the situation where

$$\xi_{GL} \ll a \ll \lambda$$

where ξ_{GL} is the intrinsic Ginzburg–Landau correlation length of the material of the grain, a is the diameter of the grains and λ is the penetration depth. In that case the amplitude $|\psi|^2$ is practically constant throughout the system, only the phase varies due to currents and fields and the corresponding term can be written

$$\gamma\left(\frac{\hbar}{i}\nabla\varphi - \frac{e^*}{c}\mathbf{A}\right)^2|\psi|^2 \tag{36}$$

and this gives directly

$$\lambda_{\text{eff}}^2 = \frac{c^2}{8\pi|\psi|^2 e^{*2}\gamma}. \tag{37}$$

Let us analyse how the phase varies on a large scale compared to the grain size if we have a current flowing in the system. Within the grains, the current density is given by

$$\mathbf{j} = \frac{e^*}{m^*}|\psi|^2\left(\hbar\nabla\varphi - \frac{e^*}{c}\mathbf{A}\right)$$

whereas at each boundary

$$\mathbf{j} \simeq J_0\left(\Delta\varphi - \frac{e^*}{\hbar c}\int \mathbf{A}\,d\mathbf{l}\right)$$

This means that if $J_0 \ll (e^*/m^*)|\psi|^2$ so that $J_0 \ll e^*/m^*|\psi|^2\hbar$ the phase varies much faster between the grains than inside, and if we look at a large scale compared to the grain size

$$\mathbf{j} = J_0 a\left(\nabla\varphi - \frac{e^*}{\hbar c}\mathbf{A}\right)$$

So the parameter

$$\gamma = \frac{J_0 a}{2e^*\hbar|\psi|^2}$$

in equations (35) and (37) and therefore

$$\lambda_{\text{eff}}^2 = \frac{\hbar c^2}{4\pi e^* J_0 a}$$

If J_0 is very small, λ_{eff} becomes very large.

Large penetration depths are indeed observed in granular superconductors [10].

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